Mixed QCD-EW corrections to vector-boson production

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Based on: 1909.08428, 2005.10221, 2009.10386, 2103.02671

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ISSP at EMFCSC
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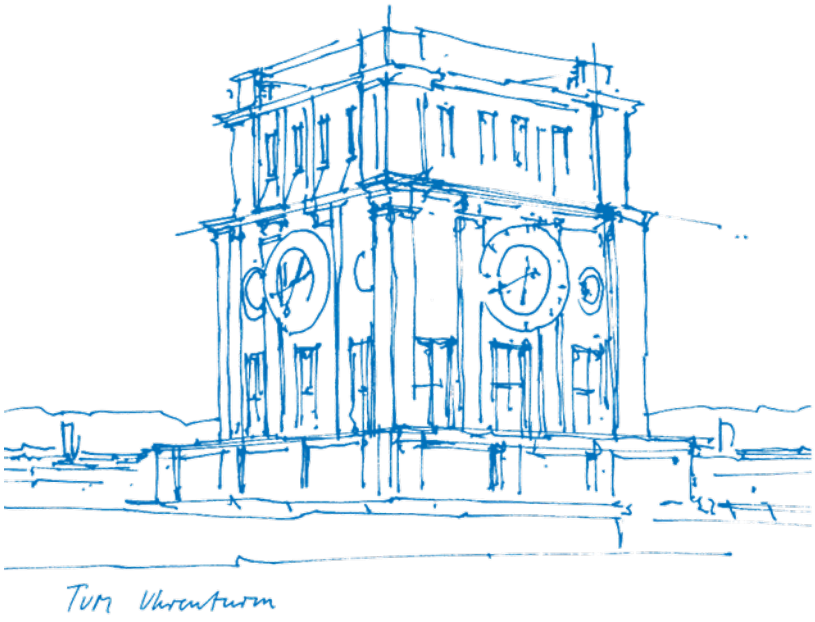


QCD computations have reached N3LO

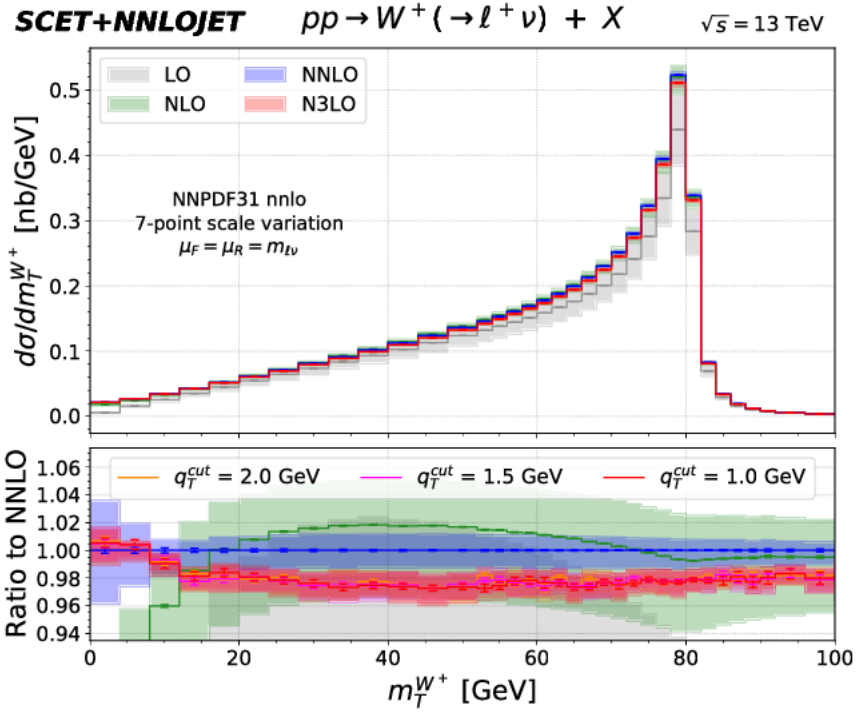
Fully-differential description of color-singlet production at the LHC has recently reached N3LO in perturbative QCD.

⇒ Higgs production [Chen et al. '21]

⇒ Drell-Yan [Chen et al. '22]

Parametrically, $\alpha_s^3 \sim \alpha_s \alpha$.
$\alpha_s(M_Z) \sim 0.1\ ,\ \alpha \sim 1/100$

These computations are vital for precise studies at the LHC.



[Chen et al. '22]

M_W as a precision test

electroweak fits

$$M_W^2(1 - M_W^2/M_Z^2) = \frac{\pi\alpha}{\sqrt{2}G_F}(1 + \Delta r)$$

$M_W = 80354 \pm 7$ MeV [Gfitter'18]

$M_W = 80359 \pm 5$ MeV [HEPfit'21]

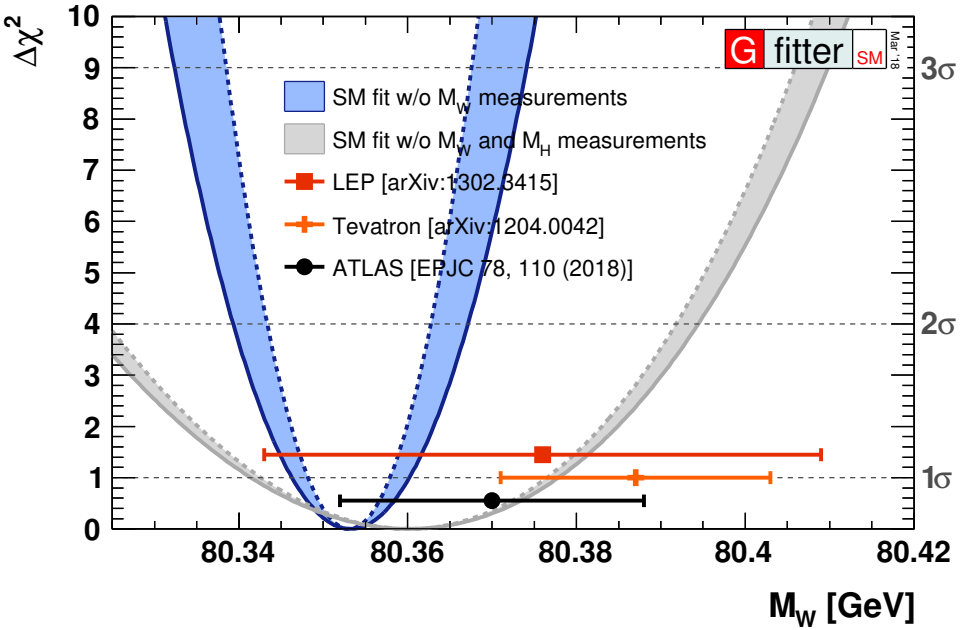
recent direct measurements

[ATLAS '17] : $80370 \pm 7_{\text{stat}} \pm 11_{\text{sys}} \pm 14_{\text{mod}}$ MeV

[LHCb '21] : $80354 \pm 23_{\text{stat}} \pm 10_{\text{exp}} \pm 17_{\text{th}} \pm 9_{\text{pdf}}$ MeV

[CDF '22] : $80433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{sys}}$ MeV

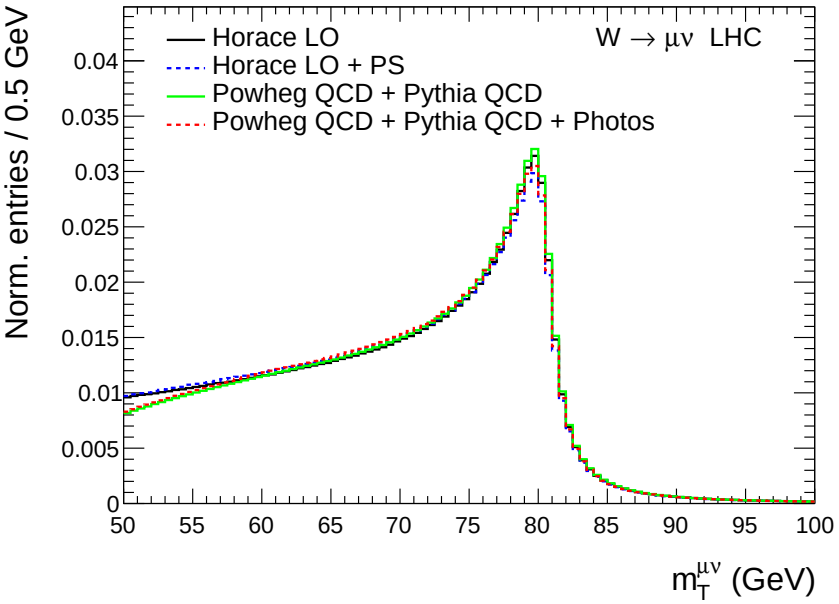
comparison

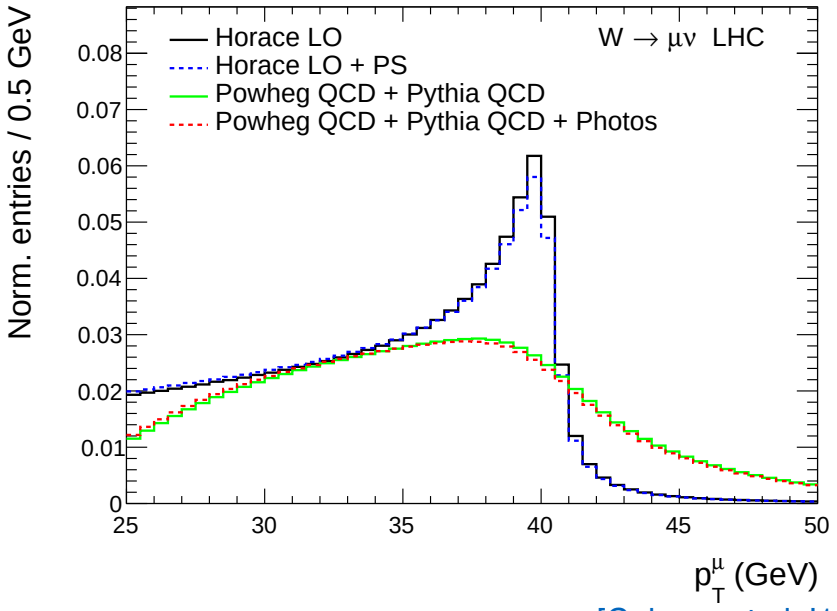


Measurement of M_W (at hadron colliders)

- through leptonic decay $pp \to W^* \to \ell\bar{\nu}$
- partonic center-of-mass energy $\hat{s}$ not known, neutrino momentum p_ν cannot be measured

  $\Rightarrow$ measurements use shape of distributions in $p_\perp^\ell$ and $m_\perp^W$





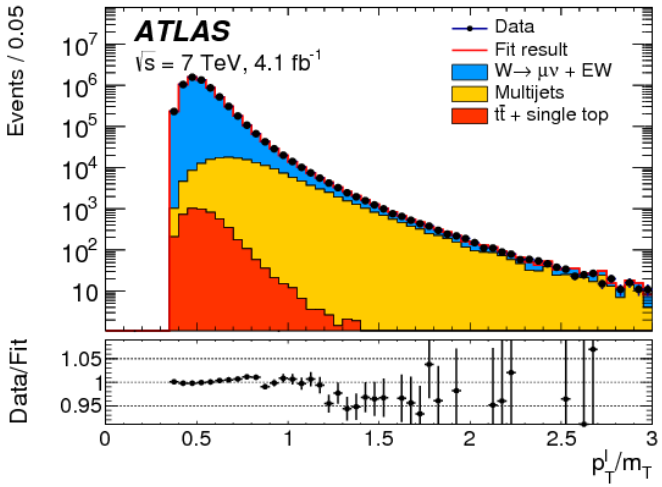
[Calame et al. '16]

- $m_\perp^W = \sqrt{2p_\perp^\ell p_\perp^{\text{miss}}(1-\cos\phi_{\ell,\text{miss}})} \leq M_W$
- W-boson width, pile-up
- see [Smith,van Neerven,Vermaseren '83]

- at LO: $p_\perp^\ell \leq m_W/2$
- perturbative corrections

Measurement of M_W (at hadron colliders)

template fit



$$8/80'000 = 0.01\%$$

collinear factorization [Collins,Soper,Sterman '88]

$$\sigma = \sum_{i,j} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_i(x_1) f_j(x_2)}_{\mathcal{O}(1\%)} \overbrace{\sigma_{ij}(x_1, x_2)}^{\mathcal{O}(1\%)} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

$\Rightarrow p_\perp^W$ spectrum can not be predicted to required precision.

- experimental analyses use Z-boson data to predict $p_\perp^W$ distribution
- hence, measurement is sensitive to effects that distinguish between W and Z
- these include PDFs, massive-quark effects, NLO EW and NNLO QCD-EW corrections

Vector-boson production at $\mathcal{O}(\alpha_s\alpha)$

- $p_\perp^W/p_\perp^Z$ distribution is dominated by the on-shell production of vector bosons

$$\mathrm{d}\sigma\Big|_{\alpha_s\alpha} = \Big| \underbrace{\alpha_s}_{\text{production}} \otimes \underbrace{\alpha}_{\text{decay}} \Big|^2 + \boxed{\Big| \underbrace{\alpha_{s\alpha}}_{\text{production}} \otimes \Big|^2} + \mathcal{O}\left(\frac{\Gamma_V}{M_V}\right)$$

[Dittmaier et al. '14 '16] [Behring et al. '19 '20]

- full, off-shell process $pp \to \ell\bar{\ell}$ was completed recently [Bonciani et al. '21] [Buccioni et al. '22]

Infrared divergences at NLO

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\mathbf{O}}\right|_{\alpha_s} = 2\,\Re\Bigg[\underbrace{\;\;}_{\sim 1/\varepsilon^2} \times \Bigg]^{\dagger} \mathscr{F}_{\mathbf{O}}\,\mathrm{d}\Phi_X + \Bigg|\;\;\Bigg|^2 \mathscr{F}^{(1)}_{\mathbf{O}}\,\mathrm{d}\Phi_{X+g} + \mathrm{d}\sigma^{gq} + \mathrm{d}\sigma^{\mathrm{pdf}}$$

- cross-section is *finite* for arbitrary “infrared-safe” observable $\mathscr{F}$

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Infrared divergences at NLO

$$\frac{1}{p_1 \cdot k_4} \sim \frac{1}{E_1 E_4 (1-\cos\theta_{14})}$$

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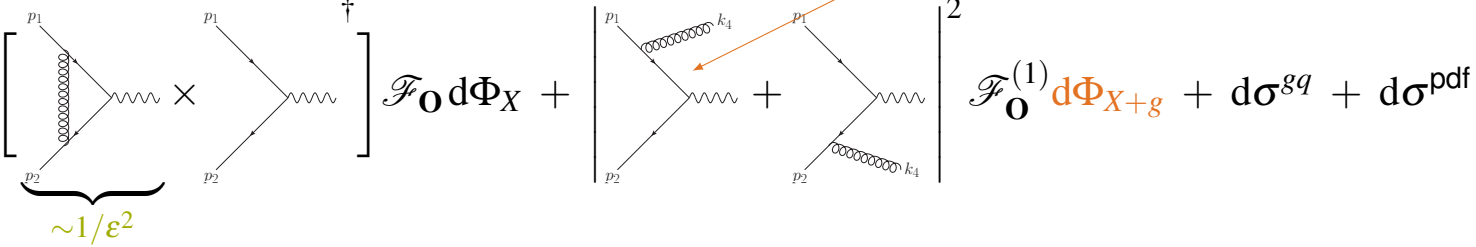

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slicing / subtraction

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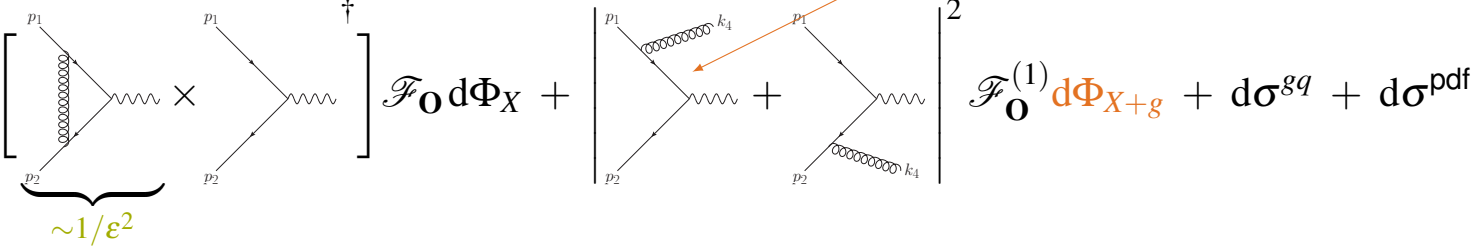

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- need to be regulated, extracted and cancelled
- only then can the limit $\varepsilon \to 0$ be taken prior to numerical simulation

$\Rightarrow$ *slicing* and *subtraction* schemes
are used to define contributions that are individually finite

slicing / subtraction

$$\mathrm{d}\sigma\Big|_{\alpha_s} = \mathrm{d}\sigma_{X+1} + \mathrm{d}\sigma_X$$

Infrared subtraction at NLO

- general idea is to subtract and add back matrix elements that approximate singular behavior

$$\mathrm{d}\sigma^{q\bar{q}}(d=4-2\varepsilon) = \underbrace{(\mathrm{d}\sigma^{r}-\mathrm{d}\widetilde{\sigma}^{r})}_{\text{regulated term}}\Big|_{d=4} + \underbrace{\mathrm{d}\widetilde{\sigma}^{r}(\varepsilon)}_{\text{subtraction term}}$$

- pole cancellation

$$\lim_{\varepsilon\to 0}\left[\mathrm{d}\sigma^{\mathrm{pdf}}+\mathrm{d}\sigma^{\mathrm{v}}(\varepsilon)+\mathrm{d}\widetilde{\sigma}^{\mathrm{r}}(\varepsilon)\right] = \text{finite}$$

- example for soft-regulated emission of a gluon

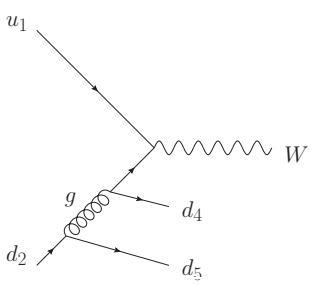
$$\mathrm{d}\sigma^{q\bar{q}} = \Bigg\{ \Bigg| \; \begin{array}{l} p_1 \\ \\ p_2 \end{array} \; \overset{k_4}{\sim\!\sim} \; \Bigg| + \Bigg| \; \begin{array}{l} p_1 \\ \\ p_2 \end{array} \; \overset{k_4}{\sim\!\sim} \; \Bigg|^2 - \frac{2g_s^2 C_F\,(p_1\cdot p_2)}{(p_1\cdot k_4)(p_2\cdot k_4)} \Bigg| \; \begin{array}{l} p_1 \\ \\ p_2 \end{array} \; \sim\!\sim \; \Bigg|^2 \Bigg\} \mathrm{d}\Phi_{V+g}$$

$$+ \underbrace{\left(\int [\mathrm{d}k_4]\frac{2g_s^2 C_F\,(p_1\cdot p_2)}{(p_1\cdot k_4)(p_2\cdot k_4)}\right)}_{\sim 1/\varepsilon^2} \times \Bigg(\Bigg| \; \begin{array}{l} p_1 \\ \\ p_2 \end{array} \; \sim\!\sim \; \Bigg|^2 \mathrm{d}\Phi_V \Bigg)$$

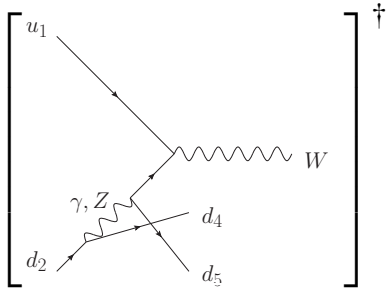
- NB similar formula for remaining collinear divergence

Quark emission at NNLO QCD-EW

$$\left|\mathscr{A}_{ud\to W^+dd}\right|^2\Big|_{\mathscr{O}(\alpha_s\alpha)} \supset$$



$\times$



$\dagger$

- only triple-collinear divergence $p_2 \| k_4 \| k_5$ due to continuous quark line

$$\begin{aligned} \mathrm{d}\sigma^{\mathrm{rr}}_{ud\to W^+dd} = &\left\langle [\mathrm{d}k_4][\mathrm{d}k_5]\,\left(I-\hat{\mathscr{C}}_2\right) F_{\mathrm{LM}}\left(1_u,2_d,W^+;4_d,5_d\right)\right\rangle && \leftarrow \text{ fully regulated} \\ &+\left\langle [\mathrm{d}k_4][\mathrm{d}k_5]\hat{\mathscr{C}}_2\, F_{\mathrm{LM}}\left(1_u,2_d,W^+;4_d,5_d\right)\right\rangle && \leftarrow \text{ subtraction term} \end{aligned}$$

- integrated triple-collinear subtraction terms were computed in the context of NNLO QCD corrections [MD,Melnikov '19], results can be re-used here

$$\begin{aligned} &\left\langle [\mathrm{d}k_4][\mathrm{d}k_5]\mathscr{C}_2\, F_{\mathrm{LM}}\left(1_u,2_d,W^+;4_d,5_d\right)\right\rangle \\ &\sim \alpha_s\,\alpha \times \int_0^1 \mathrm{d}z \left[\underbrace{\frac{f_1(z)}{\varepsilon}}_{\text{pole cancellation}} + \underbrace{f_0(z)}_{\text{physical XS}}\right] \left\langle \frac{C_F Q_d^2\, F_{\mathrm{LM}}(1_u, z\cdot 2_{\bar{d}})}{z}\right\rangle, \quad z=\frac{E_2-E_4-E_5}{E_2} \end{aligned}$$

Impact on the M_W measurement at the LHC

- goal: construct "simple" observable for M_W^{exp} that makes use of both the $p_{\ell,Z}^{\perp}$ distribution and the precisely measured Z-boson mass

$\rightarrow$ define normalized average momentum as

$$\left\langle p_{\ell,V}^{\perp}\, \theta\left[p_{\ell,V}^{\perp} - p_{\text{cut}}^{\perp}\right]\right\rangle = \frac{\int \theta^{\text{cut}}\, p_{\ell,V}^{\perp} \times \frac{\mathrm{d}\sigma_V}{\mathrm{d}p_{\ell,V}^{\perp}}\, \mathrm{d}p_{\ell,V}^{\perp}}{\int \theta^{\text{cut}}\, \mathrm{d}\sigma_V} = M_V \times f\left(\frac{p_{\text{cut}}^{\perp}}{M_V}\right)$$

$\rightarrow$ define observable M_W^{exp} as

$$M_W^{\text{exp}} = \frac{\left\langle p_{\ell,W}^{\perp}\right\rangle^{\text{exp}}}{\left\langle p_{\ell,Z}^{\perp}\right\rangle^{\text{exp}}} M_Z\, C_{\text{th}}\,, \quad C_{\text{th}} = \frac{M_W}{M_Z} \frac{\left\langle p_{\ell,Z}^{\perp}\right\rangle^{\text{th}}}{\left\langle p_{\ell,W}^{\perp}\right\rangle^{\text{th}}}$$

Impact on the M_W measurement at the LHC

- updated theoretical description shifts extracted value of M_W

$$\delta M_W^{\text{exp}} = \left[\frac{\delta \left\langle p_{\ell,Z}^{\perp} \right\rangle^{\text{th}}}{\left\langle p_{\ell,Z}^{\perp} \right\rangle^{\text{th}}} - \frac{\delta \left\langle p_{\ell,W}^{\perp} \right\rangle^{\text{th}}}{\left\langle p_{\ell,W}^{\perp} \right\rangle^{\text{th}}} \right] M_W^{\text{exp}}$$

	inclusive	ATLAS cuts	tuned cuts
NLO EW	1 MeV	3 MeV	-3 MeV
NNLO QCD-EW	-7 MeV	-17 MeV	-1 MeV

$$\delta \left\langle p_{\ell,W}^{\perp} \right\rangle^{\text{th}} \Big/ \left\langle p_{\ell,W}^{\perp} \right\rangle^{\text{th}} \times M_W^{\text{exp}} \sim \mathcal{O}(30-50) \text{ MeV}$$

$\Rightarrow$ large cancellation by one order of magnitude

$\Rightarrow$ sensitive to cuts

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ATLAS $p_{\ell,V}^{\perp}$-cuts:

$\rightarrow p_{\text{cut},Z}^{\perp} = 25$ GeV

$\rightarrow p_{\text{cut},W}^{\perp} = 30$ GeV

$\Rightarrow$ moves average momentum in decays of the lighter W boson towards higher values

Impact on the M_W measurement at the LHC

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find tuned value for $p_{\text{cut},W}^{\perp}$ such that $C_{\text{th}}^{\text{LO}} = 1$ with $p_{\text{cut},Z}^{\perp} = 25$ GeV

$$\Rightarrow p_{\text{cut},W}^{\perp} = 25.44 \text{ GeV} \left(\text{instead of } p_{\text{cut},W}^{\perp} = 30 \text{ GeV} \right)$$

Conclusion

- fully-differential description of QCD-EW corrections to on-shell vector-boson production
  - regularisation of infrared singularities
  - analytic computation of integrated subtraction terms
  - (on-shell form factor $pp \to W$, including two-loop master integrals with two internal masses)

$\Rightarrow$ QCD-EW corrections were estimated
to shift the measurement of M_W by up to $-\mathbf{17}$ MeV

Outlook

$\Rightarrow$ this result warrants further studies,
which incorporate all relevant details of experimental analyses

Backup

Breakdown LHCb uncertainties

Source	Size [MeV]
Parton distribution functions	9
Theory (excl. PDFs) total	17
Transverse momentum model	11
Angular coefficients	10
QED FSR model	7
Additional electroweak corrections	5
Experimental total	10
Momentum scale and resolution modelling	7
Muon ID, trigger and tracking efficiency	6
Isolation efficiency	4
QCD background	2
Statistical	23
Total	32

[LHCb '21]

Breakdown CDF uncertainties

Table 2. Uncertainties on the combined M_W result.

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

[CDF '22]