Integer dimensions in the Holographic Swampland

International School of Subnuclear Physics, Erice (2022)

UNIVERSITY OF OXFORD

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Based on: 2006.01021 [Joseph Conlon,FR]

mostly: 2110.06245 [Joseph Conlon,Sirui Ning,FR]

2202.09330 [Fien Apers,Joseph Conlon,Sirui Ning,FR]

Introduction and outline

Connect String Theory (ST) to experiment: an oxymoron?

["Is String Phenomenology an Oxymoron ?" Quevedo '16] [Answer: No!]

Landscape: too many vacua!

Swampland: not all EFTs admit a UV completion within ST (QG)

This talk: study constraints from AdS/CFT

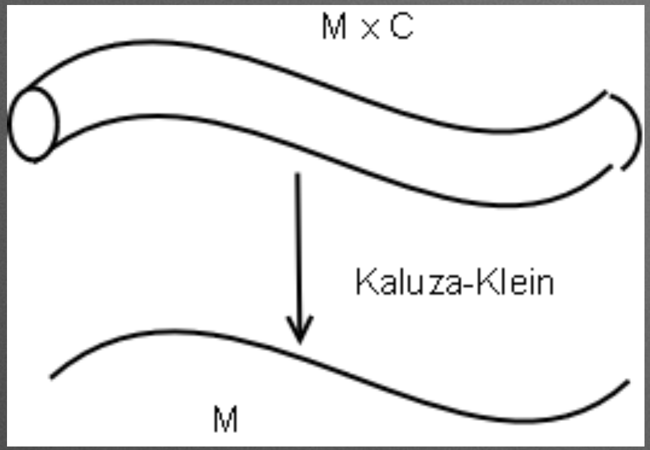
(Anti de Sitter /Conformal Field Theory correspondence)

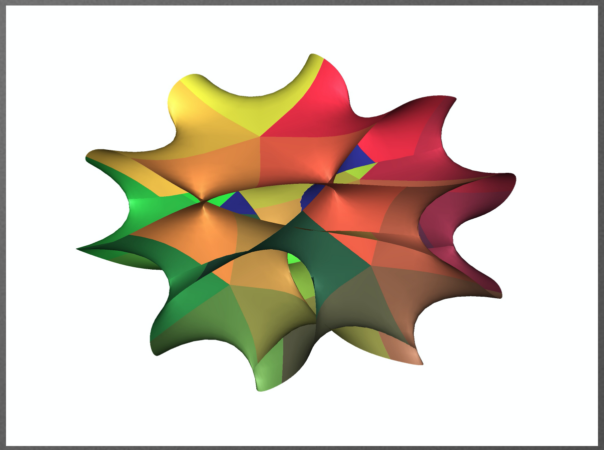
AdS/CFT

4d lagrangians derived from ST ⟷ Putative 3d CFT dual

From String Theory to reality

Compactify 10 d string theory (IIA/IIB) on Calabi Yau's





Size and shape of the extra dimensions (including internal cycles) parametrised by moduli fields

Basic requirements:

- dS? Hard! Most constructions through AdS
- All moduli stabilised
- Scale separation $R_{AdS} >> R_{KK}$

In practice

10/11 d String Theory

↓

10/11 d Supergravity + g_s, α' corrections

↓

Compactify to 4d   Insert NP effects ?

↓

Integrate out heavy moduli

↓

"Realistic" vacuum

Many successive steps of approximation, not always under control

Swampland: only a limited number of low energy EFTs compatible with ST (QG)

[Vafa '05, Ooguri Vafa '07, See Palti '18 for a comprehensive review]

The Swampland (I)

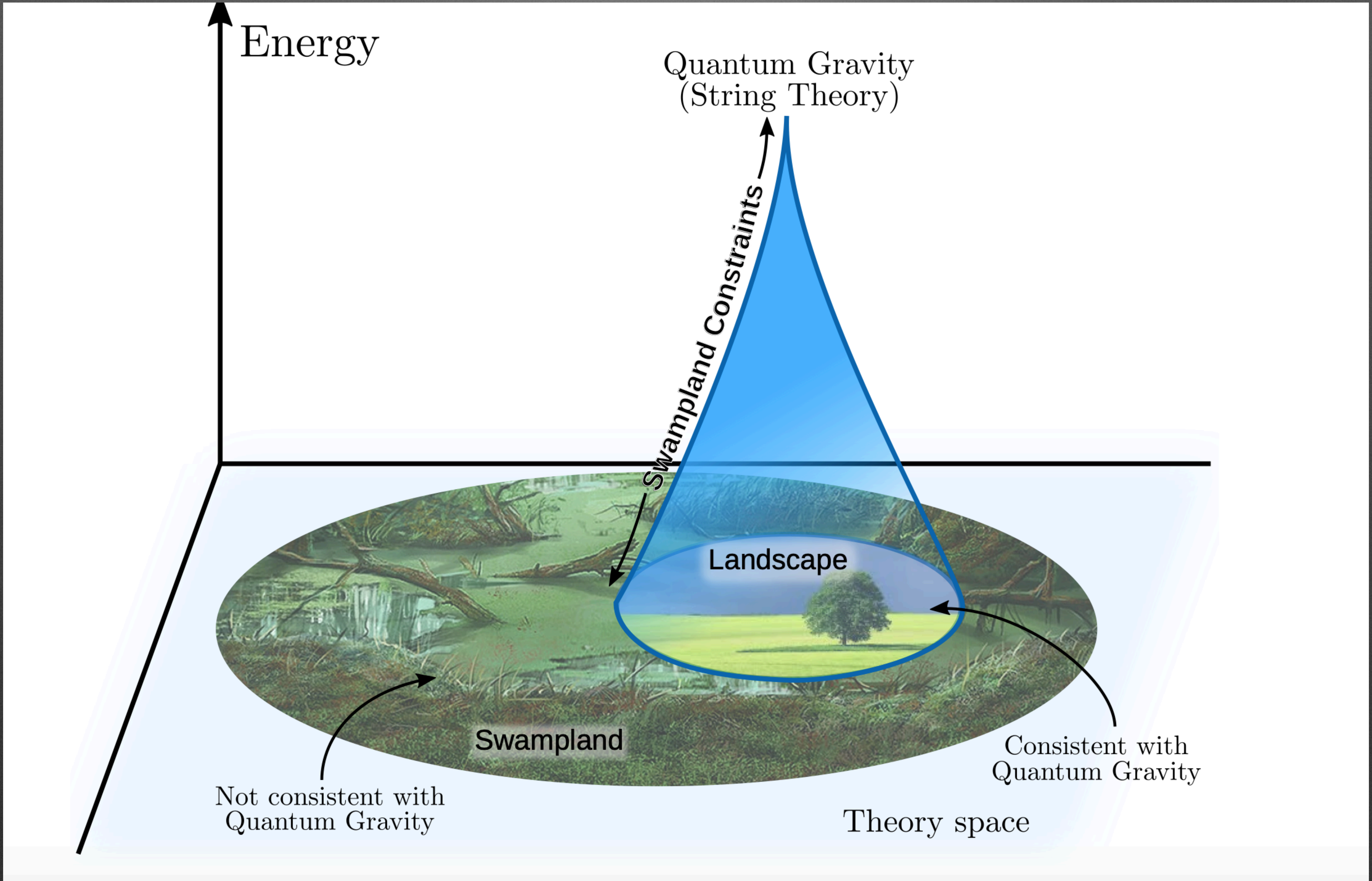


Figure taken from [van Beest,Calderòn-Infante,Mirfendereski,Valenzuela '21]

The Swampland (II)

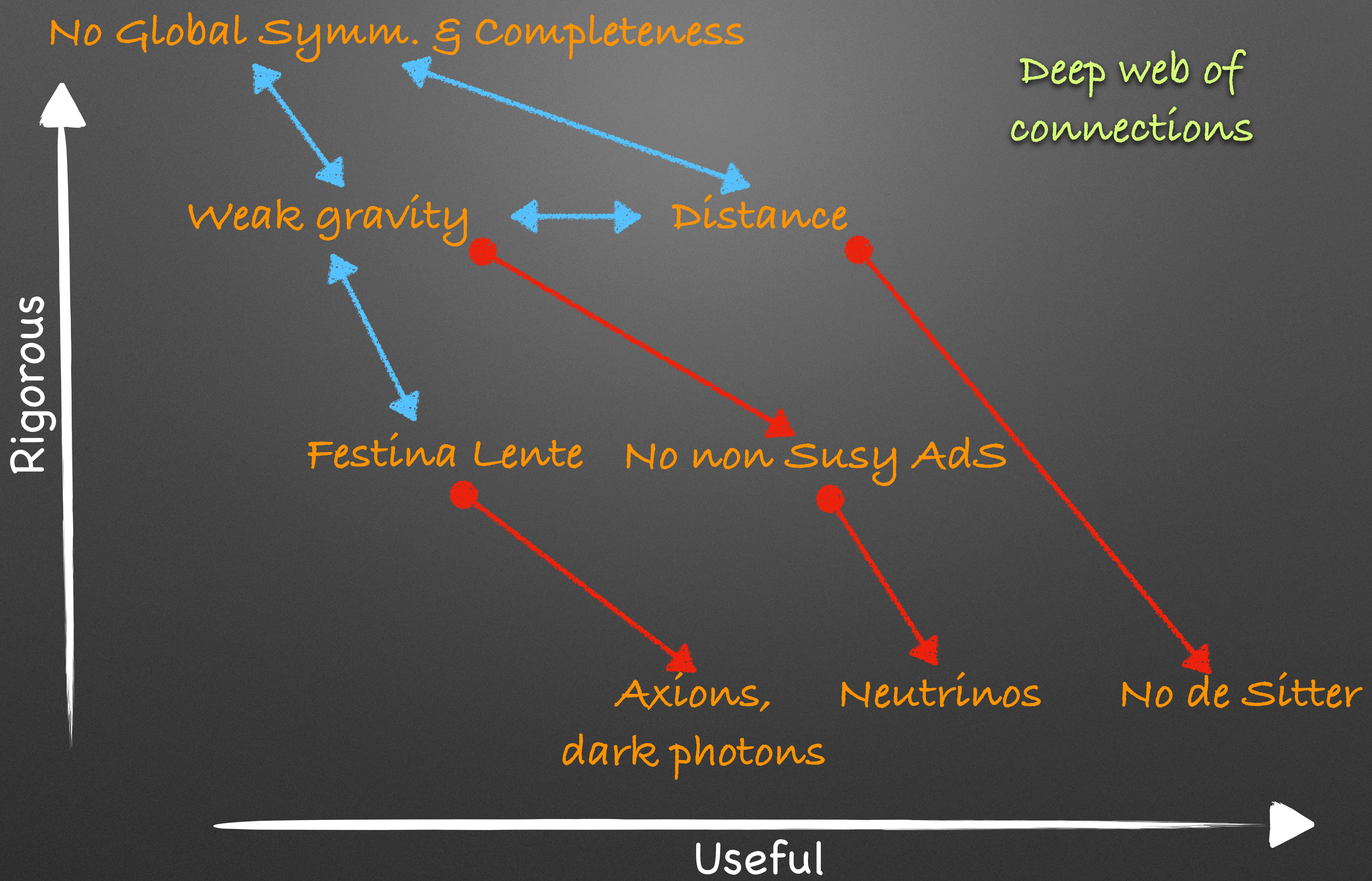


Figure Heavily based on Hiroshi Ooguri's lectures at the CERN 2019 Winter School

Swampland and AdS/CFT

No global symmetries in QG

[Harlow,Ooguri '18]
[Harlow,Ooguri '19]

Distance Conjecture

[Perlmutter,Rastelli,Vafa,Valenzuela '20]
[Baume, Calderon '20]

Weak Gravity Conjecture

[Nakayama,Nomura '15]
[Montero'19]
[Aharony,Palti '21] [Palti,Sharon '22]
+ many others (see Palti for a review)

Not much work on specific realisations - very complicated

Only very generic properties for realistic vacua

Type IIA DGKT

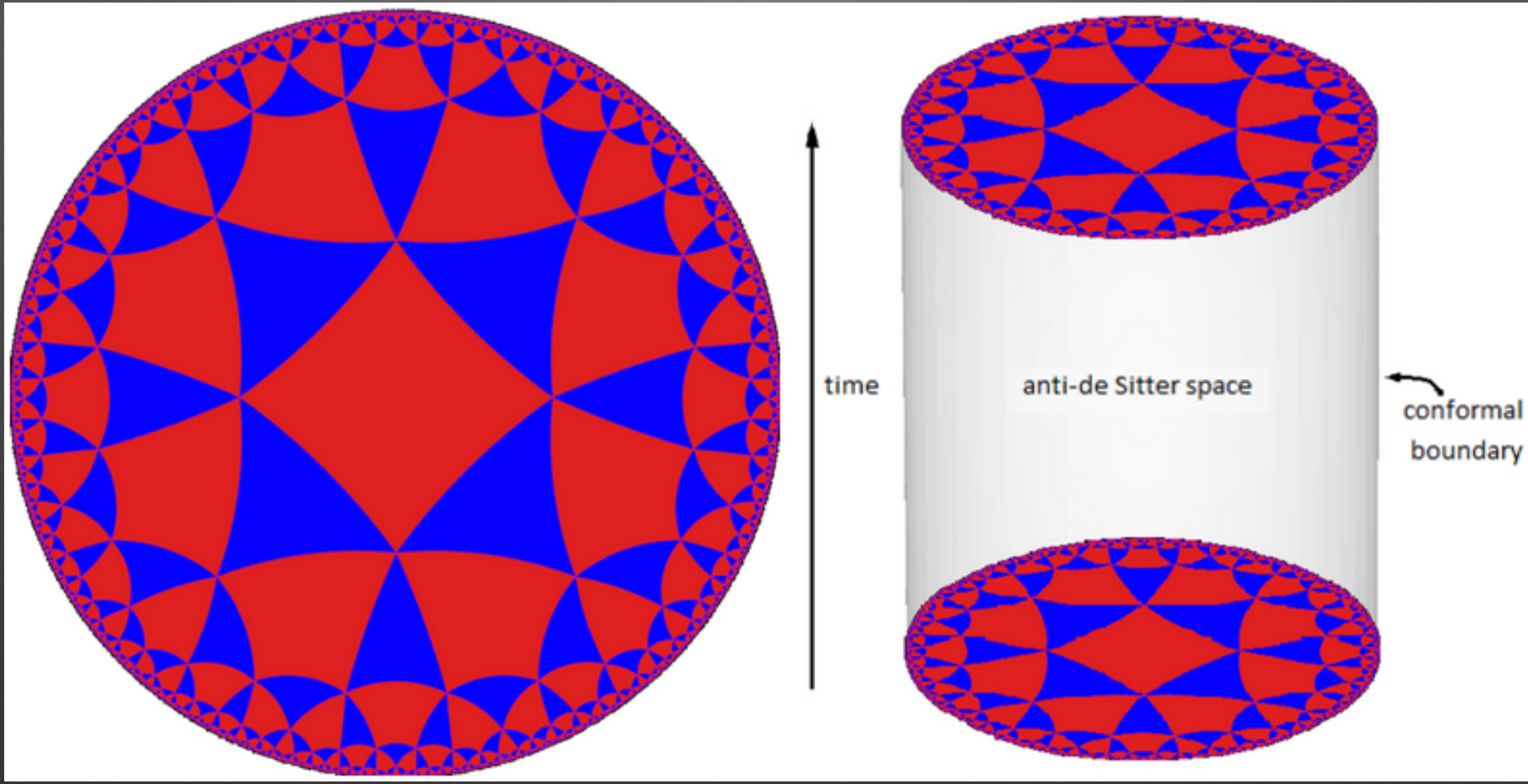
[Aharony,Antebi,Berkooz '08]

Type IIB KKLT/LVS

[de Alwis,Gupta,Quevedo,Valandro '15]

We focus on precise properties of a low energy subsector

AdS/CFT - in pictures



Conformal Field Theories

"Scale invariant" QFT

[Loosely speaking…]

$$x'_\mu = \lambda x_\mu$$

$$\mathcal{O}'(\lambda x) = \lambda^{-\Delta}\mathcal{O}(x)$$

Fully determined by set of 2 and 3-point correlators

$$\langle \mathcal{O}(x_1)\,\mathcal{O}(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2\Delta}}$$

All dynamics encoded in

$$\Delta_i, C_{ijk}$$

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}\,|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}\,|x_{13}|^{\Delta_3+\Delta_1-\Delta_2}}$$

AdS/CFT primer

Conjectured equivalence between ST(QG) on (asymptotic) AdS spacetime in d+1 dim and Conformal Field Theories (CFT) in d dim

AdS correlator ⟷ CFT correlator

Dictionary

AdS	CFT
Field φ	Operator $\mathcal{O}$
Mass m	Dimension $\Delta(\Delta - d) = m^2 R_{\mathrm{AdS}}^2$
$\left(\frac{R_{\mathrm{AdS}}}{\ell_s}\right)^d$	$\lambda = g^2 N$

The holographic Swampland

AdS/CFT

4d moduli EFT in AdS	Putative 3d CFT dual
Fields Φ_i	$m^2 R^2_{AdS} = \Delta(\Delta - 3)$

- Does the CFT interpretation offer a new perspective?

[Conlon, Ning, FR '21]
[Apers,Conlon,Ning,FR '22]

[Conlon, Quevedo '18] [Conlon, FR '20]

Can one relate <u>Swampland constraints</u> on AdS to <u>CFT consistency condition ?</u>

Bootstrap?

Example: LVS

[Balasubramanian,Berglund,Conlon,Quevedo '05]
[Conlon,Quevedo,Suruliz '05]

Swampland & scale separation

Scale separation $R_{AdS} >> R_{KK}$ + $\Delta(\Delta - d) = m^2 R_{\text{AdS}}^2$

↓

Holographic interpret: no large gap around $\Delta \sim \mathcal{O}(1)$

AdS Distance Conjecture (ADC) [35]

Take Quantum Gravity on AdS_d, with a cosmological constant Λ. A tower of states with masses

$$m \sim |\Lambda|^{\alpha} \qquad \alpha > 0. \tag{1.15}$$

should appear in the limit where $\Lambda \to 0$.

$$V_{\text{min}} = -\frac{3M_P^2}{R_{AdS}^2}$$

[Lust, Palti,Vafa '19]

Strong form (SUSY): $\alpha = \frac{1}{2}$ no scale separation

In practice

10/11 d String Theory

↓

10/11 d Supergravity + g_s, α' corrections

↓

Compactify to 4d Insert NP effects ?

↓

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"Realistic" vacuum

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Swampland: only a limited number of low energy EFTs compatible with ST (QG)

[Vafa '05, Ooguri Vafa '07, See Palti '18 for a comprehensive review]

Flux compactifications (I)

Bosonic spectrum of type IIA/IIB:

Φ	dilaton - sets string coupling		universal
$g_{\mu\nu}$	metric/graviton		universal
$B_{\mu\nu}$	$H_3 = dB_2$	Kalb-Ramond	universal

p-forms $\quad A \equiv A_{\mu_1,\ldots,\mu_p}\,, \qquad F = dA = \partial_{[\mu_{p+1}} A_{\mu_1,\ldots,\mu_p]}$

	p-forms	branes
Type IIA	F_0, F_2, F_4, F_6	$D0, D2, D4, D6$
Type IIB	F_1, F_3, F_5	$D1, D3, D5$

Flux compactifications (II)

[Free action] [Matter coupling] [Quantization condition]

$$\int \frac{1}{g^2} F_p \wedge * F_p + \int_{(p-2)-brane} A_{p-1} \qquad \int_{\gamma_p} F_p = 2\pi n$$

Landscape!!!

fluxes threading the extra dimensions generate potentials for the moduli

Toy example [Landscape, String Naturalness and Multiverse Hebecker '21]

Compactify on $S_1^A \times S_1^B$

$$\int dy_B F_1 = 1 \quad F_1 \sim \frac{1}{R_B}$$

$$S \supset -\int d^4x \int dy_A dy_B F_1 \wedge * F_1 \sim -\int d^4x \, (R_A R_B) \cdot \frac{1}{{R_B}^2} \sim -\int d^4x \frac{R_A}{R_B}$$

Type IIA moduli stabilisation

DGKT: only scale separated example in 4d

Type IIA orientifold with fluxes [Grimm, Louis '05]

Explicit example with $T^6/\mathbb{Z}_3^2$ [De Wolfe, Giryavets, Kachru, Taylor'05]

No complex structure moduli;
3 Kähler (& axions) + axiodilaton

$$\begin{cases} v_i, \phi \\ b_i, \chi \end{cases}$$

$$\mathcal{V} = \kappa\, v_1 v_2 v_3$$

Turning on F_0, H_3, F_4 stabilises all moduli

$$V = \frac{p^2}{4}\frac{e^{2\phi}}{\mathrm{vol}^2} + \frac{1}{2}\left(\sum_{i=1}^{3} {e_i}^2 v_i^2\right)\frac{e^{4\phi}}{\mathrm{vol}^3} + \frac{{m_0}^2}{2}\frac{e^{4\phi}}{\mathrm{vol}} - \sqrt{2}\,|m_0 p|\,\frac{e^{3\phi}}{\mathrm{vol}^{3/2}}$$

A first surprise

All moduli acquire universal and integer conformal dimensions

[Conlon, Ning, FR '21]

$\Delta_1 = 10$	$\Delta_{2,3,4} = 6$	SUSY
$\Delta_1 = 11$	$\Delta_{2,3,4} = 5$	$\mathrm{sgn}(m_0 e_1 e_2 e_3) > 0$
$\Delta_1 = 10$	$\Delta_{2,3,4} = 6$	non SUSY
$\Delta_1 = 1 \text{ or } 2$	$\Delta_{2,3,4} = 8$	$\mathrm{sgn}(m_0 e_1 e_2 e_3) < 0$

?

$$\Delta(\Delta - d) = m^2 R_{\mathrm{AdS}}^2$$

No dependence on fluxes or other parameters

Dual CFT at very specific points of parameter space, and independent of compactification details!

The surprise continues

Same numbers for many different toroidal orbifolds

[Apers, Montero, Van Riet, Wrase '22]

Consider a generic CY:
$h^{1,1}$ Kahler moduli and $h^{2,1}$ Complex Structure Moduli

[Apers,Conlon,Ning,FR '22]

$$\Delta_1 = 10 \qquad \Delta_{2\ldots h^{1,1}} = 6$$

$$\Delta_1 = 11 \qquad \Delta_{2\ldots h^{1,1}} = 5$$

$$\Delta_{1\ldots h^{2,1}} = 2$$

$$\Delta_{1\ldots h^{2,1}} = 3$$

Same for all SUSY vacua

Extension to non-SUSY vacua [Quirant '22]

A short proof (technical...)

[Apers,Conlon,Ning,FR '22]

$$\mathcal{L}_{\rm kin} \supset K_{i\bar{j}} \partial_\mu T^i \partial^\mu T^{\bar{j}}$$

$$V = e^K \left(\sum_{t_i, U_\alpha, S} K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right)$$

N=1 SUGRA

K, W

from dimensional reduction

[Grimm,Louis '05,+...]

[De Wolfe, Giryavets, Kachru, Taylor'05]

$$D_{t_i} W = 0 \quad D_S W = 0 \quad D_{U_\alpha} W = 0$$

$$V_{\rm min} = -\frac{3M_P^2}{R_{AdS}^2}$$

$$m_{ab}^2 R_{AdS}^2 \sim \frac{\partial_a \partial_b V_{\rm min}}{V_{\rm min}} = \ldots$$

Integer dimensions!

[See also Marchesano, Quirant '19 + related work]

Only ingredients: no-scale relations for K and explicit form of W

$$K^{ab} K_a K_b = 3 \quad K^{ab} K_b = -v_a$$

Conclusions and outlook

CFT techniques promising tool to address AdS stringy EFTs

- Swampland properties related to CFT conditions
- universal, integer dimensions for type IIA (DGKT), for any CY

Open questions

- Integers very surprising - begging for an explanation
- Smells like N=2 SUSY… but no 10d picture
- Related to scale separation? [Suggestive that scale separated vacua may be dual to special CFTs]

Long term goal: invert the relationship to “navigate” the Swampland

Thank you for your attention!

Back-up slides

Holographic CFTs

Gravity dual is weakly coupled and amenable to perturbative analysis

$$\lambda = g^2 N \longrightarrow (R/\ell_S)^d >> 1$$

Expansion in large - N parameter or equivalent

Large gap in the spectrum $\Delta_{gap} >> 1$

Single trace primaries $\mathcal{O}_1, \mathcal{O}_2$ Dimension Δ_1, Δ_2

OPE: $\mathcal{O}_1 \times \mathcal{O}_2 \supset \mathbb{1}, \mathcal{O}_1, \ \mathcal{O}_2, \ [\mathcal{O}_1\mathcal{O}_2]_{n,l}$

Double trace operators: $[\mathcal{O}_1\mathcal{O}_2]_{n,l} \sim \mathcal{O}_1 \Box^n \partial_{\mu_1}\partial_{\mu_2}\ldots\partial_{\mu_l}\mathcal{O}_2$

$\Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,l)$ ⟶ anomalous dimension

The bootstrap

conformal blocks

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_1(x_2)\,\mathcal{O}_2(x_3)\,\mathcal{O}_2(x_4)\rangle = \sum_{\mathcal{O}} C_{11\mathcal{O}} C_{22\mathcal{O}} \frac{G_{\Delta,\ell}(u,v)}{|x_{12}|^{2\Delta_1}\,|x_{34}|^{2\Delta_2}}$$

OPE coefficients

Bootstrap Equation:

$$u^{\Delta}\left(1+\sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(v,u)\right) = v^{\Delta}\left(1+\sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(u,v)\right)$$

No constraints from crossing symmetry alone

[Heemskerk,Penedones,
Polchinski,Sully '05]

Solution to bootstrap equations ⟷ Quartic (derivative) vertices

CFT positivity bounds

Minimal twist operators dominate Lorentzian OPE on the light cone

$\tau = \Delta - \ell$

$$\frac{\tau^*_{\ell_3} - \tau^*_{\ell_1}}{\ell_3 - \ell_1} \leq \frac{\tau^*_{\ell_2} - \tau^*_{\ell_1}}{\ell_2 - \ell_1}$$

→ $\gamma(0,\ell)$ for identical operators

convex & negative for $\ell \geq \ell_c$

$\ell_c \geq 2$

Analytical bootstrap [Komargodski, Zhiboedov '12]

Inversion formula [Caron-Huot '17] + [Costa, Hansen, Penedones '17]

Causality arguments in the CFT:

$$\gamma(0,2) \leq 0$$

[Hartman, Jain, Kundu '16]

→ $\mathcal{L} = \frac{g}{\Lambda^4}(\nabla\varphi)^4 \quad g > 0$

on AdS

Generalization of flat space S-matrix bounds [Adams,Arkani-Hamed,Dubovsky, Nicolis,Rattazzi '06]

The Large Volume Scenario (I)

Motivating scenario: Low energy dynamics of moduli in ST

LVS: Type IIB flux compactification, with all moduli stabilised at an exponentially large volume

$$V = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right) \qquad \Phi = \sqrt{\frac{2}{3}}\ln\mathcal{V}$$

canonically normalised volume modulus

AdS vacuum:

$$V_{\rm min} = -3M_P^2 R_{AdS}^{-2} \qquad m_\Phi = \sqrt{\frac{3}{2}}\frac{\lambda}{R_{AdS}}$$

[Balasubramanian,Berglund,Conlon,Quevedo '05]

[Conlon,Quevedo,Suruliz '05]

The Large Volume Scenario (2)

"Big" and "small" Kähler moduli T_b, T_s

$$\mathcal{V} = \frac{1}{\kappa}\left(\tau_b^{3/2} - \tau_s^{3/2}\right)$$

$$K = -2\ln\left(\frac{1}{\kappa}\left(\left(\frac{T_b+\bar{T}_b}{2}\right)^{3/2} - \left(\frac{T_s+\bar{T}_s}{2}\right)^{3/2}\right) + \frac{\xi}{g_s^{3/2}}\right)$$

$$W = W_0 + A_s e^{-a_s T_s}$$

α'^3 correction

Minimum for an exponentially large volume $\langle\mathcal{V}\rangle \sim e^{a_s\langle\tau_s\rangle}$

$$V_{eff} = \frac{1}{\mathcal{V}^3}\left(-A(\ln\mathcal{V})^{3/2} + \frac{B}{g_s^{3/2}}\right) \qquad \langle\tau_s\rangle \sim \frac{\zeta^{2/3}}{g_s}$$

LVS Effective Lagrangian

All interactions fixed in term of R_{AdS} only-
uniquely determined theory in large $\mathcal{V}$ limit

$$\mathcal{L}_{(\delta\Phi)^n} = (-\lambda)^n \frac{3M_P^2}{R_{AdS}^2} \frac{n-1}{n!} \left(\frac{\delta\Phi}{M_P}\right)^n \left(1 + \mathcal{O}\left(\frac{1}{\lambda\langle\Phi\rangle}\right)\right)$$

$$\mathcal{L}_{(\delta\Phi)^{n-2}aa} = \left(+\sqrt{\frac{8}{3}}\right)^{(n-2)} \frac{1}{2(n-2)!} \left(\frac{\delta\Phi}{M_P}\right)^{n-2} \partial_\mu a \partial^\mu a$$

Some modifications certainly in the Swampland

Sign flip in axion kinetic term → Divergent f_a as $\mathcal{V} \to \infty$

$$\mathcal{L} \supset \frac{3}{4} e^{+\sqrt{\frac{8}{3}}\frac{\Phi}{M_P}} \partial_\mu a \partial^\mu a$$

What happens to the CFT?

Holographic reformulation of LVS

AdS/CFT

4d moduli EFT in AdS

Crucial property of LVS:

$m_\phi << m_{3/2}$ as $\mathcal{V} \to \infty$

+

Putative 3d CFT dual

$$m^2 R^2_{AdS} = \Delta(\Delta - 3)$$

Small number of low lying operators:

Mode	Spin	Parity	Conformal dimension
$T_{\mu\nu}$	2	+	3
a	0	-	3
Φ	0	+	$8.038 = \frac{3}{2}\left(1+\sqrt{19}\right)$

[Conlon,Quevedo '18]

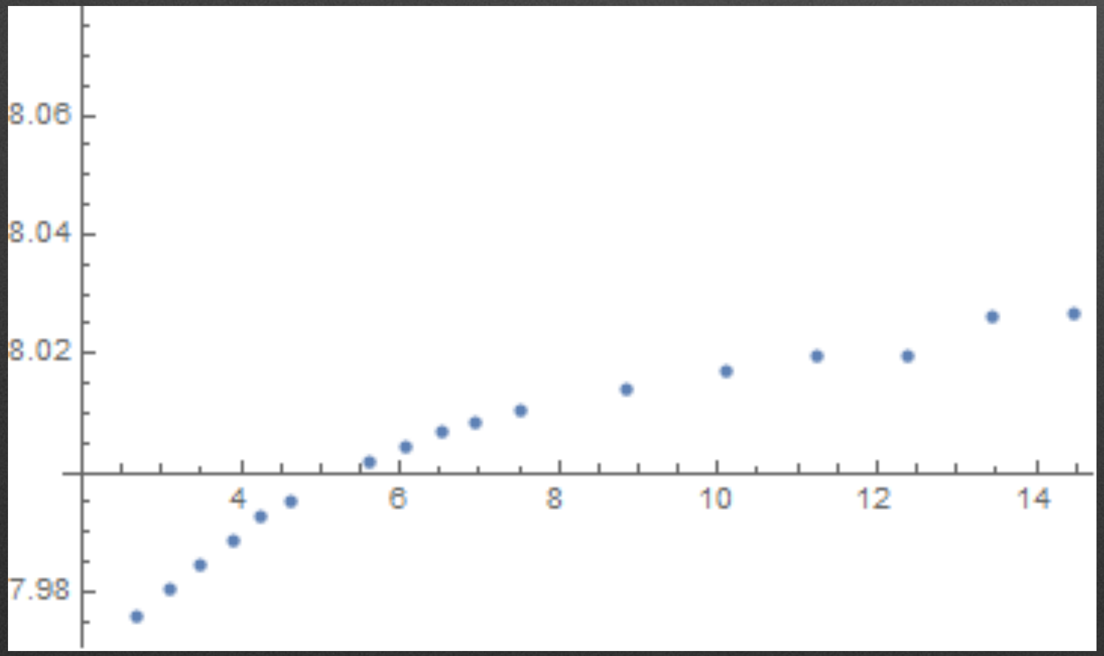
LVS at finite volume

At small volumes, Δ_φ can in principle drop below 8

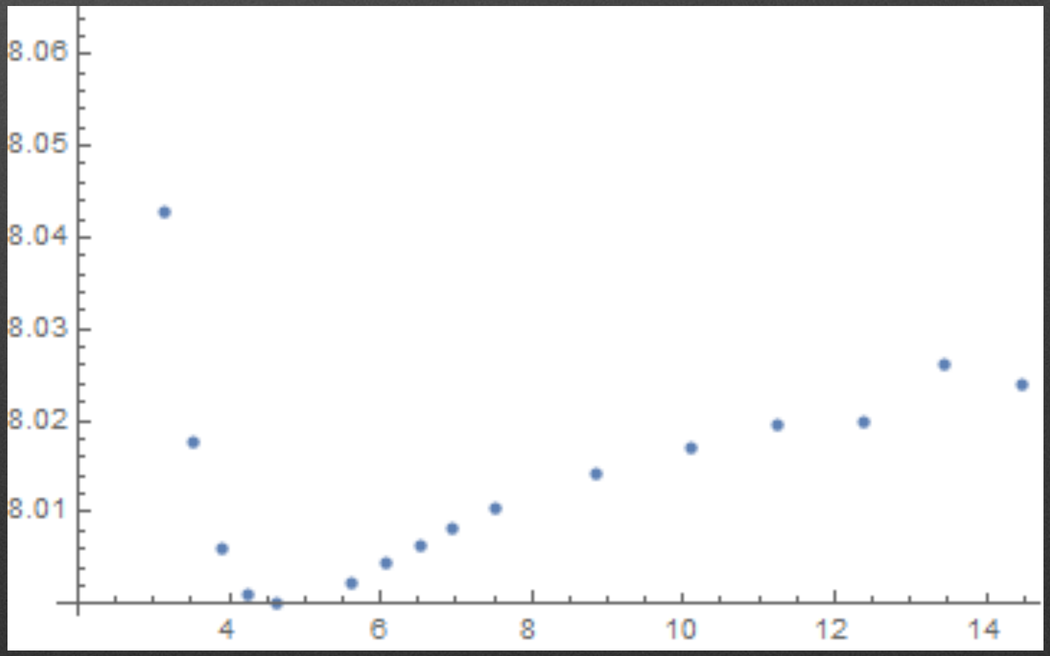
$$\Delta = \frac{3(1+\sqrt{19})}{2}\left(1 - \sqrt{\frac{2}{27}}\frac{1}{\langle\Phi\rangle} + \mathcal{O}\left(\frac{1}{\langle\Phi\rangle}\right)^2\right)$$

However, subleading volume corrections become important

$$K = -2\ln(\mathcal{V}+\xi) \rightarrow K = -2\ln\left(\mathcal{V}+\xi+\frac{\xi^2}{\mathcal{V}}\right)$$



Pure LVS



With corrections

KKLT

Well known example of dS vacuum - we consider it before uplifting

$$\mathcal{K} = -3\log(-i(T-\bar{T})) \qquad \mathcal{W} = W_0 + Ae^{-\alpha T}$$

[Kachru,Kallosh, Linde,Trivedi '03]

SUSY vacuum

$$\frac{W_0}{A} = -e^{-a\sigma_c}(1+\frac{2}{3}a\sigma_c)$$

$\sigma_c a >> 1$

$\sigma >> 1$

Potential for a ⟶ $\Delta_a > 3$

$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma\partial_\mu a \partial^\mu a$

} qualitatively different

Negative anomalous dimensions in its regime of validity

$$\gamma^{\sigma a}(0,\ell) \;\; \propto -\Big[a\sigma_c(a\sigma_c+2)(2a\sigma_c+3)$$
$$+2\Delta_\varphi(a\sigma_c) - 2\Delta_a^2(a\sigma_c) + 4\Delta_a(a\sigma_c) - 6\Big]$$