Dynamics and freeze-out of fluctuations near the QCD critical point

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Maneesha Pradeep[1*], Krishna Rajagopal[2], Misha Stephanov[1], Yi Yin[3]

1 University of Illinois at Chicago, 2 Massachusetts Institute of Technology, 3 Institute of Modern Physics, Lanzhou

*mprade2@uic.edu, Speaker

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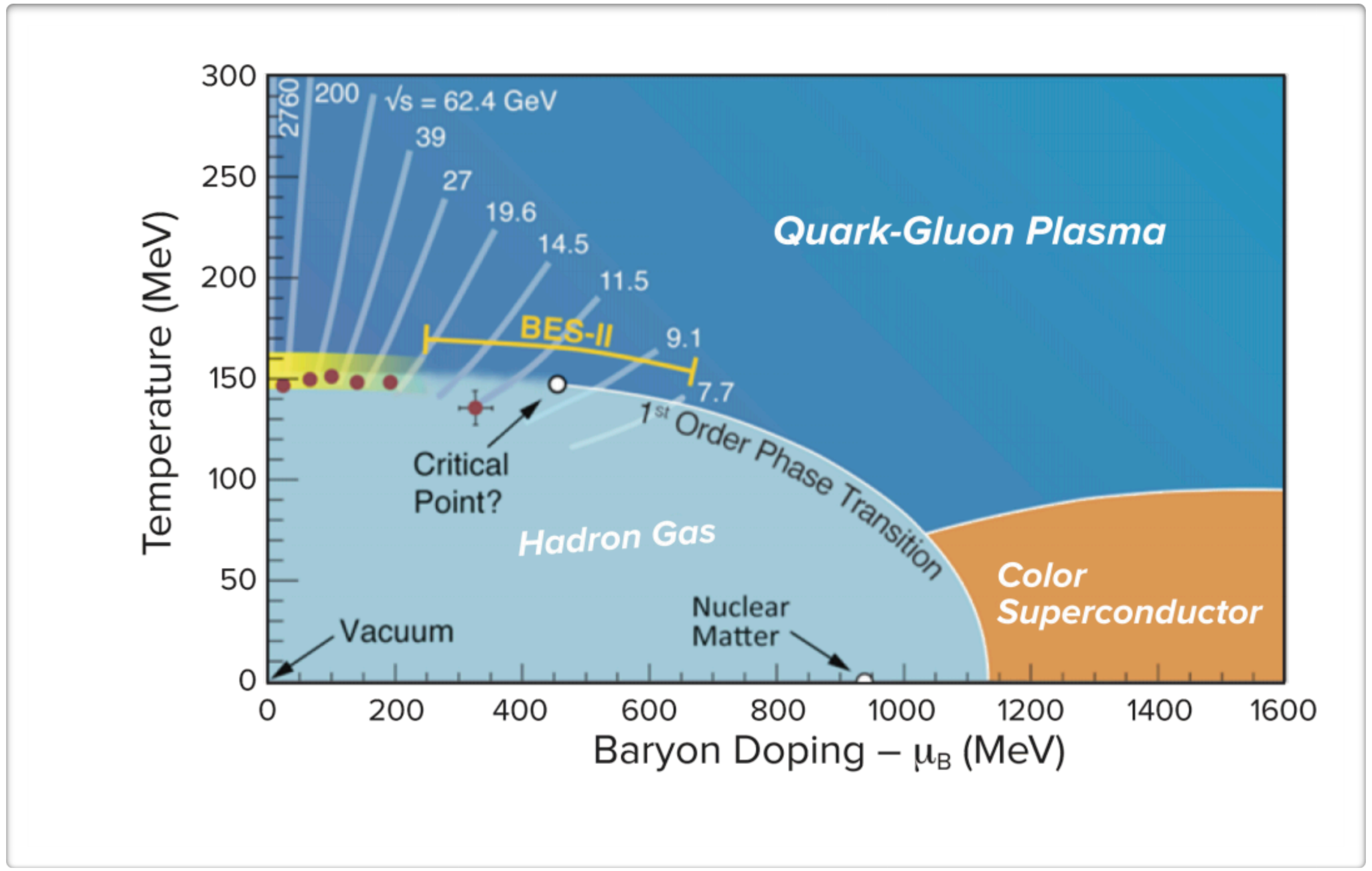
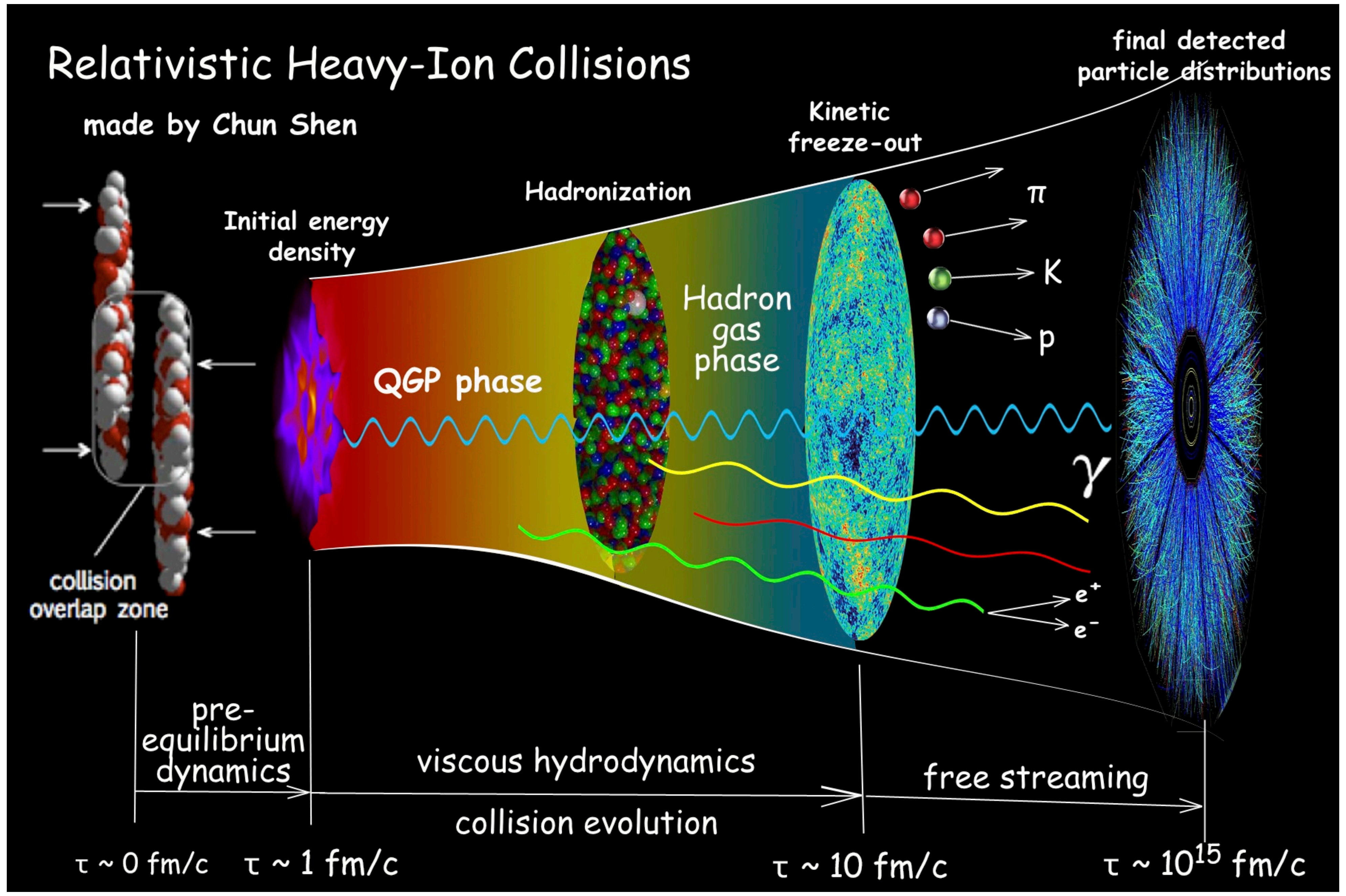
Heavy-ion collisions as laboratory to discover the QCD critical point

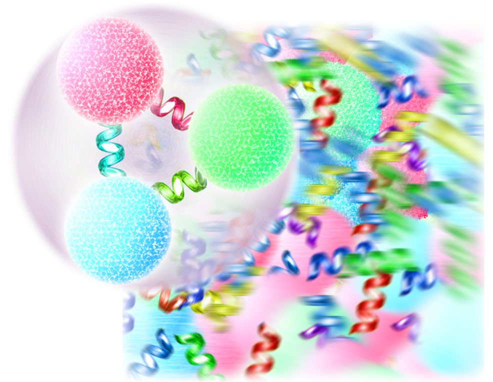




In this talk we'll focus on the stage of hydrodynamic evolution and the eventual freeze-out into a gas of hadrons

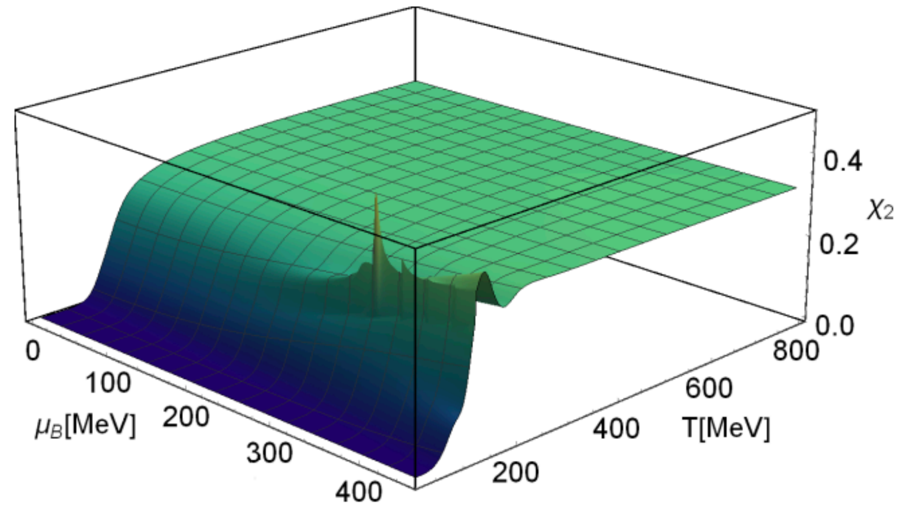
Overview

STAR Collaboration, 21



Hydrodynamic fluctuations of QGP

Parotto et al., 18



$$\langle \delta N^k \rangle = \left\langle \left(N - \langle N \rangle\right)^k \right\rangle_c$$

Stephanov.,08

$$\langle \delta N^2 \rangle \sim \xi^2 \,,\ \langle \delta N^3 \rangle \sim \xi^{4.5} \,,\quad \langle \delta N^4 \rangle_c \sim \xi^7$$

Conservation laws

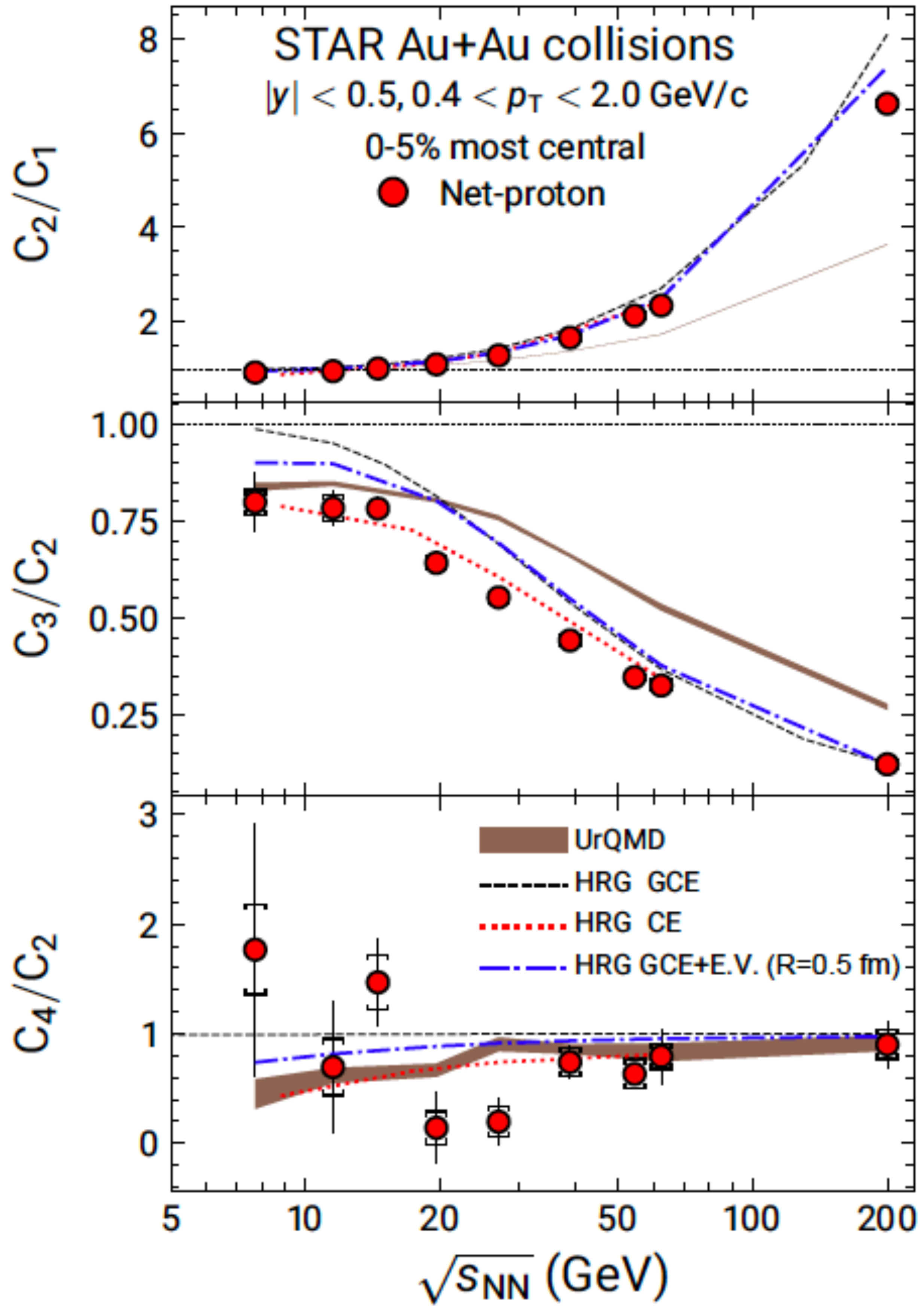
Finite time

Critical slowing down

Fluctuations as observables for the QCD critical point

Non-monotonic deviation from baseline is suggestive of the presence of a critical point!

This talk : Freeze-out of Gaussian fluctuations



Cumulants of particle multiplicities

Dynamics of fluctuations near the critical point

There are both stochastic and deterministic approaches to describing critical fluctuations.

Stochastic approach

$$\partial_t \breve{\psi} = -\nabla \cdot \left(\textbf{flux}\,[\breve{\psi}] + \textbf{noise} \right) \quad \textbf{(conservation)}$$

$$\langle \textbf{noise}(x)\textbf{noise}(y) \rangle \sim \delta^{(4)}(x-y) \quad \textbf{FDT}$$

- \+ Only one equation
- \- Noise gets larger for smaller lattice spacing

Bluhm et al., 2020

Deterministic approach

$$\partial_t \psi = -\nabla \cdot \textbf{flux}\,[\psi, G],$$

$$\partial_t G = \textbf{relaxation}\left[G - G^{\textbf{eq}}; \psi\right]$$

- \+ Deterministic equations
- \- Multiple equations to solve

Rajagopal et al, 19, Du et al. 20

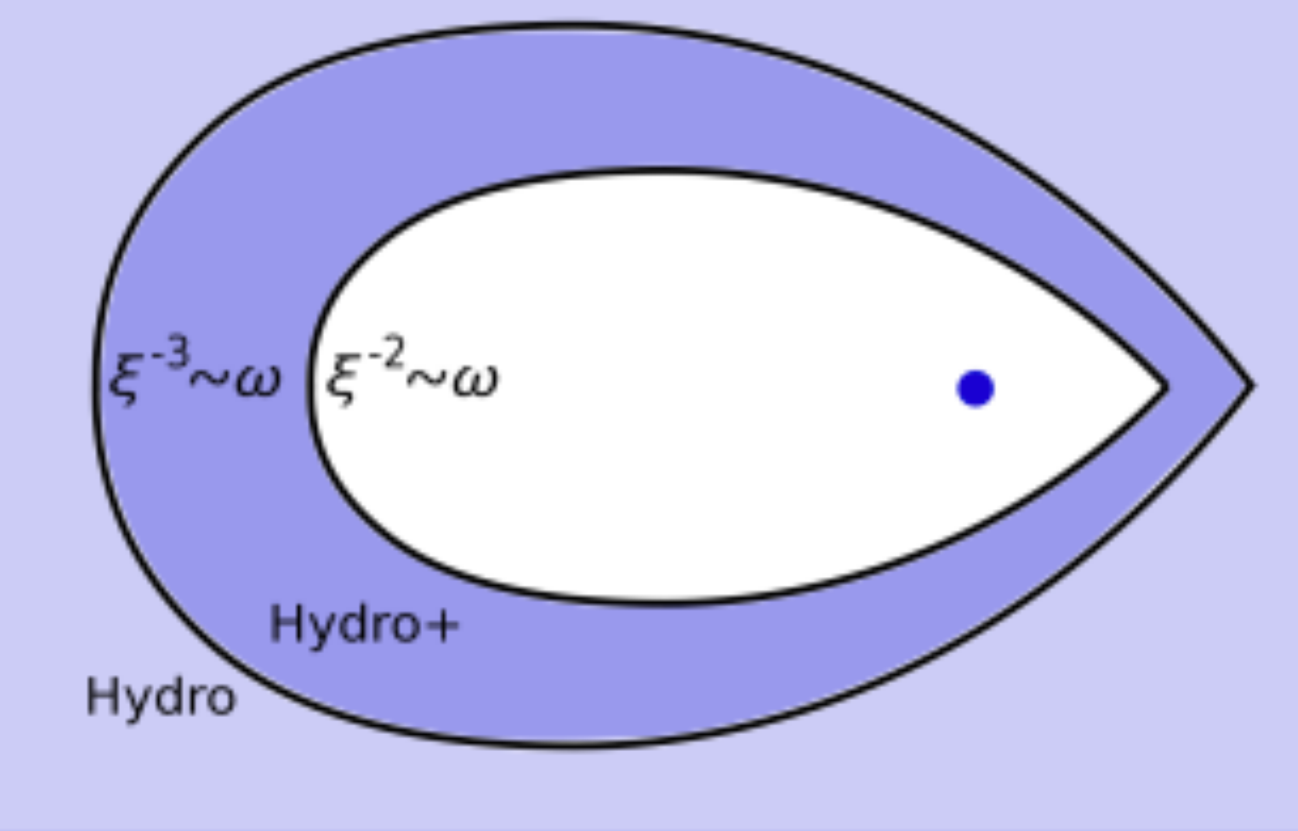
We use Hydro+ framework. We'll demonstrate the freeze-out in one of the available Hydro+ simulation.

Hydro+

Stephanov and Yin, 17

Hydro+ is a deterministic approach to studying the dynamics of fluctuations.

Taken from Xin An's talk, QM 2019



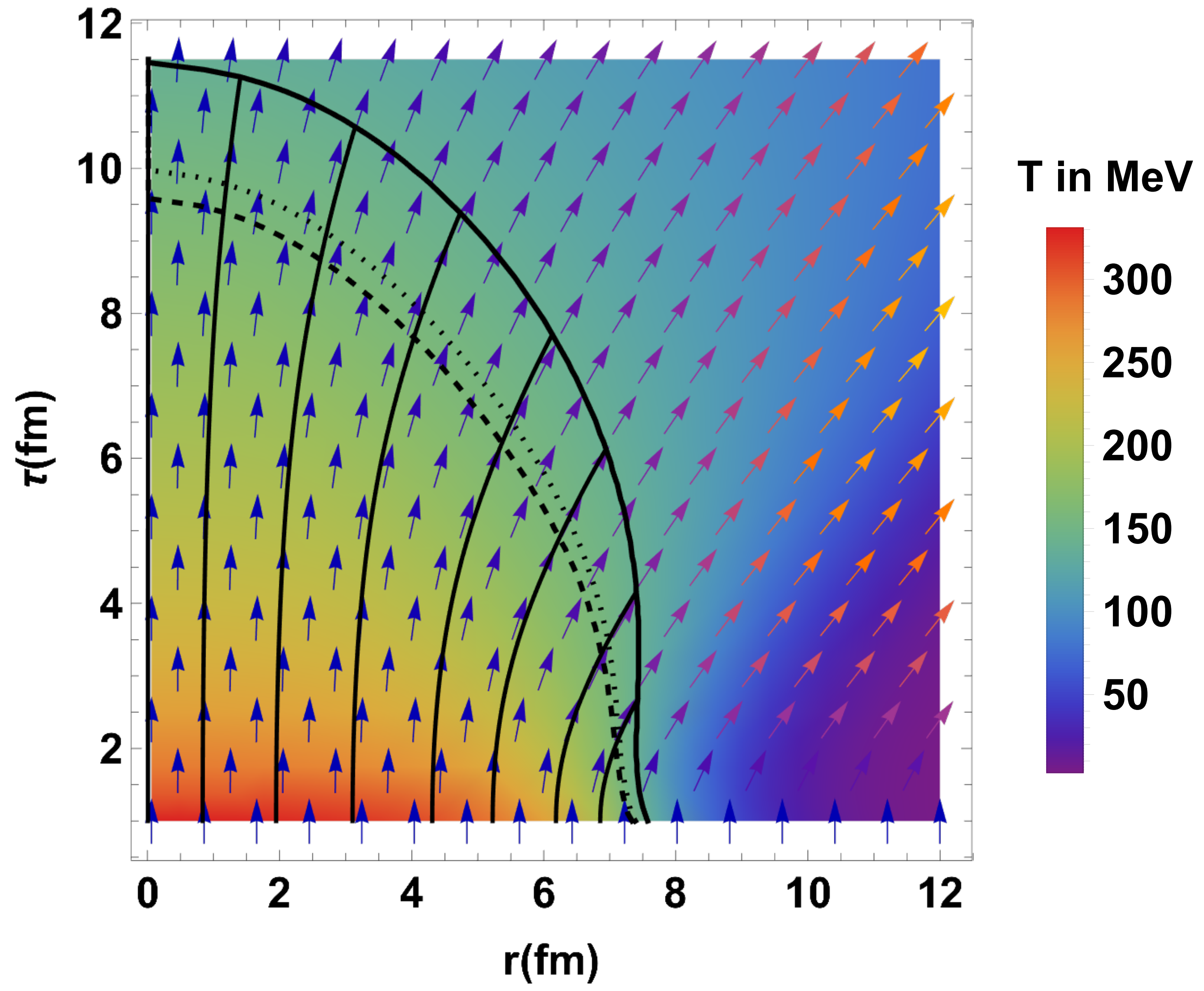
Hydro breaks down when relaxation rate of the slowest non-hydro mode becomes comparable to the expansion rate

* The fluctuations of $\hat{s} \equiv s/n$ which relaxes parametrically as $\Gamma \sim \xi^{-3}$ is the slowest non-hydrodynamic mode
* Dynamics governed by hydrodynamics + relaxation equations for the two point correlations of $\hat{s}$

Hydro+ simulation

* Hydrodynamics + relaxation equation for the slowest non-hydrodynamic mode

Stephanov & Yin, 2017



Baier and Romatschke, 2007

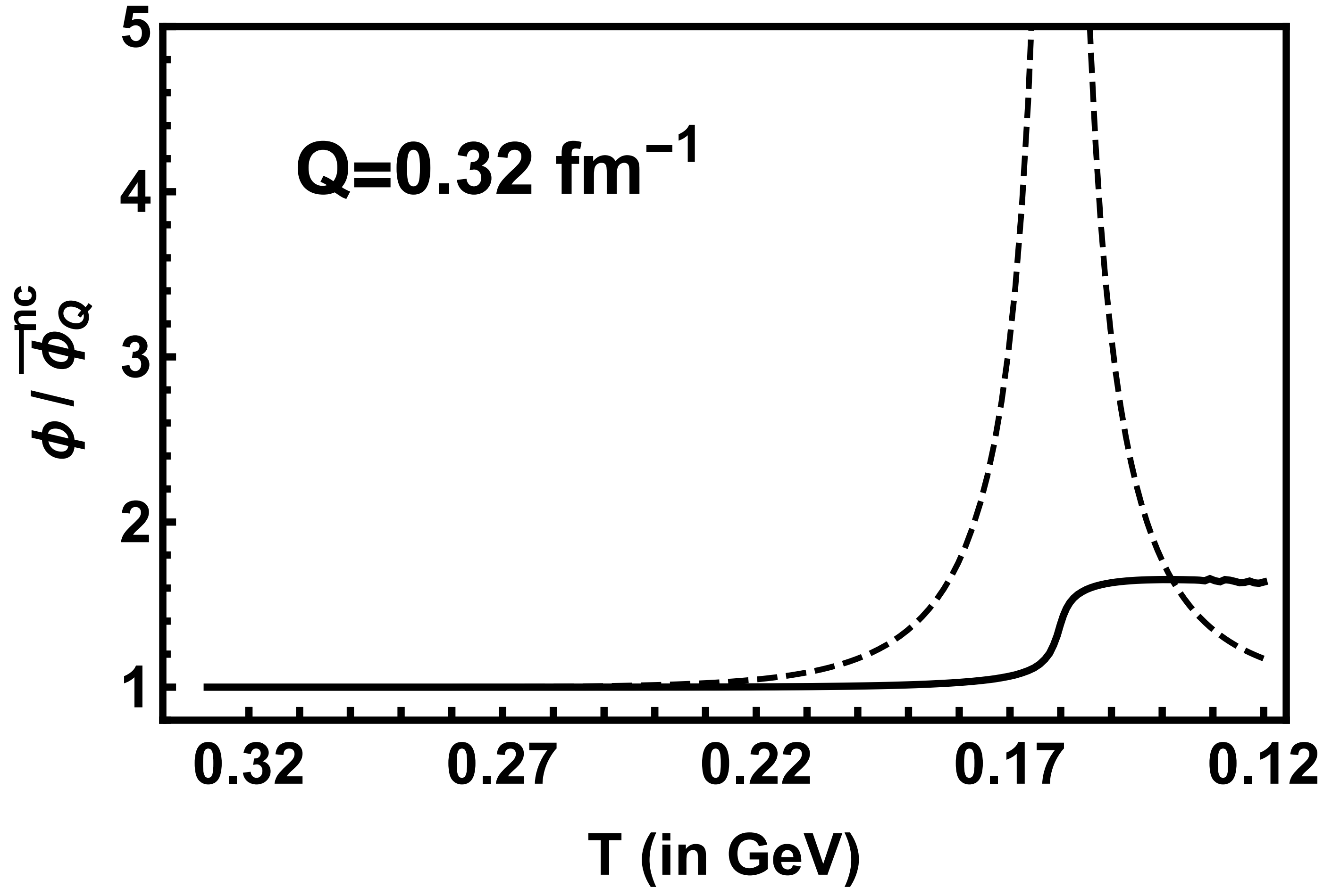
Back reaction of out-of-equilibrium fluctuations on the EoS neglected as they have been found to be less than sub-percent level in Rajagopal et al, 19, Du et al, 20

This talk :

Azimuthally symmetric, boost invariant hydrodynamic background with radial expansion with fluctuations discussed in Rajagopal, Ridgway, Weller, Yin, 19

Evolution of fluctuations

Stephanov & Yin, 2017



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$$\phi_{\mathbf{Q}} = \int_{\Delta\mathbf{x}} e^{-i\,\mathbf{Q}\cdot\Delta\mathbf{x}} \left\langle \delta\hat{s}(x_+)\,\delta\hat{s}(x_-)\right\rangle$$

Zero mode doesn't evolve

- The slowest and the most singular mode near the critical point corresponds to fluctuations of $\hat{s} \equiv \dfrac{s}{n}$
- The relaxation rate $\Gamma \sim \xi^{-3}$
- Equilibrium fluctuations $\propto C_p \sim \xi^2$

$$u \cdot \partial\phi_{\mathbf{Q}} = -\Gamma\left(\mathbf{Q}\right)\left(\phi_{\mathbf{Q}} - \bar{\phi}_{\mathbf{Q}}\right)$$

$$\Gamma(\mathbf{Q}) = \frac{2D_0\xi_0}{\xi^3}K(|\mathbf{Q}\xi|),\, K(x) \sim x^2 \text{ for } x << 1$$

Demonstrating critical slowing down

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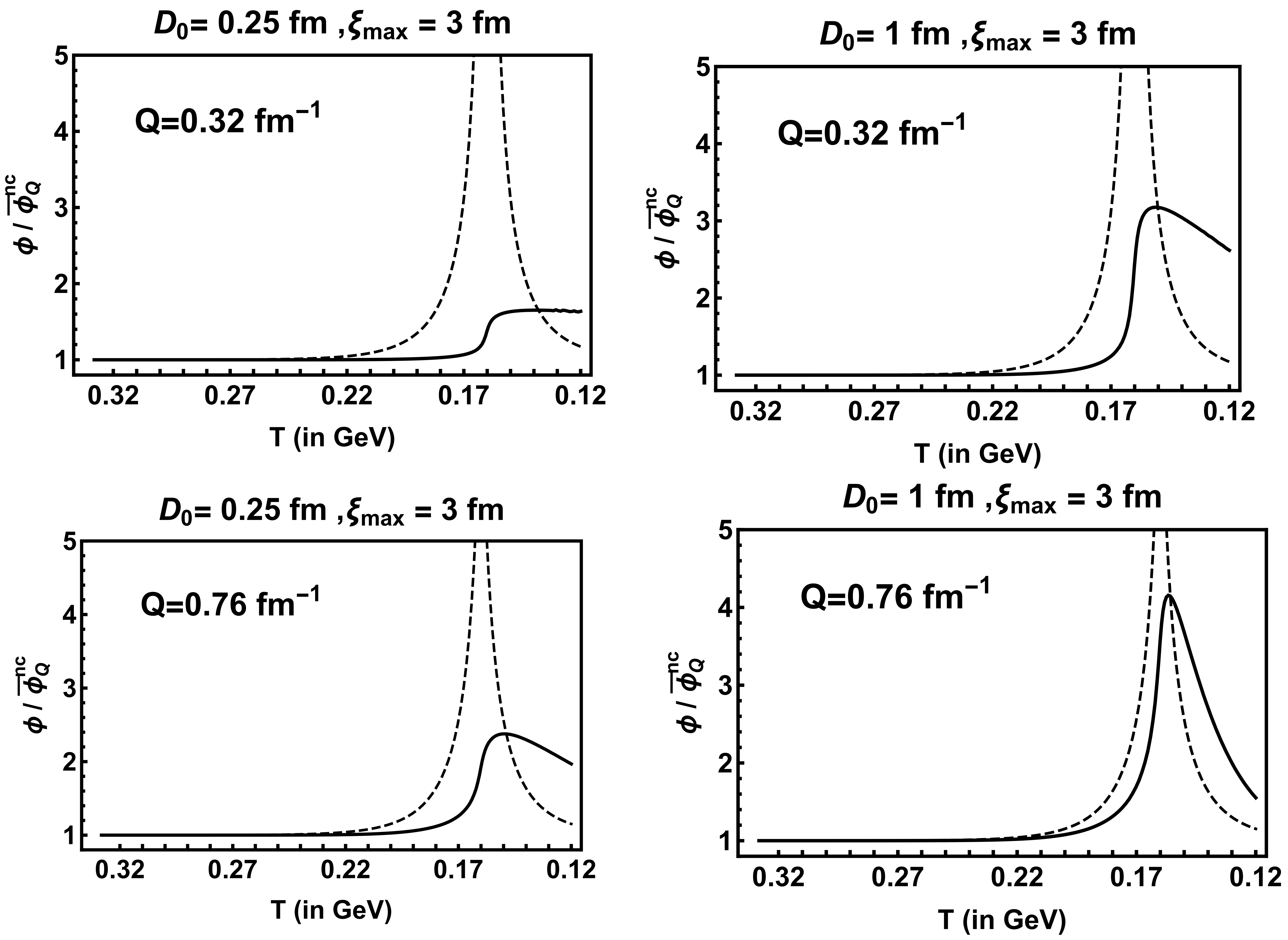
Lower Q modes are suppressed strongly due to conservation and relax more slowly

$$\bar{\phi}^{nc}_{\mathbf{Q}} \sim \frac{\xi_0^2}{1+(\mathbf{Q}\xi_0)^2}$$

Normalized out-of-equilibrium fluctuations for two Q modes and two relaxation rates



Critical correlations in space

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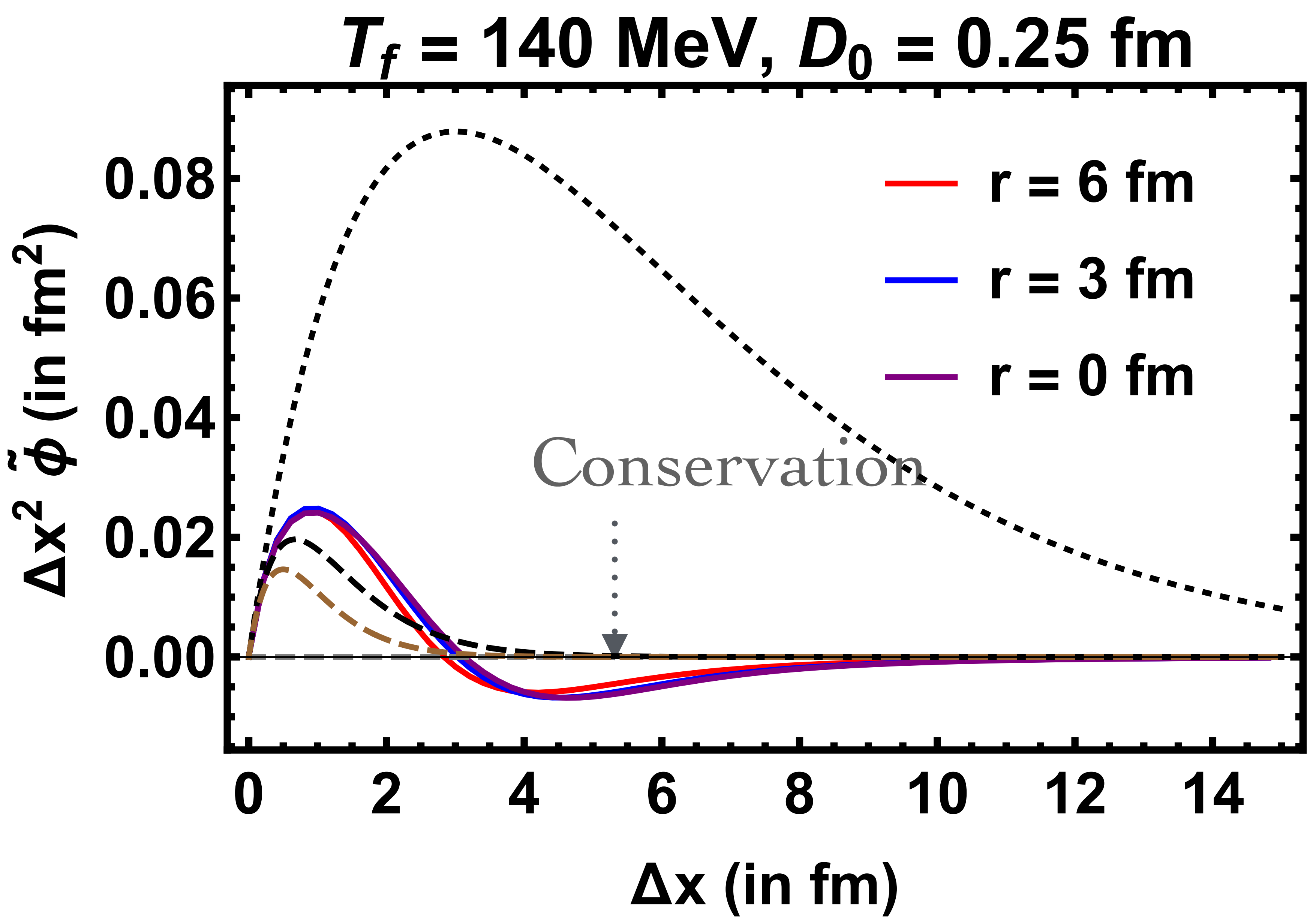
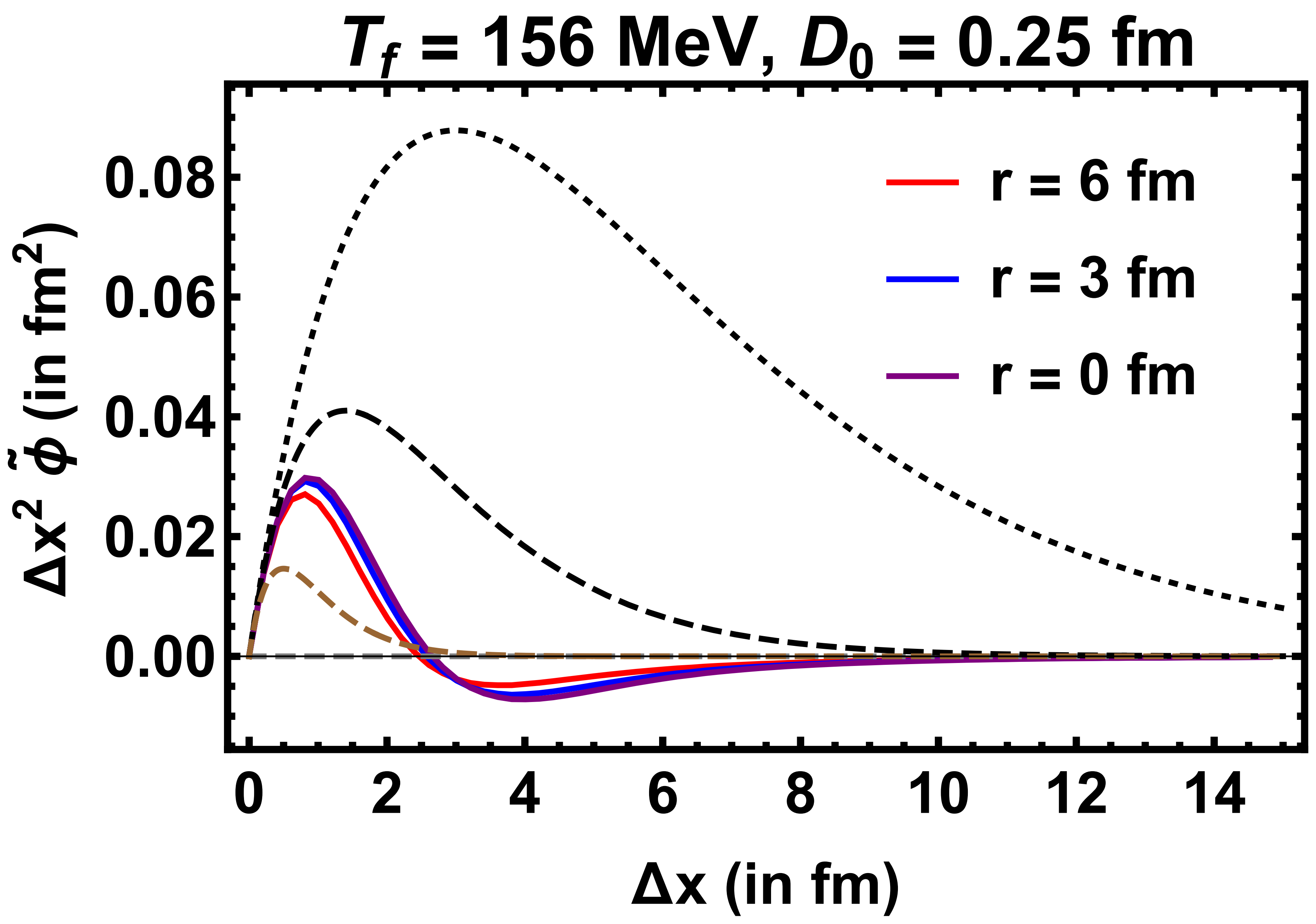
We consider two isothermal freeze-out scenarios: T=140 MeV and T=156 MeV





Memory

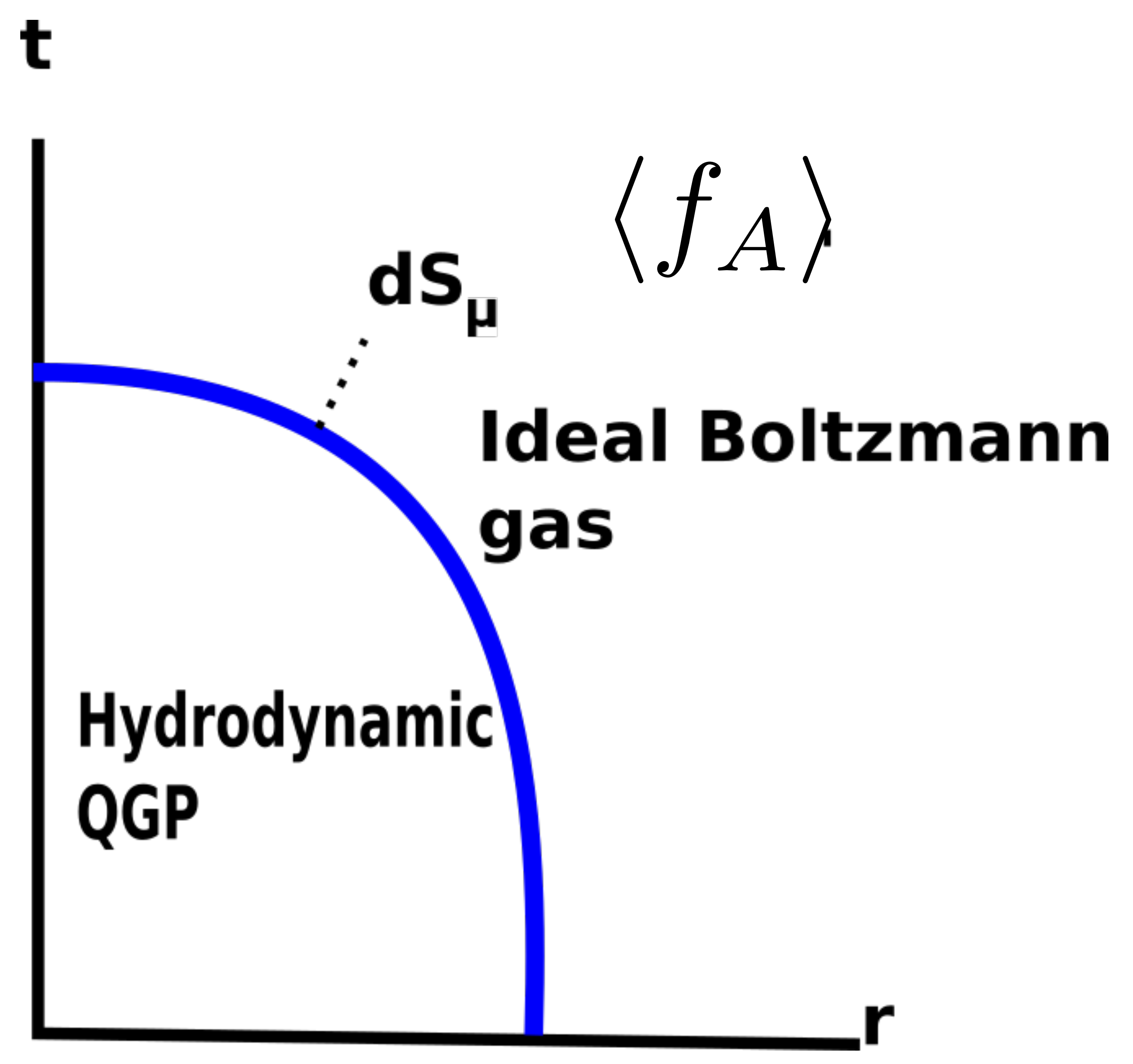
Out-of equilibrium fluctuations "remember" their past, so the difference between the two freeze-out scenarios is not too large

Conservation

$$\int d\Delta x\, \Delta x^2\, \tilde{\phi}(\Delta x) = \phi_0$$

Zero mode doesn't evolve

Traditional Cooper-Frye freeze-out procedure



$$\langle N_A \rangle = \int dS_\mu \int Dp\, p^\mu \, \langle f_A(x,p) \rangle$$

Matches the averages of conserved densities before (hydrodynamic) and after (hadron resonance gas) freeze_out

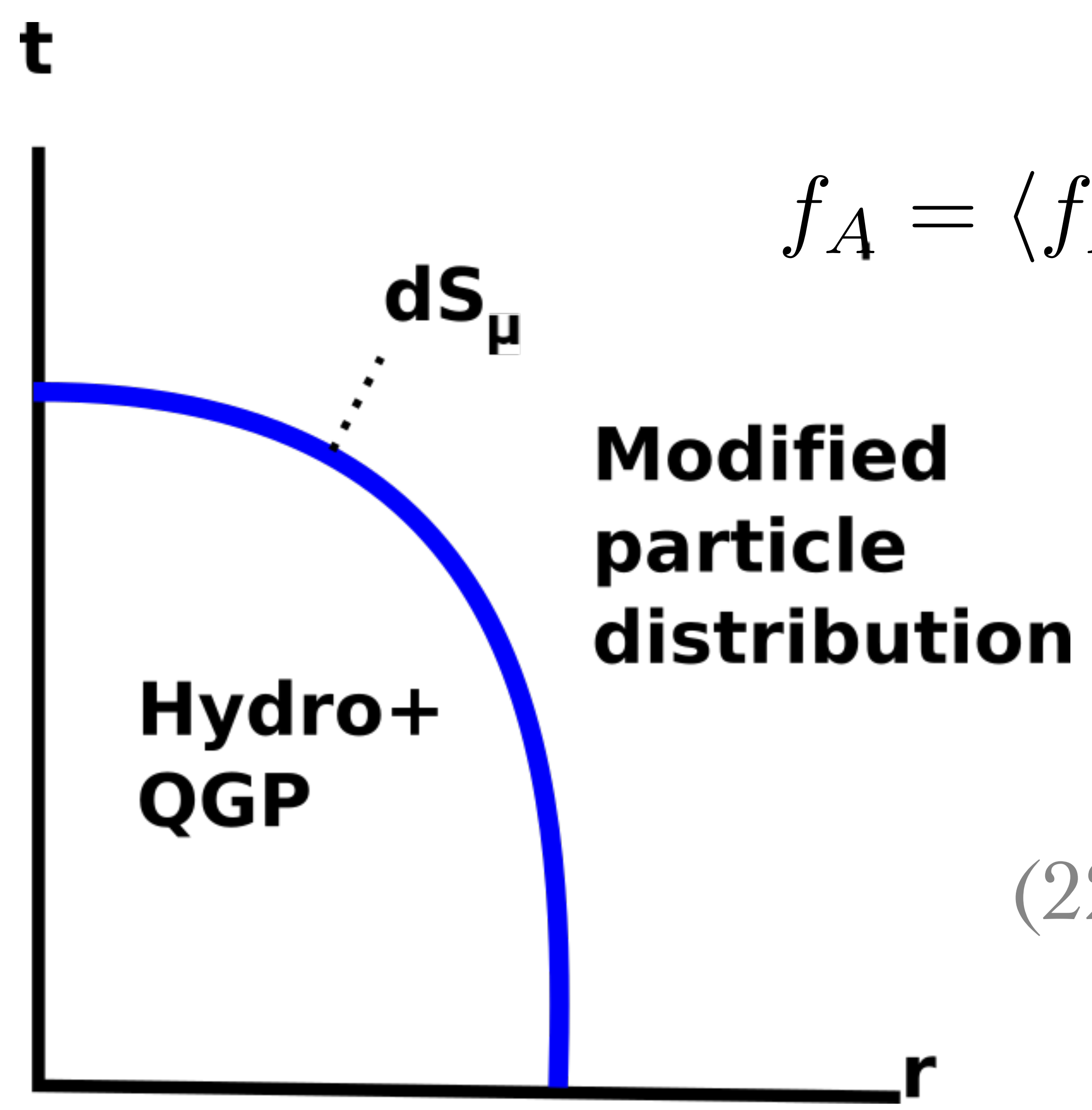
Does not describe fluctuations

Cooper and Frye, *74*

Critical fluctuations in hadron resonance gas

* We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with a critical sigma field

$$\delta m_A \approx g_A \sigma$$



$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$

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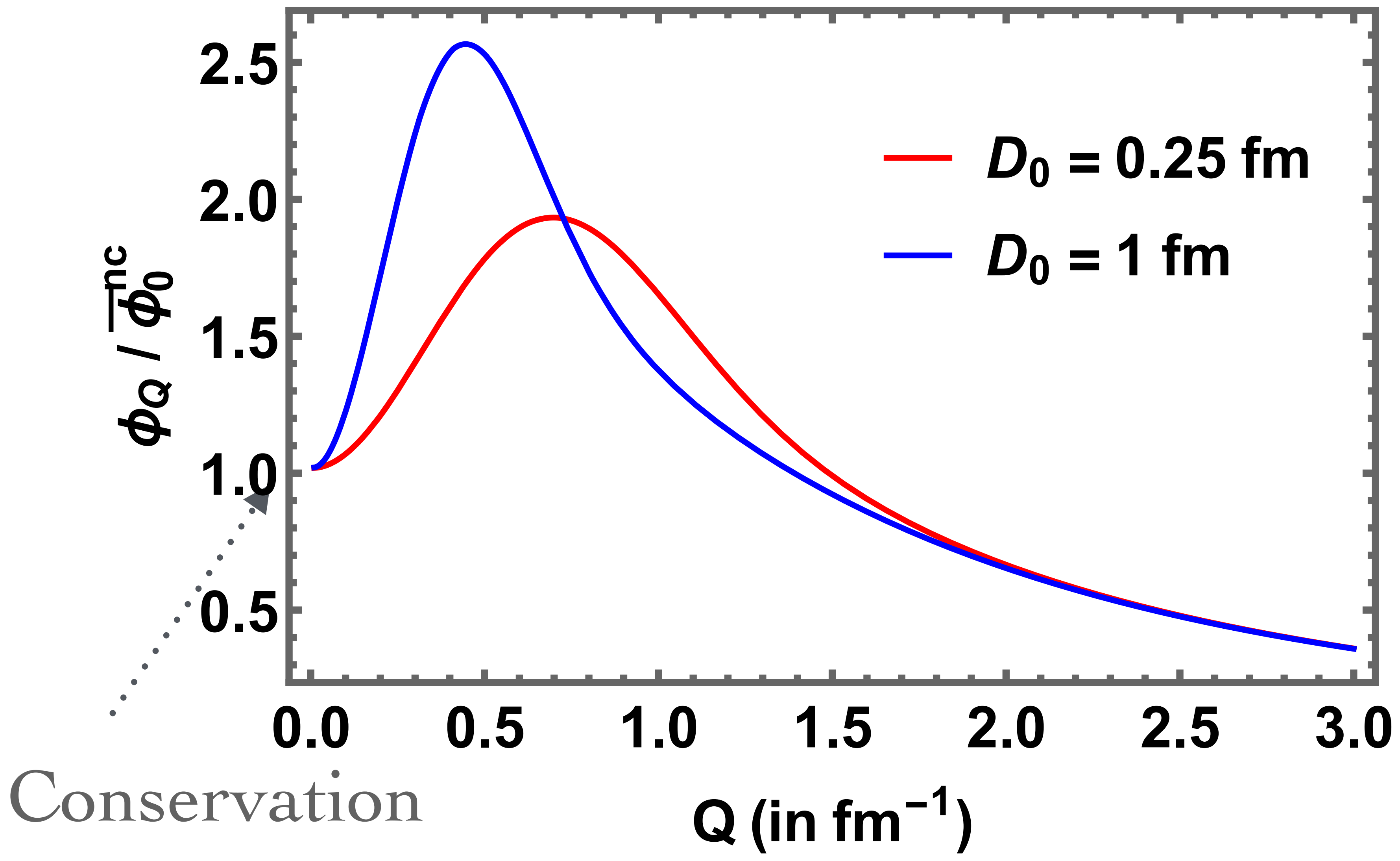
$$\left\langle \delta N_A^2 \right\rangle = \langle N_A \rangle + \left\langle \delta N_A^2 \right\rangle_\sigma$$

We match the two point function of σ to the two point function of the Hydro+ mode, $\hat{s} \equiv s/n$

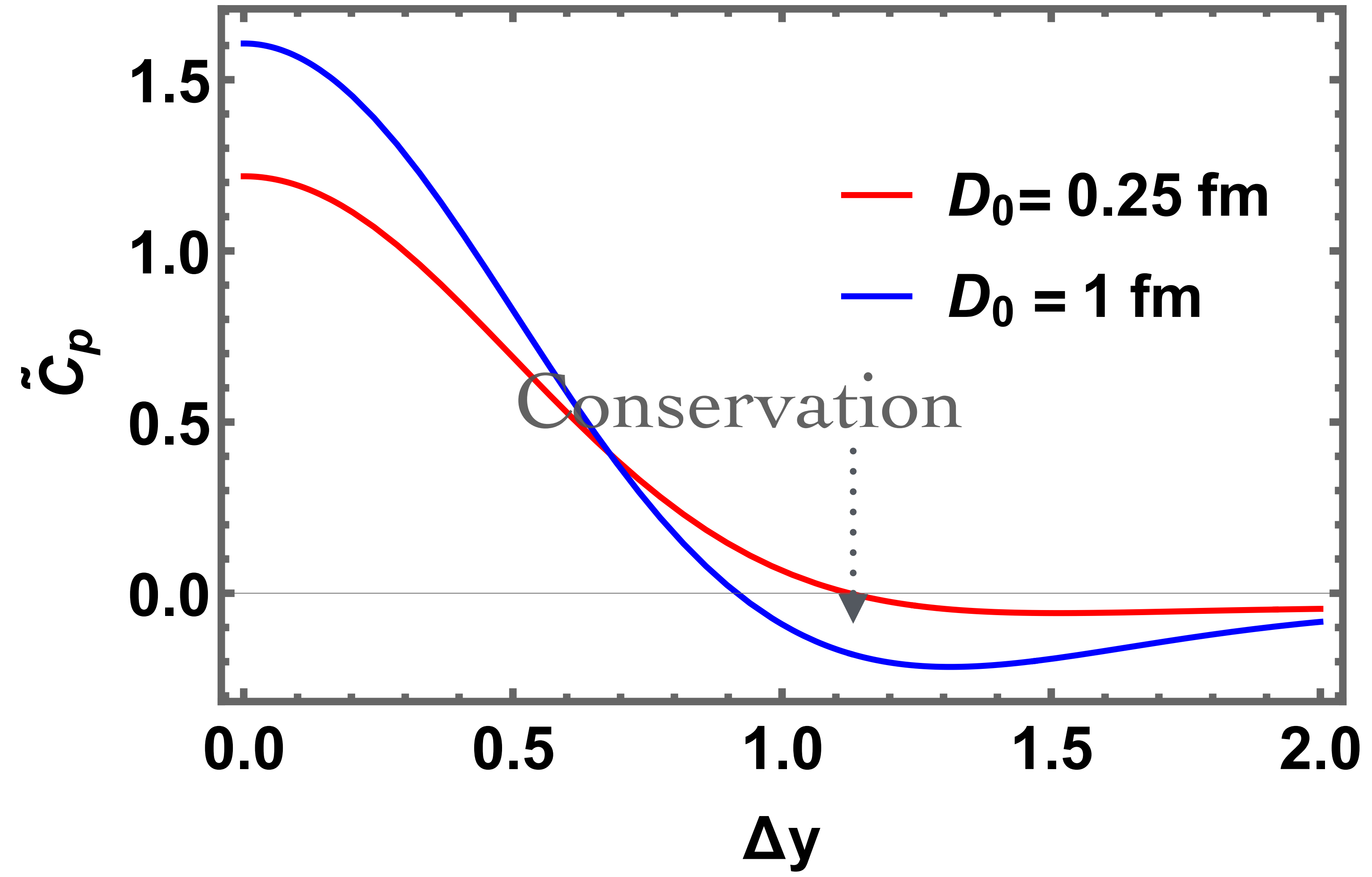
$$\left\langle \sigma(x_+)\sigma(x_-) \right\rangle \approx Z^{-1} \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

$$\left\langle \delta N_A^2 \right\rangle_\sigma = g_A^2 Z^{-1} \int dS_\mu J_A^\mu(x_+) \int dS_\nu J_A^\nu(x_-) \left\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-) \right\rangle$$

Effect of conservation laws on particle (anti)correlations at freeze-out



$$C_A(y_+, y_-) = \left\langle \delta \frac{dN_A}{dy_+} \delta \frac{dN_A}{dy_-} \right\rangle$$



Enhancement at low Δy, anti-correlations at large Δy

The low Q modes contribute the most to rapidity correlations

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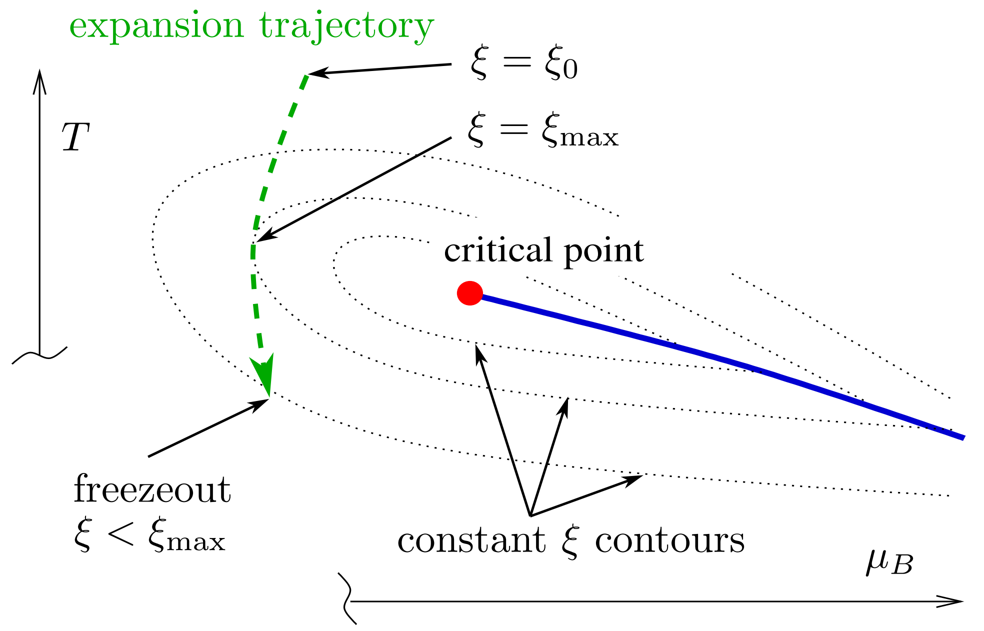
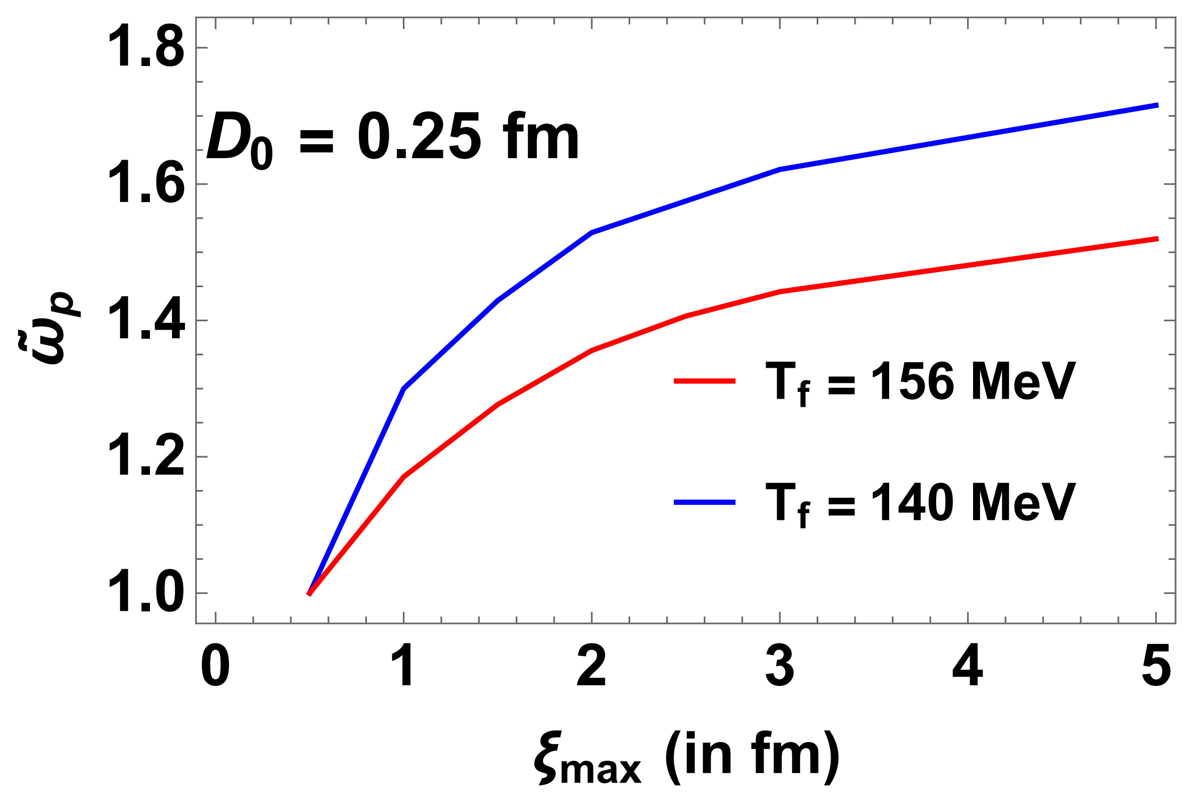
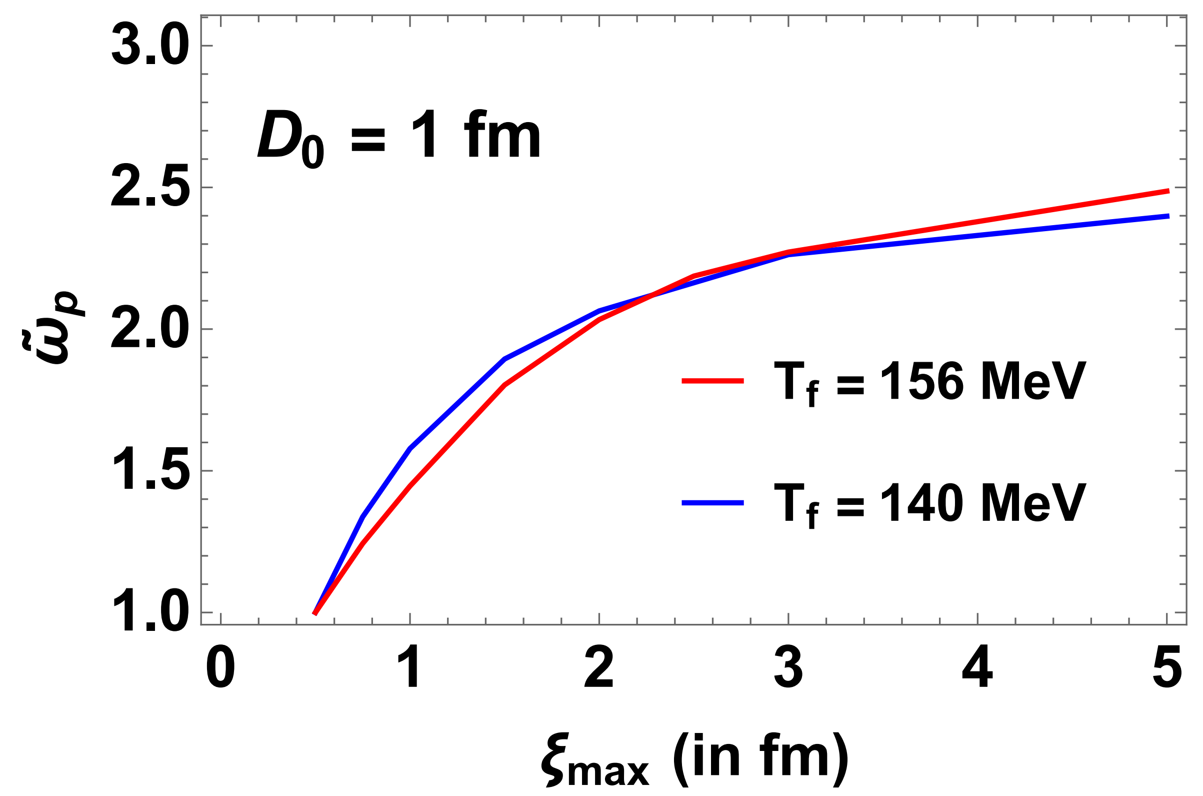
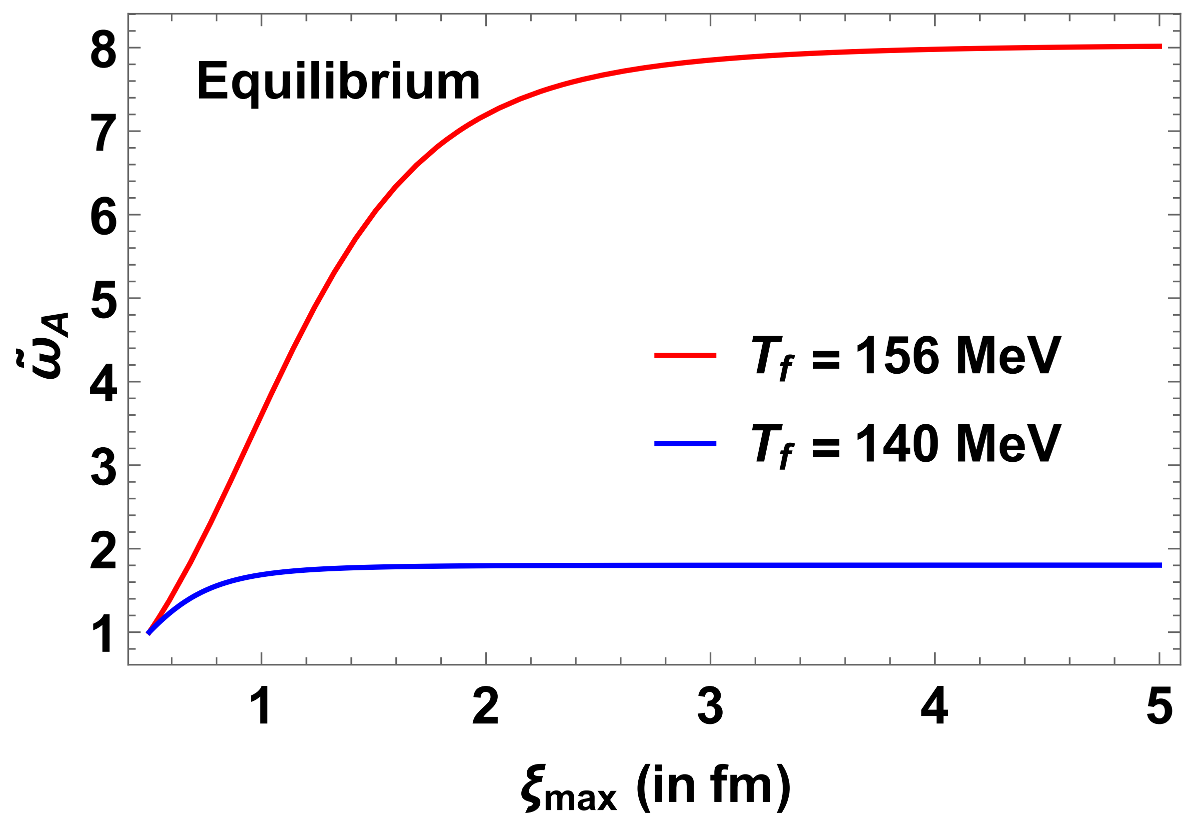
Critical contribution to variance of proton multiplicities

$$\omega_p \equiv \frac{\left\langle \delta N_p^2 \right\rangle_\sigma}{\left\langle N_p \right\rangle}$$









$$\tilde{\omega}_p \equiv \frac{\omega_p}{\omega_p^{\mathbf{nc}}}$$

- The fluctuations are reduced relative to equilibrium value (due to conservation laws)
- The fluctuations are found to increase with D_0 (faster diffusion)
- Compared to the equilibrium scenario, the fluctuations are less sensitive to freeze-out temperature

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Summary

* We have generalized the Cooper-Frye freeze-out procedure so that not only the averages, but also the critical fluctuations of the conserved densities are matched on the freeze-out hypersurface

* We have demonstrated the freeze-out in a semi-realistic scenario and estimated the dynamical effects for the critical contribution to the Gaussian cumulants of proton multiplicity

* The fluctuations are less sensitive to the freeze-out temperature in an out-of-equilibrium scenario unlike in an equilibrium case

Outlook

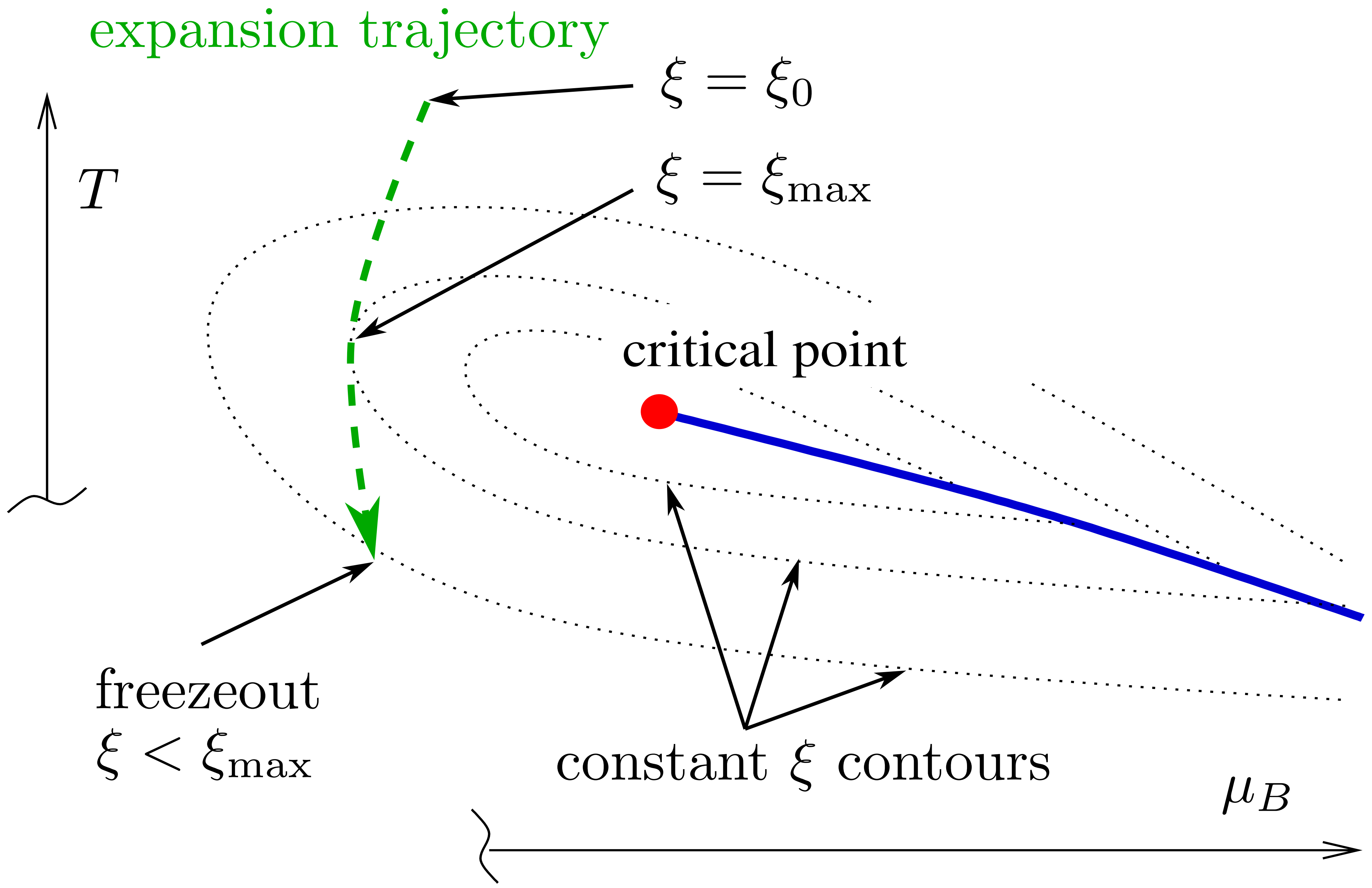
* The freeze-out procedure developed here can already be integrated into the full numerical simulation of heavy ion collisions relevant for BES program

* Freeze-out of higher point fluctuations needs to be implemented and analyzed

* The procedure can be improved by adding less singular contributions and modes which are not critical

Thank you!

Ratio of observables



$$\omega_A \equiv \frac{\langle \delta N_A^2 \rangle_\sigma}{\langle N_A \rangle}$$

$$\tilde{\omega}_A \equiv \frac{\omega_A}{\omega_A^{\mathbf{nc}}}$$

$\omega_A^{\mathbf{nc}}$ is the non-critical estimate obtained by assuming $\xi_{\mathbf{max}} = \xi_0$

$$\tilde{\omega}_A^{\mathbf{eq}} = \frac{\xi^2(T_f)}{\xi_0^2}$$

The normalized fluctuation measure $\tilde{\omega}_A$ is independent of g_A, Z and largely insensitive to acceptance cuts