Modified gravity: a playground for non-standard cosmologies and wormholes

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- **A discovery of the accelerated expansion of the present Universe:** the cosmological constant does the job but implies strong fine-tuning.
- **Inflation in the early Universe:** numerous inflationary models rely on a scalar field, which in some cases is non-minimally coupled.
- **Academic interest:** our attempts to modify gravity give us deeper understanding of GR.

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Non-minimal coupling $f(\varphi)$ does the job:

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- How far can we go with this kind of modifications?

Potential problems

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- There exist a wide class of healthy scalar-tensor theories (STT) with 2nd derivatives: Horndeski theories and their generalisations.

  Horndeski (1974)

  Deffayet, Deser, Esposito-Farese (2009)

  Gleyzes, Langlois, Piazza, Vernizzi (2015)

- The most general class today - Degenerate Higher Order Scalar-Tensor theories or DHOST).

  Ben Achour, Langlois, Noui (2016)

Horndeski theories

General form of the Lagrangian

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\varphi, X),$$

$$\mathcal{L}_3 = K(\varphi, X)\Box\varphi,$$

$$\mathcal{L}_4 = -G_4(\varphi, X)R + 2G_{4X}(\varphi, X)\left[(\Box\varphi)^2 - \varphi_{;\mu\nu}\varphi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\varphi, X)G^{\mu\nu}\varphi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\varphi)^3 - 3\Box\varphi\varphi_{;\mu\nu}\varphi^{;\mu\nu} + 2\varphi_{;\mu\nu}\varphi^{;\mu\rho}\varphi_{;\rho}^{\ \ \nu}\right],$$

$\varphi_{;\mu\nu} = \nabla_\nu\nabla_\mu\varphi$, $\Box\varphi = g^{\mu\nu}\nabla_\nu\nabla_\mu\varphi$, $X = g^{\mu\nu}\varphi_{;\mu}\varphi_{;\nu}$, $G_{iX} = \partial G_i/\partial X$.

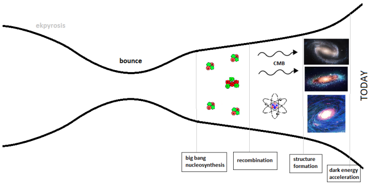
- Equations of motion (EOM) are the 2nd order (no Ostrogradsky ghost).
- Generalizations of Horndeski: add more functions into Lagrangian + constraints.
- Some extensions of Horndeski theories give rise to the 3rd order EOMs.

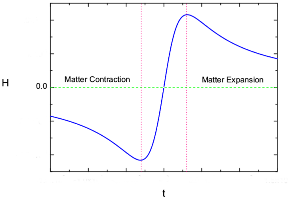
Applications of Horndeski theories in cosmology

- Dark Energy modelling
- Modelling inflation

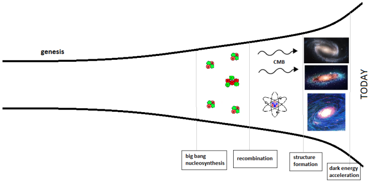
Applications of Horndeski theories in cosmology

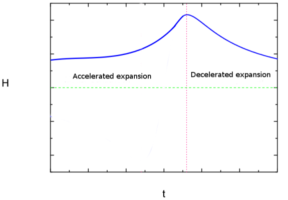
- Non-standard cosmological models without the initial singularity aka Big Bang:
  - a Universe with a bounce





  - a Universe starting off with Genesis





Non-standard cosmological models:

- a Universe with a bounce
- a Universe starting off with Genesis

- Both scenarios require **violation of the Null Energy Condition** (NEC):

For a homogeneous stationary fluid: $p + \rho > 0$

because NEC ensures that the Hubble parameter never grows

$$\dot{H} = -4\pi G(p + \rho) < 0$$

and the energy density in standard cosmologies always decreases

$$\frac{d\rho}{dt} = -3H(\rho + p) < 0.$$

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- Further development: adding other matter components (extra matter considerably affects stability).

Applications of Horndeski theories: gravitational objects

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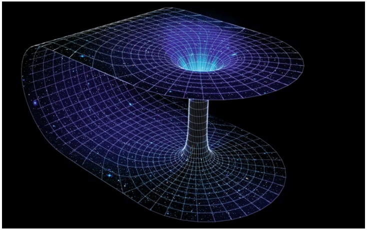
- Black holes (a well developed topic)
- Wormholes (our current project)



  - Traversable wormholes require an exotic matter to support its throat
  - Stability at the perturbed level is an issue (much more involved than in the cosmological setting)
  - A completely stable solution still does not exist (the work is in process)

Mironov, Rubakov, VV (2018)
Franciolini, Hui, Penco, Santoni, Trincherini (2018)

Bakopoulos, Charmousis, Kanti (2021)

Conclusion

- Scalar-tensor theories of modified gravity are widely used in cosmology and usually belong to Horndeski class of theories.
- Horndeski theories and their extensions enable one to construct viable non-singular cosmological scenarios - bouncing Universe and Universe with Genesis.
- (Beyond) Horndeski theories may potentially support a traversable wormhole.

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Thank you for your attention!

Cubic Horndeski theory

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$= ...$terms with the 2nd derivative at most