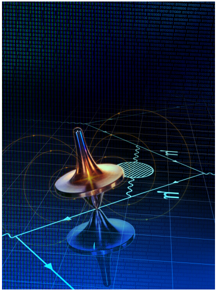
The challenge of $g-2$

Laurent Lellouch
CNRS & Aix-Marseille U.



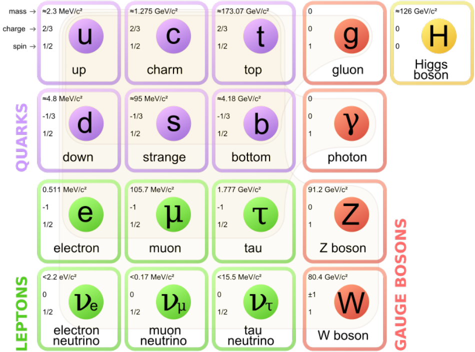
Muon $g-2$ collab., PRL 126 (2021) 141801 (Featured in Physics) → FNAL '21
Aoyama et al., Phys. Rep. 887 (2020) 1-166 → WP '20
BMW collab., Nature 593 (2021) 51, online 7 April 2021 → BMWc '20
BMW collab., PRL 121 (2018) 022002 (Editors' Suggestion) → BMWc '17

Introduction and motivation

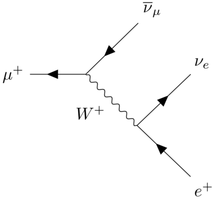
The place of the muon in the Standard Model

The muon (μ) is a cousin of the electron (e):

- both are charged leptons
  - $\rightarrow$ they don't feel the strong interaction
  - $\rightarrow$ same spin $= \frac{1}{2}$
  - $\rightarrow$ same electric charge $= -e$
  - $\rightarrow$ same electromagnetic and weak interactions
- BUT muon couples 207 times more strongly to the Higgs
  - $\rightarrow$ 207 times more massive
  - $\rightarrow$ 2 μs lifetime



(Wikimedia)



Interaction with an external EM field: QM

Dirac eq. w/ minimal coupling (1928):

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\vec{\alpha}\cdot\left(c\frac{\hbar}{i}\vec{\nabla} - e_\ell\vec{A}\right) + \beta c^2 m_\ell + e_\ell A_0\right]\psi$$

nonrelativistic limit $\downarrow$ (Pauli eq.)

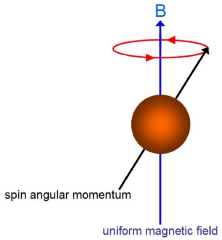
$$i\hbar\frac{\partial\phi}{\partial t} = \left[\frac{\left(\frac{\hbar}{i}\vec{\nabla} - \frac{e_\ell}{c}\vec{A}\right)^2}{2m_\ell} - \underbrace{\frac{e_\ell\hbar}{2m_\ell}\vec{\sigma}\cdot\vec{B}}_{\vec{\mu}_\ell\cdot\vec{B}} + e_\ell A_0\right]\phi$$

with

$$\vec{\mu}_\ell = g_\ell\left(\frac{e_\ell}{2m_\ell}\right)\vec{S}, \qquad \vec{S} = \hbar\frac{\vec{\sigma}}{2}$$

and

$$g_\ell\big|_{\text{Dirac}} = 2$$



"That was really an unexpected bonus for me, completely unexpected." (P.A.M. Dirac)

Interaction with an external EM field: RQFT

Assuming Poincaré invariance and current conservation ($q^\mu J_\mu = 0$ with $q \equiv p' - p$):

$$\langle \ell(p')|J_\mu(0)|\ell(p)\rangle = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i}{2m_\ell}\sigma_{\mu\nu}q^\nu F_2(q^2) - \gamma_5\sigma_{\mu\nu}q^\nu F_3(q^2)\right.$$

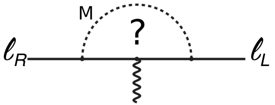
$$\left. +\gamma_5(q^2\gamma_\mu - 2m_\ell q_\mu)F_4(q^2)\right] u(p)$$

$F_1(q^2)$	$\rightarrow$	Dirac form factor: $F_1(0) = 1$
$F_2(q^2)$	$\rightarrow$	Pauli form factor, magnetic dipole moment: $F_2(0) = a_\ell = \dfrac{g_\ell - \overbrace{2}^{g_\ell\vert_{\text{Dirac}}}}{2}$
$F_3(q^2)$	$\rightarrow$	$\not{P}$, $\not{T}$, electric dipole moment: $F_3(0) = d_\ell/e_\ell$
$F_4(q^2)$	$\rightarrow$	$\not{P}$, anapole moment: $\vec{\sigma}\cdot(\vec{\nabla}\times\vec{B})$

- q^2 dependence of $F_1(q^2)$ and non-zero $F_2(q^2)$ & $F_{3,4}(q^2)$ come from loops
- nevertheless UV finite once lagrangian parameters are renormalized (in a renormalizable theory)

  $\rightarrow$ parameter-free predictions

Why are a_ℓ special?



$\longrightarrow \qquad \mathcal{L}_{\rm eff} = -\frac{Qe}{2}\frac{a_\ell}{2m_\ell}F^{\mu\nu}[\bar\ell_L\sigma_{\mu\nu}\ell_R] + \text{hc}$

- $a_{e,\mu}$ are parameter-free predictions of the SM that can be calculated & measured very precisely $\Rightarrow$ excellent tests of SM
- Loop induced $\Rightarrow$ sensitive to new dofs that may be too heavy or too weakly coupled to be produced directly
- Flavor and CP conserving, chirality flipping $\Rightarrow$ complementary to: EDMs, FCNCs (e.g. s and b decays), LHC direct searches, ...
- Chirality flipping $\Rightarrow$ generic contribution of particle w/ $M \gg m_\ell$

$$a_\ell^{\rm M} = C\left(\frac{\Delta_{LR}}{m_\ell}\right)\left(\frac{m_\ell}{M}\right)^2$$

- In EW theory, $M = M_W$, chirality flipping from Yukawa, i.e.

$$\Delta_{LR} = m_\ell \qquad \text{and} \qquad C \sim \frac{\alpha}{4\pi\sin^2\theta_W}$$

- In BSM, can have chiral enhancement: e.g. SUSY $M = M_{\rm SUSY}$ and $C \sim \alpha/\left(4\pi\sin^2\theta_W\right)$ & $\Delta_{LR} = \left(\mu/M_{\rm SUSY}\right) \times \tan\beta \times m_\ell$; or radiative m_ℓ model, $\Delta_{LR} \simeq m_\ell$, $C \sim 1$ and $M = M_{\rm N\Phi}$

Why is a_μ special?

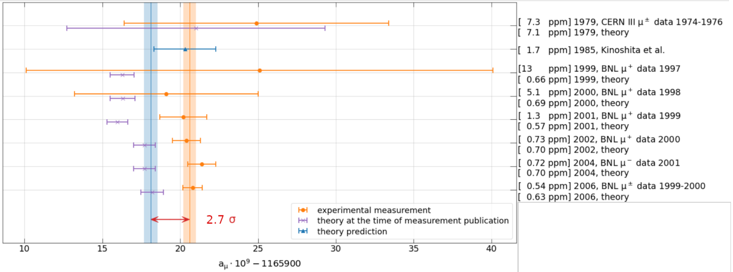
$m_e : m_\mu : m_\tau = 0.0005 : 0.106 : 1.777\,\mathrm{GeV} \qquad \tau_e : \tau_\mu : \tau_\tau = \text{“}\infty\text{”} : 2.{\cdot}10^{-6} : 3.{\cdot}10^{-15}\,\mathrm{s}$

- a_μ typically $(m_\mu/m_e)^2 \sim 4. \times 10^4$ times more sensitive to new Φ than a_e
- a_τ is even more sensitive to new Φ, but is too shortly lived
- τ_μ small but manageable

Brief history of a_μ

- 1956 : Berestetskii notes that sensitivity of a_ℓ to contributions of heavy particles w/ $M \gg m_\ell$ typically goes like $\sim (m_\ell/M)^2$
- 1960 : despite $\tau_\mu \sim 2\,\mu$s, Garwin et al manage to measure $g_\mu \simeq 2$
- $>$ 1960 : measurement of a_μ progressed in // with the development of the SM

[Adapted from G. Venanzioni. Bands not relevant here. 2.7σ is between 2006 experiment and theory]



- 2006 : 2.7σ discrepancy was **too small** to claim new physics, but **too large** to ignore

Muon: recent history

To decide on possible presence of new fundamental physics:

Improve the measurement

Move BNL apparatus to FNAL & significantly ugprade experiment to reduce measurement error by factor of 4

$\Rightarrow$ presentation & publication on April 7 2021 of first results (only 6% of planned data) [Abi et al, PRL 126 (2021)]

$\longrightarrow$ tour de force measurement w/ already improved precision over BNL result

Improve the SM prediction

Important theoretical effort to improve SM prediction to comparable level of precision

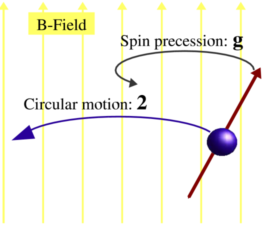
$\Rightarrow$ White Paper from the muon $g-2$ Theory Initiative w/ reference SM prediction [Aoyama et al '20 = WP '20]

$\Rightarrow$ Several onging, *ab initio* supercomputer calculations of nonlinear corrections from quarks and gluons that give leading contribution to error in SM prediction

$$a_\mu^{\rm exp} = a_\mu^{\rm SM} \ ?$$

Experimental measurement of a_μ

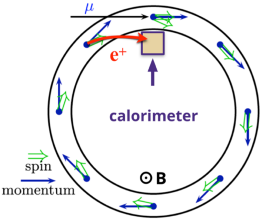
Measurement principle for a_μ



Precession determined by

$$\vec{\mu}_\mu = 2(1+a_\mu)\frac{Qe}{2m_\mu}\vec{S}$$

$$\vec{d}_\mu = \eta_\mu \frac{Qe}{2m_\mu}\vec{S}$$



$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta \simeq -\frac{Qe}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right] - \eta_\mu \frac{Qe}{2m_\mu}\left[\vec{E} + \vec{\beta}\times\vec{B}\right]$$

- Experiment measures very precisely $\vec{B}$ with $|\vec{B}| \gg |\vec{E}|$ &

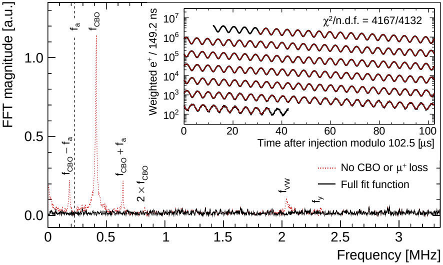
$$\Delta\omega \equiv \omega_S - \omega_C \simeq \sqrt{\omega_a^2 + \omega_\eta^2} \simeq \omega_a$$

since $d_\mu = 0.1(9)\times 10^{-19} e\cdot\mathrm{cm}$ (Benett et al '09)

- Consider either magic $\gamma = 29.3$ (CERN/BNL/Fermilab) or $\vec{E}=0$ (J-PARC)

$$\to \Delta\omega \simeq a_\mu B \frac{e}{m_\mu}$$

Fermilab E989 @ magic γ: Run 1 measurement



More high-E e^+ emitted along μ^+ spin axis (PV)
→ select e^+ with $E \geq E_{th}$
→ get ω_a from decay e^+ time counts

$$N_{e^+}(t) = N_0\eta_N(t)e^{-t/\gamma\tau_\mu}\,[1 + A\eta_A(t)\times \cos\left(\omega_a t + \phi(t) + \eta_\phi(t)\right)]$$

where the η_i account for beam oscillations and μ^+ loss

Using $B = \omega_p/(2\mu_p)$ and $e = 4m_e\mu_e/g_e$, rewrite

$$a_\mu = \frac{\bar{\omega}_a}{\bar{\omega}_p(T_r)}\,\frac{\mu_p(T_r)}{\mu_e(H)}\,\frac{\mu_e(H)}{\mu_e}\,\frac{m_\mu}{m_e}\,\frac{g_e}{2}$$

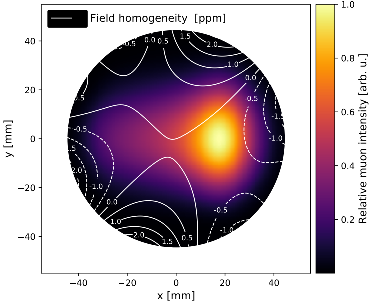
w/ clock blinding (± 25 ppm) and

$$\bar{\omega}_a = f_{clock}\,\omega_a(1 + C_{beam}) \quad \& \quad \bar{\omega}_p = f_{calib.}\langle\omega_p\rangle(1 + B_{B\text{-field}})$$

where $C_{beam} = 4$ beam-dynamics corrections & $B_{B\text{-field}} = 2$ transient magnetic field corrections

All other quantities measured or calculated very precisely, to 25 ppb precision

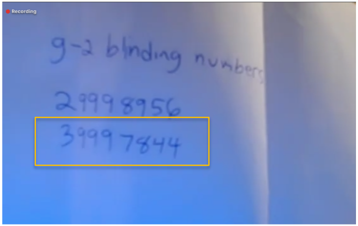
$$\langle\omega_p\rangle \equiv \langle\omega_p(x, y, \phi)M(x, y, \phi)\rangle$$



a_μ: Unblinding

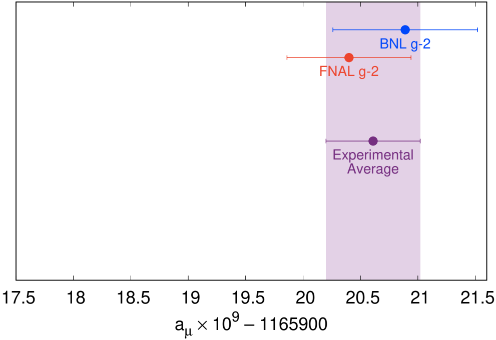
On February 25 the collaboration met for the unblinding:

1) The box was opened
2) The number was plugged in two independent programs
3) And the result was....



Secret offset

a_μ: present experimental status



$$a_\mu = 11\,659\,206.1\,(4.1) \times 10^{-10} \qquad [0.35\,\text{ppm}]$$

Bathroom scale sensitive to the weight of a single eyelash !!!

Based on only 6% of expected FNAL data! $\rightarrow$ aim $\delta a_\mu = 0.14\,\text{ppm}$

Reference standard model calculation of a_μ

[Aoyama et al '20 = WP '20]

At needed precision: all three interactions and all SM particles

$$\begin{aligned} a_\mu^{\text{SM}} &= a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{EW}} \\ &= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{e}{4\pi \sin\theta_W}\right)^2 \left(\frac{m_\mu}{M_W}\right)^2\right) \\ &= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right) \end{aligned}$$

QED contributions to a_ℓ

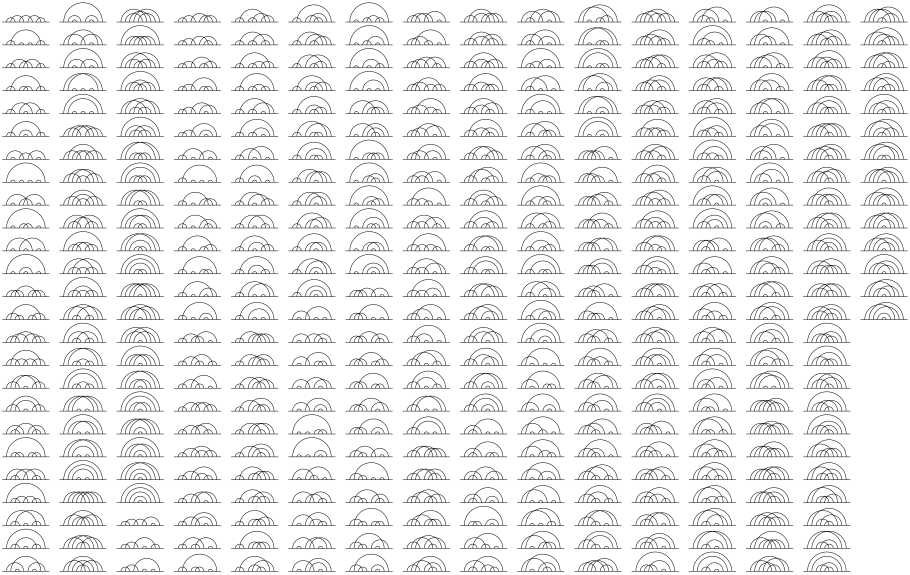
Loops with only photons and leptons: can expand in $\alpha = e^2/(4\pi) \ll 1$

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

- $A_1^{(2)}, A_1^{(4)}, A_1^{(6)}, A_2^{(4)}, A_2^{(6)}, A_3^{(6)}$ known analytically (Schwinger '48; Sommerfield '57, '58; Petermann '57; ...)
- $O((\alpha/\pi)^3)$: 72 diagrams (Laporta et al '91, '93, '95, '96; Kinoshita '95)
- $O((\alpha/\pi)^4; (\alpha/\pi)^5)$: 891;12,672 diagrams (Laporta '95; Aguilar et al '08; Aoyama, Kinoshita, Nio '96-'18)
  - Automated generation of diagrams
  - Numerical evaluation of loop integrals
  - Not all contributions are fully, independently checked

5-loop QED diagrams



(Aoyama et al '15)

QED contribution to a_μ

From Cs recoil measurement [Mueller et al '18]:

$$\alpha = 137.035\,999\,046(27)\ [0.2\text{ ppb}]$$

Then:

					% of a_μ	order
$a_\mu^{\text{QED}} \times 10^{10}$	$=$		11 614 097.3321	(23)	99.6133%	α
		$+$	41 321.7626	(7)	0.3544%	α^2
		$+$	3 014.1902	(33)	0.0259%	α^3
		$+$	38.1004	(17)	0.0003%	α^4
		$+$	0.5078	(6)	$4 \cdot 10^{-6}$	α^5
	$=$		11 658 471.8931	$(7)_{m_\tau}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_\alpha$ [0.9 ppb]		

(Aoyama et al '12, '18, '19)

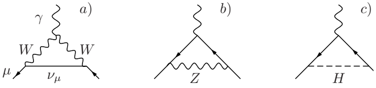
99.994% of a_μ are due to QED contributions!

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} = 734.2(4.1) \times 10^{-10}$$

$$\stackrel{?}{=} a_\mu^{\text{EW}} + a_\mu^{\text{had}}$$

Electroweak contributions to a_μ: Z, W, H, etc. loops

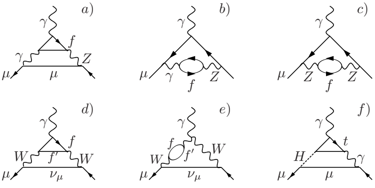
1-loop



$$a_\mu^{\mathrm{EW},(1)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\right) = 19.479(1) \times 10^{-10}$$

(Gnendiger et al '15, Aoyama et al '20 and refs therein)

2-loop



$$a_\mu^{\mathrm{EW},(2)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\frac{\alpha}{\pi}\right) = -4.12(10) \times 10^{-10}$$

(Gnendiger et al '15 and refs therein)

$$a_\mu^{\mathrm{EW}} = 15.36(10) \times 10^{-10}$$

Hadronic contributions to a_μ: quark and gluon loops

$$a_\mu^{\rm exp} - a_\mu^{\rm QED} - a_\mu^{\rm EW} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\rm had}$$

- Clearly right order of magnitude:

$$a_\mu^{\rm had} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\rm had} = 650(50) \times 10^{-10}$)

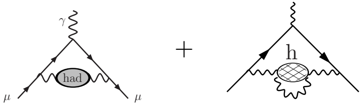
- However, involves quarks and gluons at low energies

  $\Rightarrow$ must be able to describe the highly nonlinear dynamics of confinement
  $\Rightarrow$ cannot rely on the perturbative methods used for QED and weak corrections
  $\Rightarrow$ need methods that allow computations to all orders in α_s with fully controlled uncertainties

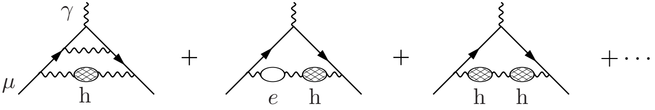
- Decompose:

$$a_\mu^{\rm had} = a_\mu^{\rm LO\text{-}HVP} + a_\mu^{\rm HO\text{-}HVP} + a_\mu^{\rm HLbyL} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

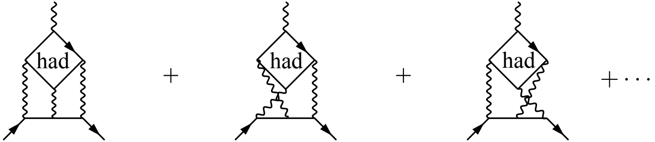
Hadronic contributions to a_μ: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$



$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$



$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic vacuum polarization (HVP)

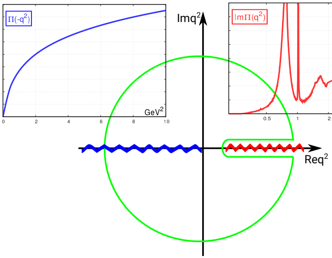
- $\Pi_{\mu\nu}(q) =$ 

$= (q_\mu q_\nu - g_{\mu\nu}q^2)\,\Pi(q^2)$

- On shell renormalization of α: $\Pi(q^2) \rightarrow \hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$

- For $a_\mu^{\text{LO-HVP}}$ need $\hat{\Pi}(q^2)$ for spacelike $q^2 = -Q^2$ and $Q^2 \in [0, \infty[$

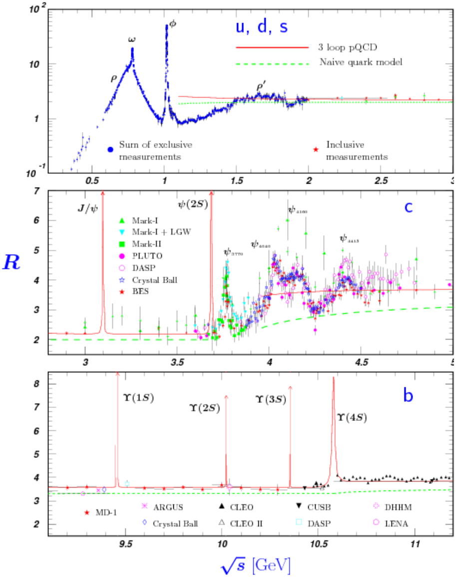
- $\hat{\Pi}(q^2)$ is real and analytic except for cut along real, positive q^2 axis



$\gamma^* \rightarrow$ hadrons

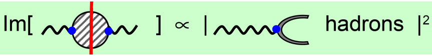
- Can get $\hat{\Pi}(-Q^2)$ from $\mathrm{Im}\Pi(q^2)$ via contour integral $\rightarrow$ dispersion relation

HVP from $e^+e^- \to$ had (or $\tau \to \nu_\tau +$ had)



(PDG compilation)

Use [Bouchiat et al 61] optical theorem **(unitarity)**



$$\mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}, \quad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

and a once subtracted dispersion relation **(analyticity)**

$$\hat{\Pi}(Q^2) = \int_0^\infty ds\, \frac{Q^2}{s(s+Q^2)} \frac{1}{\pi} \mathrm{Im}\Pi(s)$$
$$= \frac{Q^2}{12\pi^2} \int_0^\infty ds\, \frac{1}{s(s+Q^2)} R(s)$$

$\Rightarrow$ $\hat{\Pi}(Q^2)$ & $a_\mu^{\text{LO-HVP}}$ from data: sum of exclusive 32 relevant channels (e.g. $\pi^+\pi^-$) from CMD-2, SND, BES, KLOE '08,'10&'12, BABAR '09, etc., plus pQCD at higher $\sqrt{s}$

Can also use $I(J^{PC}) = 1(1^{--})$ part of $\tau \to \nu_\tau +$ had and isospin symmetry + corrections, but corrections not controlled well enough at present

LO-HVP from $e^+e^- \to$ had

- $O(\alpha^2)$ but includes some $O(\alpha^3)$ corrections:
  - final state radiation (FSR): $\sigma(e^+e^- \to \text{had} + \gamma_{\text{FSR}})$, e.g. $\pi^+\pi^-\gamma$
  - radiative modes, e.g. $\pi^0\gamma$ & $\eta\gamma$

- Three recent values:

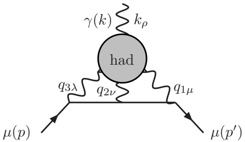
$$a_\mu^{\text{LO-HVP}} = 692.78(2.42) \times 10^{-10} \quad [3.5‰] \qquad \text{(KNT '19)}$$
$$= 693.9(4.0) \times 10^{-10} \quad [5.8‰] \qquad \text{(DHMZ '19)}$$
$$= 692.3(3.3) \times 10^{-10} \quad [4.8‰] \qquad \text{(CHHKS '19)}$$

where the latter 2 account for overall BaBar vs KLOE difference and are *systematic* dominated

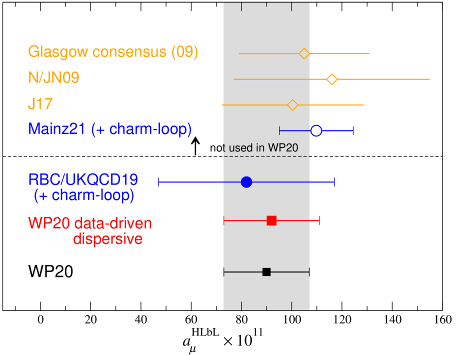
- Higher orders:

$$a_\mu^{\text{NLO-HVP}} = -9.83(0.07) \times 10^{-10} \qquad \text{(Kurz et al '14, Jgerlehner '16, WP'20)}$$
$$a_\mu^{\text{NNLO-HVP}} = 1.24(0.01) \times 10^{-10} \qquad \text{(Kurz et al '14, Jegerlehner '16)}$$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq$ 10% instead of $\simeq$ 0.2%
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - → Phenomenology: dispersive, data-driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,... '15-'20]
  - → Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average and conservative error estimate [WP '20]
- $a_\mu^{\text{HLbL}} = 9.0(1.7) \times 10^{-10}$ [19%]



[Colangelo '21]

Reference standard model prediction and comparison to experiment

Reference SM result vs experiment

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	rel. err.	Ref.
QED [5 loops]	11658471.8931 ± 0.0104	[0.9 ppb]	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[0.7%]	[Gnendiger '15, WP '20]
HVP Tot. (R-ratio)	684.5 ± 4.0	[0.6%]	[WP '20]
HLbL Tot.	9.2 ± 1.8	[20%]	[WP '20]
SM	11659181.0 ± 4.3	[0.37 ppm]	[WP '20]

$$a_\mu|_{\text{exp.}} = 0.00116592061(41)$$
$$a_\mu|_{\text{ref.}} = 0.00116591810(43)$$
$$\text{diff.} = 0.00000000251(59)$$

- Comparable errors but 4.2σ disagreement: probability $\lesssim 1/40\,000$

  $\Rightarrow$ evidence for BSM physics

- Particle physicists require probability $\lesssim 1/2\,000\,000$ to claim discovery (5σ)

Important to check most uncertain contribution (HVP) w/ fully independent methods

$\rightarrow$ *ab initio* calculations of contribution using **lattice quantum chromodynamics (QCD)**