ETTORE MAJORANA FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE

TO PAY A PERMANENT TRIBUTE TO ARCHIMEDES AND GALILEO GALILEI, FOUNDERS OF MODERN SCIENCE AND TO ENRICO FERMI, THE "ITALIAN NAVIGATOR", FATHER OF THE WEAK FORCES

International School of Subnuclear Physics, Erice, 2022

Event-by-event analysis of the two-particle source function in heavy-ion collisions with EPOS

Entropy 24 (2022) 3, 308

Correspondence: kincses@ttk.elte.hu

DÁNIEL KINCSES

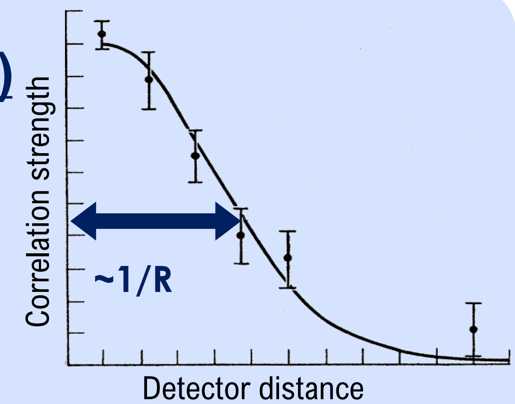
IN COLLABORATION WITH

M. STEFANIAK, M. CSANÁD

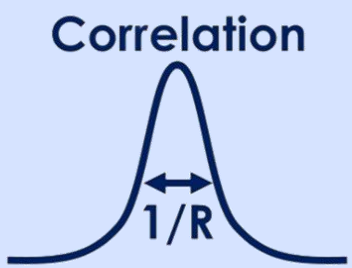
EÖTVÖS LORÁND UNIVERSITY | BUDAPEST

The HBT effect and the idea of femtoscopy

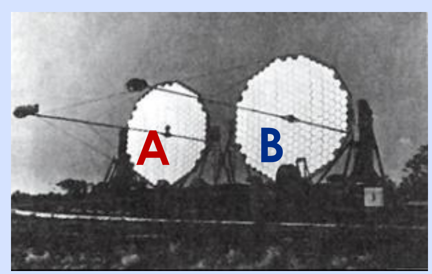
- **R. Hanbury Brown & R. Q. Twiss (Radio-astronomy)**
  - Intensity corr. vs detector dist. ⇒ source size



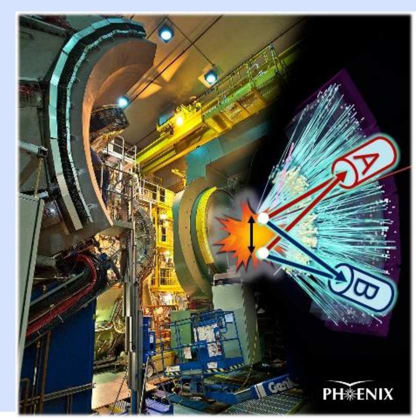
- **Goldhaber et al: analogy in high energy physics**
  - Distant star ⇔ Quark-Gluon Plasma
  - Light ⇔ particles from freeze-out
  - Intensity corr. of light ⇔ Momentum corr. of identical (bosonic) particles
  - **Measuring source shape on the fm scale!**



$$C(Q) = 1 + |\tilde{S}(Q)|^2$$



⇔



Basic definitions of femtoscopical correlation functions

- **Single particle distribution:** $N_1(p) = \int dx\, S(x,p)$ → phase-space density
- **Pair momentum distr.:** $N_2(p_1,p_2) = \int dx_1 dx_2 S(x_1,p_1)S(x_2,p_2)|\psi(x_1,x_2)|^2$
- **Correlation function:** $C(p_1,p_2) = \frac{N_2(p_1,p_2)}{N_1(p_1)N_2(p_2)}$
- **Pair source/spatial correlation:** $D(r,K) = \int d^4\rho S\left(\rho+\frac{r}{2},K\right)S\left(\rho-\frac{r}{2},K\right)$ → relative coordinate

$$C(Q,K) = \frac{\int D(r,K)\left|\psi_Q(\mathrm{r})\right|^2 dr}{\int D(r,K)dr}$$

relative pair momentum

average pair momentum

Pair wave function

- **Experiments: measuring C(Q) to gain information about D(r)**

The two-particle source function (spatial correlations)

$$D(r,K) = \int d^4\rho S\left(\rho + \frac{r}{2}, K\right) S\left(\rho - \frac{r}{2}, K\right)$$

- **Experiments – no direct access to pair-source**
  - Assumption on the **shape of the $D(r)$ pair-source function**
  - Proper description of FSI in $\psi_Q(\mathrm{r})$ **symmetrized pair wave function**
  - Calculating C(Q), then testing the assumption on experimental data
  - Experimental indications – **power-law tail for pions, Lévy-type sources?**
- **Event generator models (like EPOS) – direct access to pair-source!**
  - Phenomenological investigations of $D(r)$ possible

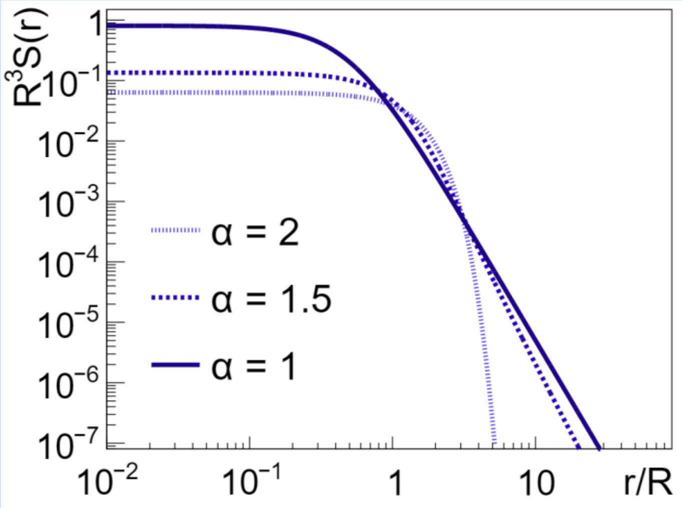
Csörgő, Hegyi, Zajc, Eur.Phys.J. C36
Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828
Metzler, Klafter, Physics Reports 339 (2000) 1-77,
Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002
Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36

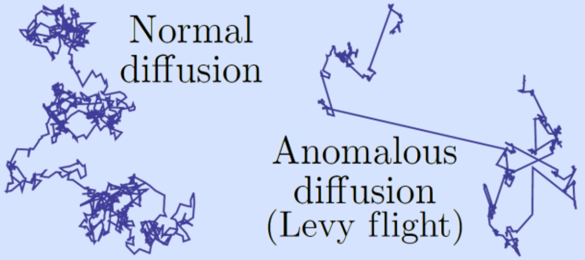
What is the shape of the source?
Gaussian vs. Lévy distributions in heavy ion physics

$$S(r,K) = \mathcal{L}(\alpha(K),R(K);r) = \frac{1}{(2\pi)^3}\int d^3q\, e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

- **Possible (competing) reasons for the appearance of Lévy-type sources:**
  1. **Proximity of the critical endpoint**
  2. **Anomalous diffusion**
  3. **Jet fragmentation**
  4. **Event averaging (different shapes)?**
- **Symmetric Lévy-stable distribution:**
  - From generalized central limit theorem, power-law tail (if $\alpha < 2$) ~ $r^{-(1+\alpha)}$
  - $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy
  - Retains the same α under convolution

$$S(r) = \mathcal{L}(\alpha,R;r) \Rightarrow D(r) = \mathcal{L}(\alpha,2^{1/\alpha}R;r)$$





The EPOS model

- **E**nergy conserving quantum-mechanical multiple scattering approach, based on **P**artons (parton ladders), **O**ff-shell remnants, and **S**plitting of parton ladders.

- The model is based on **Monte-Carlo techniques**
- Theoretical framework: **parton-based Gribov-Regge theory** (PBGRT)
- Three main parts of the model:
  - **Core-Corona division** (based on dE/dx of string segments)
  - **Hydrodynamical evolution** (vHLLE 3D+1 viscous hydro)
  - **Hadronic cascades** (UrQMD afterburner)

Details of the analysis

- $\sqrt{s_{\mathrm{NN}}}$ **= 200 GeV Au+Au** collisions generated by **EPOS359**
- Observable:
  angle-avg. radial source distribution of like-sign pion pairs

$$D\left(r_{1,2}^{LCMS}\right) = \int d\Omega dt D(r)$$

- Investigated cases:

1. **CORE, primordial pions – Gaussian source shape***
2. CORE, decay products incl. – power-law structures appear*
3. CORE+CORONA+UrQMD, primordial pions – Lévy-shape*
4. CORE+CORONA+UrQMD, decay products incl. – Lévy-shape

$$r_{1,2}^{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2};\ \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}};\ \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

*for details see backup slide!

Details of the analysis

- $\sqrt{s_{\mathrm{NN}}}$ **= 200 GeV Au+Au** collisions generated by **EPOS359**
- Observable:
  angle-avg. radial source distribution of like-sign pion pairs

$$D\left(r_{1,2}^{LCMS}\right) = \int d\Omega dt D(r)$$

- Investigated cases:

1. CORE, primordial pions – Gaussian source shape*
2. **CORE, decay products incl. – power-law structures appear***
3. CORE+CORONA+UrQMD, primordial pions – Lévy-shape*
4. CORE+CORONA+UrQMD, decay products incl. – Lévy-shape

$$r_{1,2}^{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

***for details see backup slide!**

Details of the analysis

- $\sqrt{s_{\mathrm{NN}}}$ **= 200 GeV Au+Au** collisions generated by **EPOS359**
- Observable:
  angle-avg. radial source distribution of like-sign pion pairs

$$D\left(r_{1,2}^{LCMS}\right) = \int d\Omega dt D(r)$$

- Investigated cases:

1. CORE, primordial pions – Gaussian source shape*
2. CORE, decay products incl. – power-law structures appear*
3. **CORE+CORONA+UrQMD, primordial pions – Lévy-shape***
4. CORE+CORONA+UrQMD, decay products incl. – Lévy-shape

$$r_{1,2}^{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

*for details see backup slide!

Details of the analysis

- $\sqrt{s_{\mathrm{NN}}}$ **= 200 GeV Au+Au** collisions generated by **EPOS359**
- Observable:
  angle-avg. radial source distribution of like-sign pion pairs

$$D\left(r_{1,2}^{LCMS}\right) = \int d\Omega dt D(r)$$

- Investigated cases:

1. CORE, primordial pions – Gaussian source shape*
2. CORE, decay products incl. – power-law structures appear*
3. CORE+CORONA+UrQMD, primordial pions – Lévy-shape*
4. **CORE+CORONA+UrQMD, decay products incl. – Lévy-shape**

$$r_{1,2}^{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

*for details see backup slide!

Details of the analysis

- $\sqrt{s_{\mathrm{NN}}}$ **= 200 GeV Au+Au** collisions generated by **EPOS359**
- Observable:
  angle-avg. radial source distribution of like-sign pion pairs

$$D\left(r_{1,2}^{LCMS}\right) = \int d\Omega dt D(r)$$

- Investigated cases:

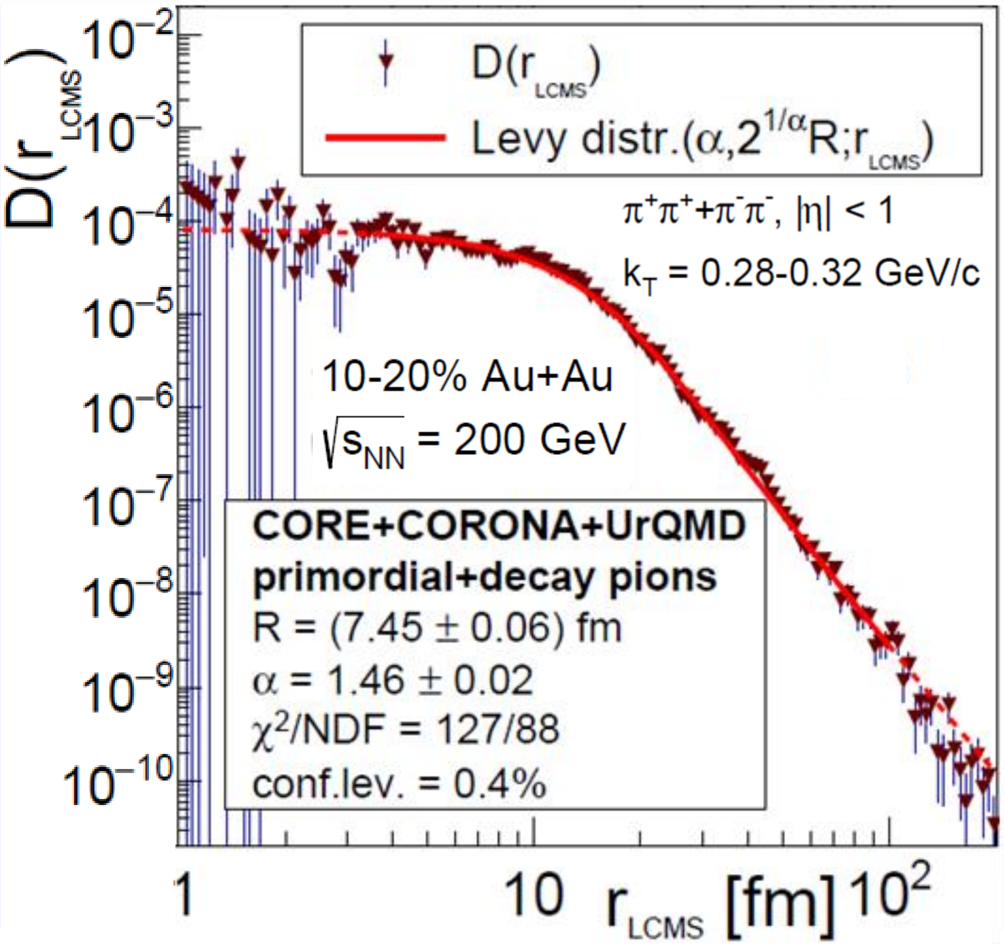
1. CORE, primordial pions – Gaussian source shape*
2. CORE, decay products incl. – power-law structures appear*
3. CORE+CORONA+UrQMD, primordial pions – Lévy-shape
4. **CORE+CORONA+UrQMD, decay products incl. – Lévy-shape**

$$r_{1,2}^{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2}; \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}}; \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

*for details see backup slide!

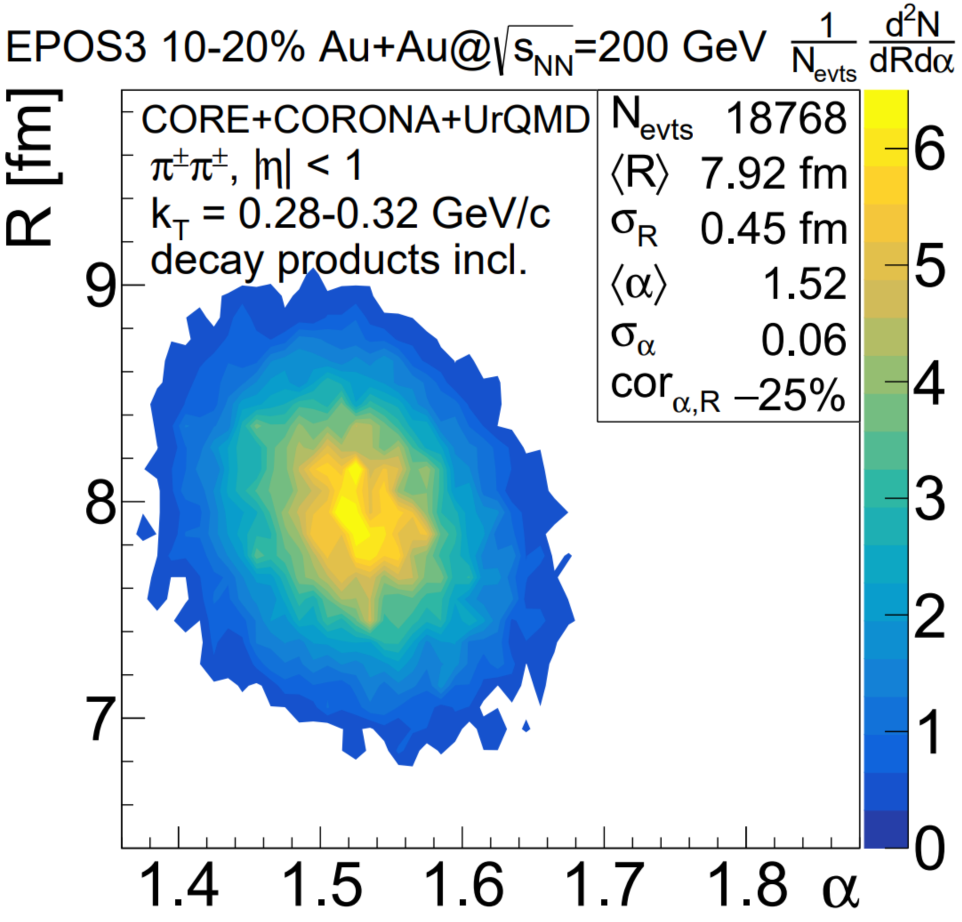
Example single evt. fit – CORE+CORONA+UrQMD with primordial + decay pions

- Investigating D(r) **event-by-event**
- **Lévy-fits** provide good description (2-100 fm range)
- Let's repeat such fits for thousands of events
- Extract α, R distribution



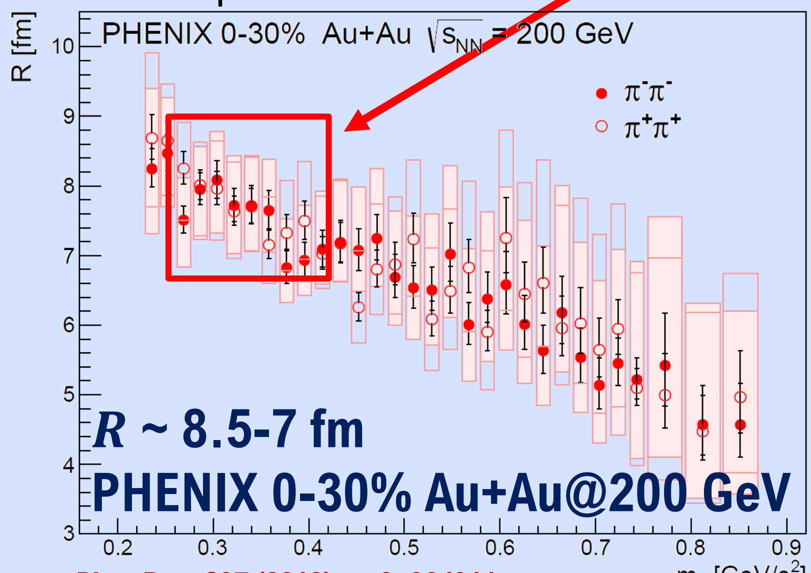
Example α, R distribution – CORE+CORONA+UrQMD with primordial + decay pions

- **Normal distr. of α, R** for given centrality & kT
- Extract **mean** and **std.dev**, investigate **centrality** and **mT dependence**
- kT dependence investigated around the peak of the pair-kT distr. to have adequate stat.



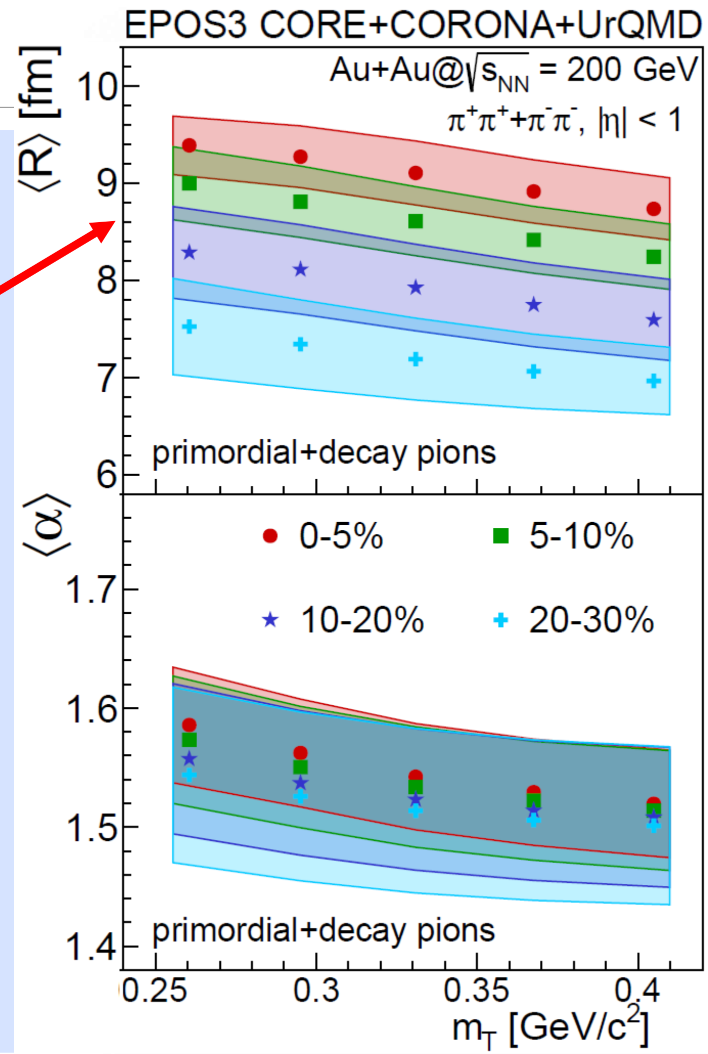
$\langle\alpha\rangle, \langle R\rangle$ $vs.$ m_T, centr.
CORE+CORONA+UrQMD
primordial + decay pions

- **Trends, magnitudes of R similar to experimental results**
- Higher magnitudes of α than experimental results



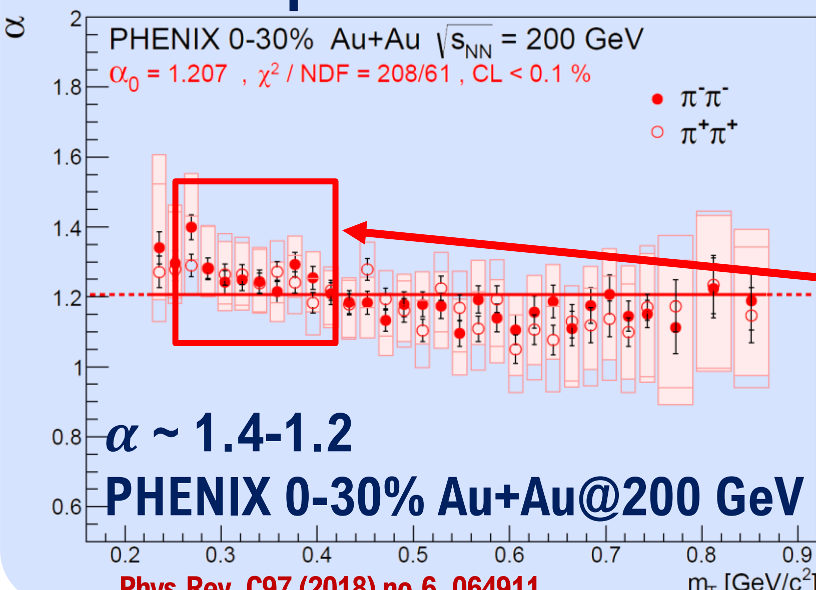
***R* ~ 8.5-7 fm**
PHENIX 0-30% Au+Au@200 GeV

Phys.Rev. C97 (2018) no.6, 064911

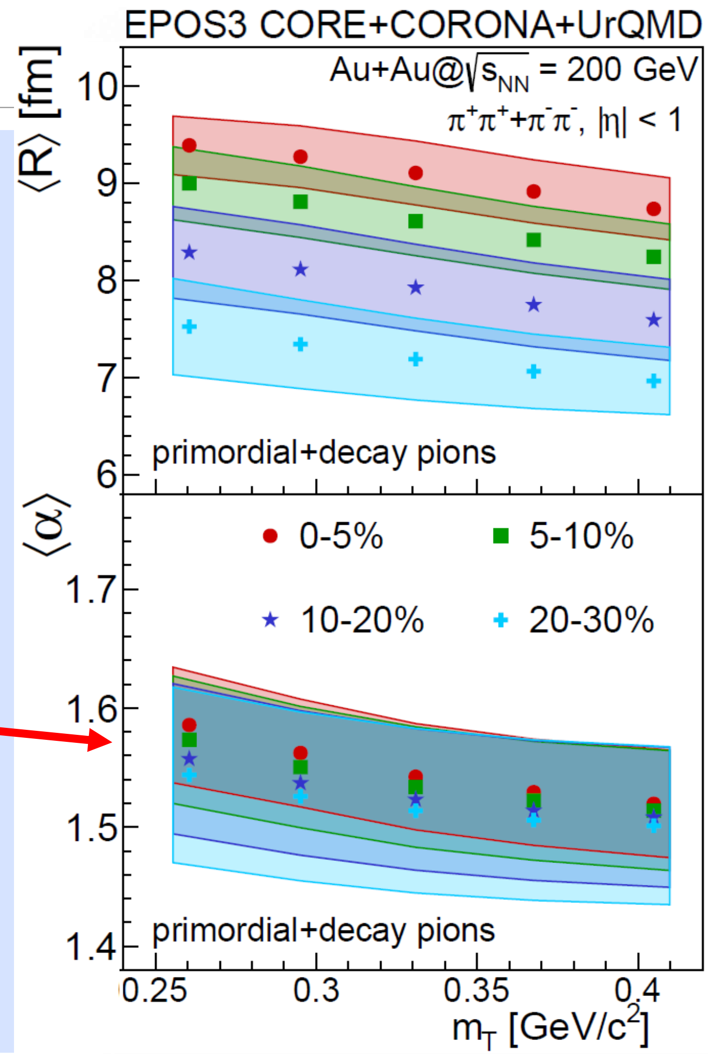


$\langle\alpha\rangle, \langle R\rangle$ $vs.$ m_T, centr.
CORE+CORONA+UrQMD
primordial + decay pions

- Trends, magnitudes of R similar to experimental results
- **Higher magnitudes of α than experimental results**



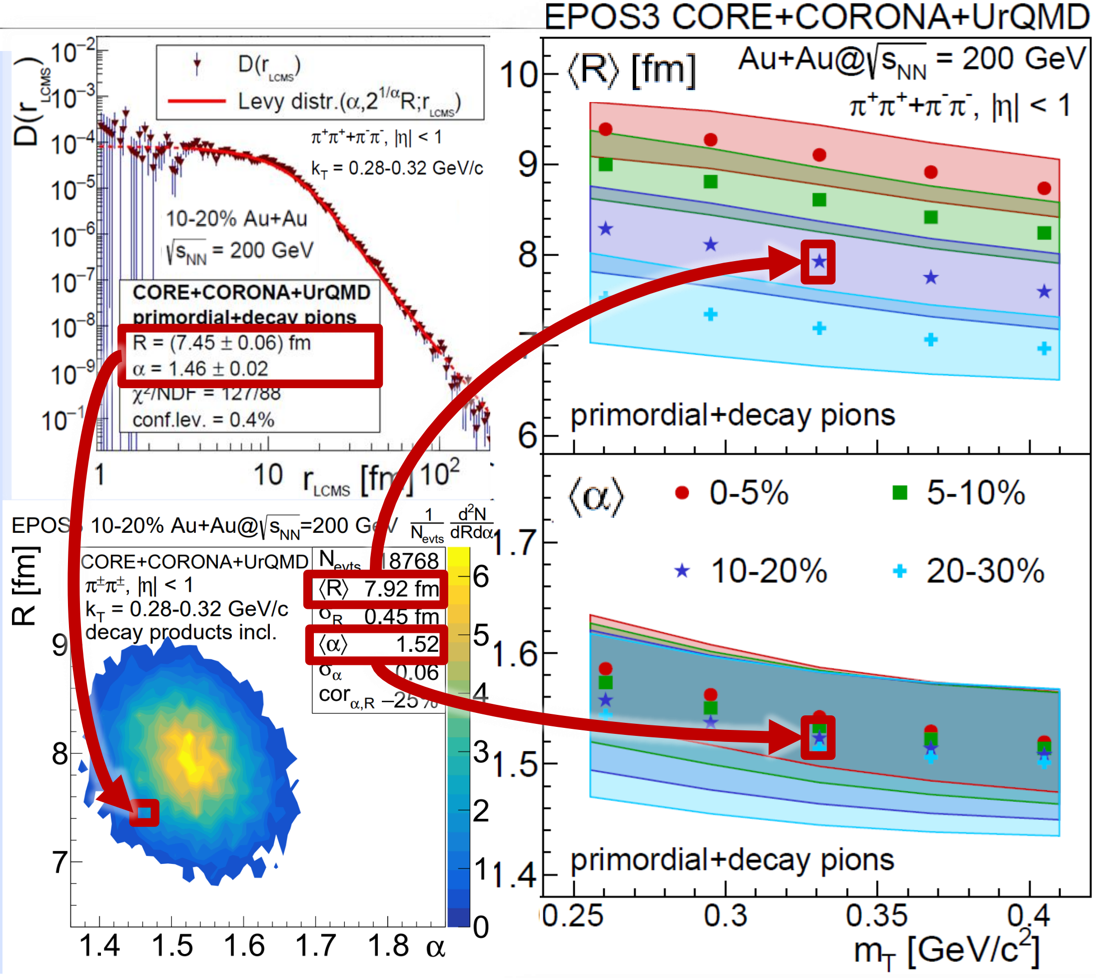
Phys.Rev. C97 (2018) no.6, 064911



Summary – event by event analysis of the pion pair-source in EPOS 200 GeV Au+Au collisions

1. Single event Levy fits to angle-averaged D(r) – **event-by-event non-Gaussianity**
2. Extracting the **mean, std.dev.** of R, α distr.
3. Investigating mT & centr. dependence

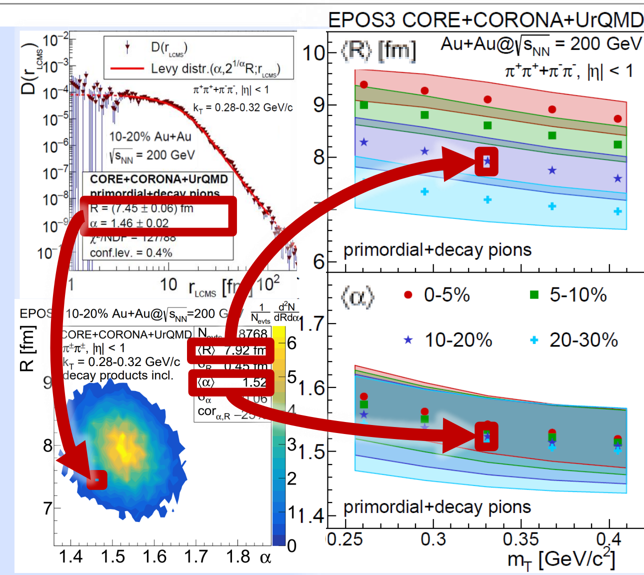
- **Lévy fits provide good descr., power-law tail strongly affected by rescattering, decays**



Outlook

Thank you for your attention!

- Investigating the pair-source in multiple dimensions
- Investigating the pair-source of different particles (kaons, protons)
- Reconstruct correlation func. from measured pair-source
- If you are interested in similar topics come to the Zimányi Winter School! http://zimanyischool.kfki.hu/22/



See details in Entropy 24 (2022) 3, 308

Correspondence: kincses@ttk.elte.hu

ETTORE MAJORANA FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE
TO PAY A PERMANENT TRIBUTE TO ARCHIMEDES AND GALILEO GALILEI, FOUNDERS OF MODERN SCIENCE AND TO ENRICO FERMI, THE "ITALIAN NAVIGATOR", FATHER OF THE WEAK FORCES
International School of Subnuclear Physics, Erice, 2022

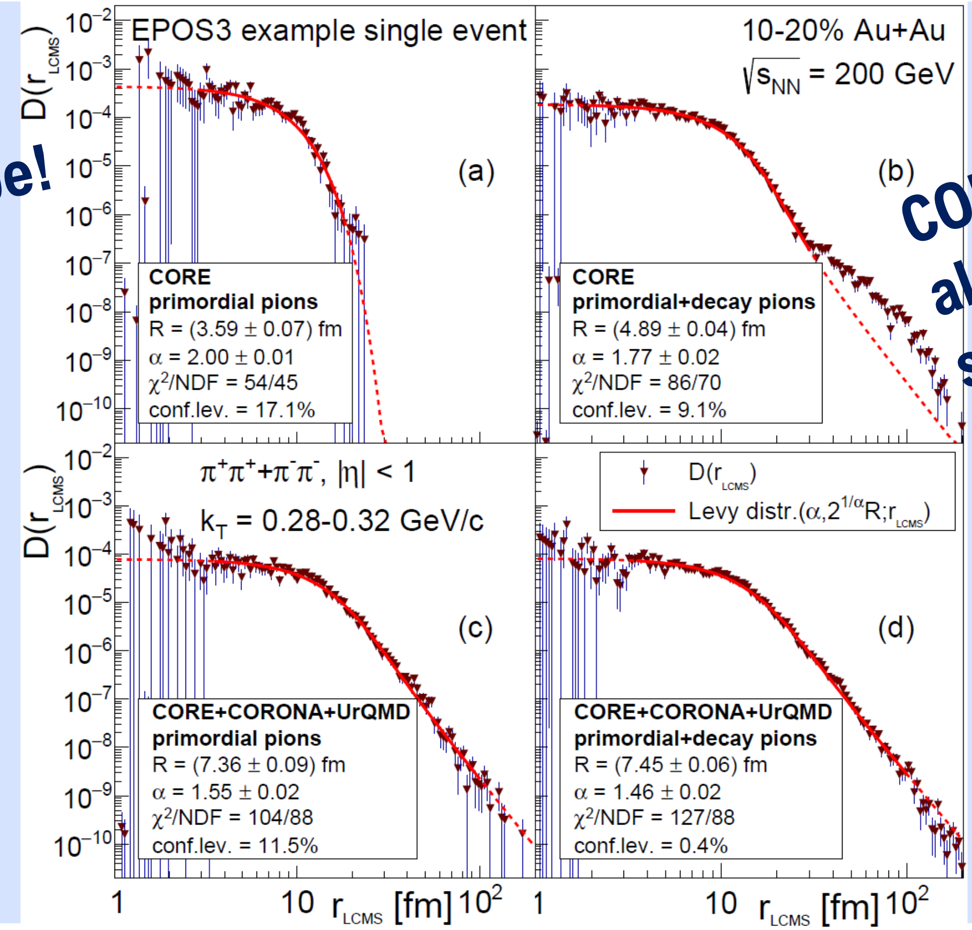
Event-by-event analysis of the two-particle source function in heavy-ion collisions with EPOS

BACKUP SLIDES

DÁNIEL KINCSES
IN COLLABORATION WITH
M. STEFANIAK, M. CSANÁD
EÖTVÖS LORÁND UNIVERSITY | BUDAPEST

Backup

CORE only: Gaussian source shape!



CORE + decays: already power-law structures!

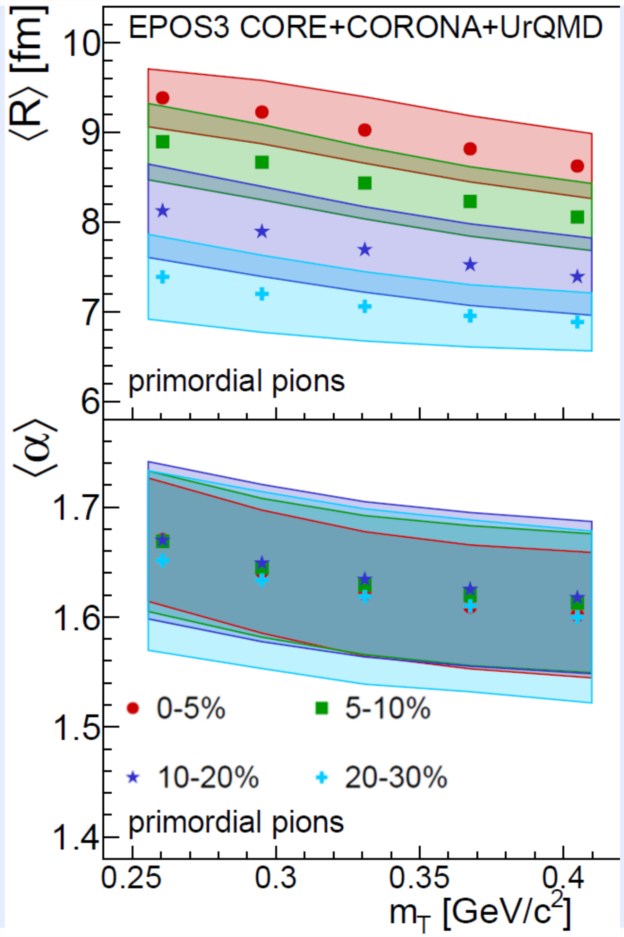
$\langle\alpha\rangle, \langle R\rangle$ $vs.$ m_T, centr.
CORE+CORONA+UrQMD
decay pions excluded(!)

Backup

- **Removing decay pions decreases R, increases α** (but still far from Gaussian)

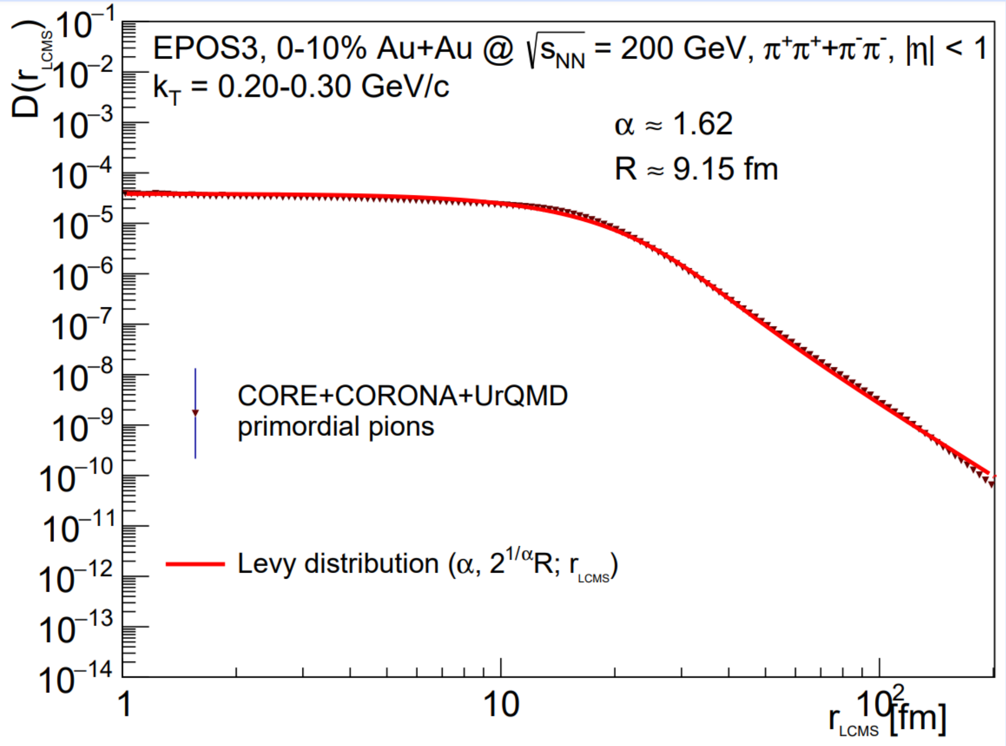
- **Decays and rescattering both play an important role** in the appearance of the power-law behavior

see also other phenomenological studies e.g. Universe 5, 148, Phys. Part. Nucl. 51(3), 282–287



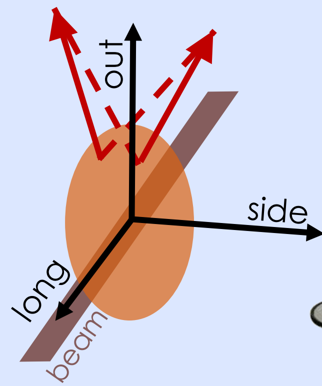
Backup

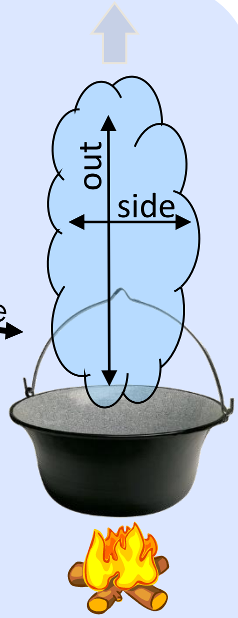
- Event-averaged source not perfectly Lévy
- Nevertheless, very similar parameters
  - Event averaged: $\alpha \approx 1.62$, $R \approx 9.15$ fm
  - Event-by-event: $\alpha \approx 1.66$, $R \approx 8.96$ fm
- More reasonable approach for kaons



Backup - HBT and the phase transition

- C(q) usually measured in the Bertsch-Pratt pair coordinate-system
  - **out:** direction of the average transverse momentum
  - **long:** beam direction
  - **side:** orthogonal to the latter two
- $R_{out}, R_{side}, R_{long}$: HBT radii
- $\Delta\tau$ emission duration, i.e. $S(r,\tau) \sim e^{-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}}$
- From a simple hydro calculation:
  $R^2_{\text{out}} = \frac{R^2}{1+u_T^2 m_T/T_0} + \beta_T^2 \Delta\tau^2, \quad R^2_{\text{side}} = \frac{R^2}{1+u_T^2 m_T/T_0}$
- RHIC, 200 GeV: $R_{out} \approx R_{side} \rightarrow$ no strong 1st order phase trans.
- Plus lots of other details: pre-equilibrium flow, initial state, EoS, ...





S. Chapman, P. Scotto, U. Heinz, Phys.Rev.Lett. 74 (1995) 4400; T. Csörgő and B. Lörstad, Phys.Rev. C54 (1996) 1390; S. Pratt, Nucl.Phys. A830 (2009) 51C

Backup – 2nd order phase transition?

- Second order phase transitions: **critical exponents**
  - **Near the critical point**
    - Specific heat ~ $((T - T_c)/T_c)^{-\alpha}$
    - Order parameter ~ $((T - T_c)/T_c)^{-\beta}$
    - Susceptibility/compressibility ~ $((T - T_c)/T_c)^{-\gamma}$
    - Correlation length ~ $((T - T_c)/T_c)^{-\nu}$
  - **At the critical point**
    - Order parameter ~ (source field)$^{1/\delta}$
    - **Spatial correlation function** ~ $r^{-d+2-\eta}$
  - Ginzburg-Landau: $\alpha = 0, \beta = 0.5, \gamma = 1, \eta = 0.5, \delta = 3, \eta = 0$
- QCD ↔ 3D Ising model
- Can we measure the η power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- **What distribution has a power-law exponent? Levy-stable distribution!**

Backup – Properties of univariate stable distributions

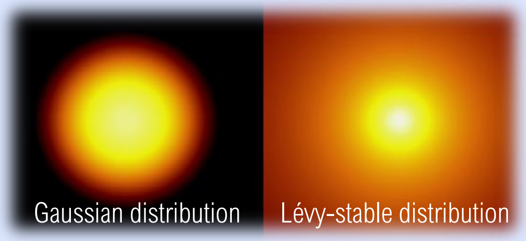
- **Univariate stable distribution:** $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi(q)e^{-ixq}dq\,,$

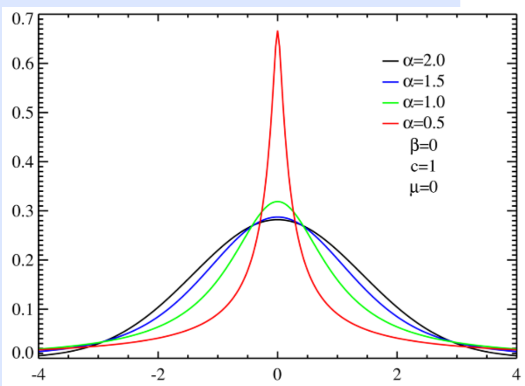
  where the characteristic function:

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), \alpha \neq 1 \\ -\frac{2}{\pi}\log|q|, \alpha = 1 \end{cases}$$

- $\varphi(q;\alpha,\beta,R,\mu) = \exp(iq\mu - |qR|^{\alpha}(1 - i\beta\,\mathrm{sgn}(q)\Phi))$
- α: index of stability
- β: skewness, symmetric if $\beta = 0$
- R: scale parameter
- μ: location, equals the median,

  if $\alpha > 1$: $\mu =$ mean





- **Important characteristics of stable distributions:**
  - The distributions retain the same α and β under convolution of random variables
  - Any moment greater than α isn't defined

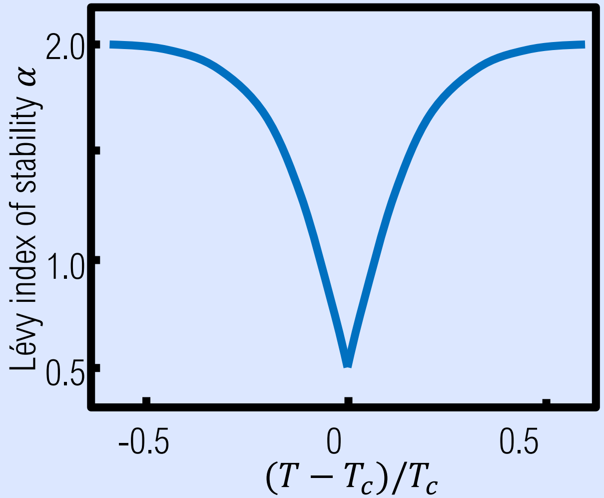
Backup - Lévy index as a critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\boldsymbol{\alpha \Leftrightarrow \eta}$?

  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising

  Halasz et al., Phys.Rev.D58 (1998) 096007

  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- **Change in α_{Levy} - proximity of CEP?**



- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distr.: anomalous diffusion, QCD jets, …