

INTRO TO PHENOMENOLOGY WITH MASSIVE NEUTRINOS IN 2022

Concha Gonzalez-Garcia

(YITP-Stony Brook & ICREA-University of Barcelona)

58th International School of Subnuclear Physics

Erice, June 16th, 2022



Global fit to neutrino
oscillation data



Hunting Invisibles: Dark matter, Dark water and Neutrinos



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OUTLINE

- Historic Introduction to the SM of Massless Neutrinos
- Introducing ν mass: Dirac vs Majorana, Lepton mixing, Flavour Oscillations
- Summary of Flavour Oscillation Observations
- Status of 3ν global description
- Explorations beyond 3ν 's: steriles, NSI's, Z's...

Discovery of ν 's

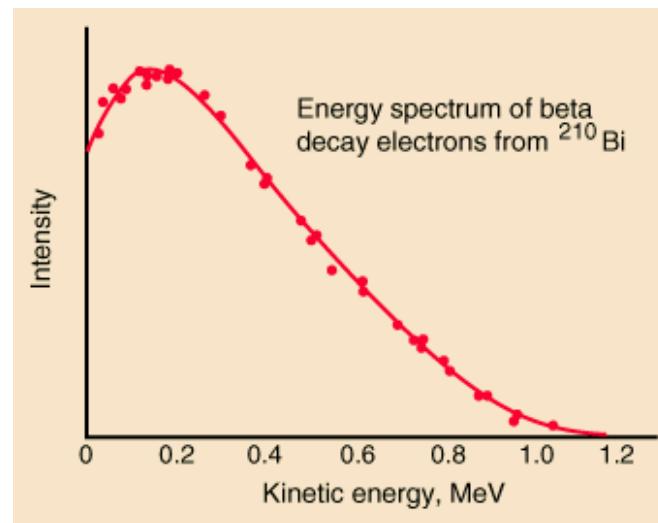
- At end of 1800's radioactivity was discovered and three types identified: α , β , γ
 β : an electron comes out of the radioactive nucleus.
- Energy conservation $\Rightarrow e^-$ should have had a fixed energy

$$(A, Z) \rightarrow (A, Z + 1) + e^- \Rightarrow E_e = M(A, Z + 1) - M(A, Z)$$

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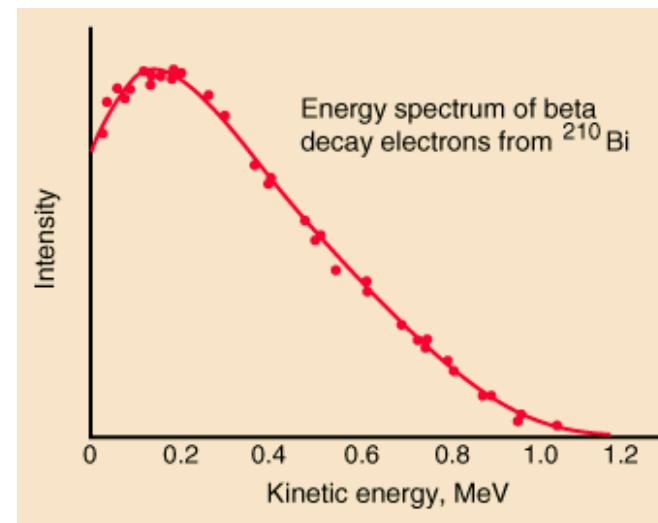
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Do we throw away the energy conservation?

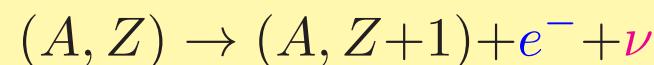
Bohr: we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β ray disintegrations

Discovery of ν 's

- The idea of the **neutrino** came in 1930, when **W. Pauli** tried a desperate saving operation of "the energy conservation principle".



In his letter addressed to the *Liebe Radioaktive Damen und Herren* (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tübingen. He put forward the hypothesis that a new particle exists as *constituent of nuclei, the neutron ν* , able to explain the continuous spectrum of nuclear beta decay



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$$(A, Z) \rightarrow (A, Z+1) + e^- + \nu$$

- The ν is **light** (in Pauli's words:
 m_ν should be of the same order as the m_e),
neutral and has **spin 1/2**

Neutrino Detection

Fighting Pauli's “Curse”:

*I have done a terrible thing, I have postulated a particle
that cannot be detected.*

Sources of ν 's

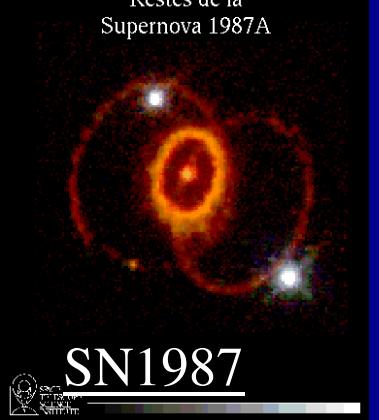


The Big Bang

$$\rho_\nu = 330/\text{cm}^3$$

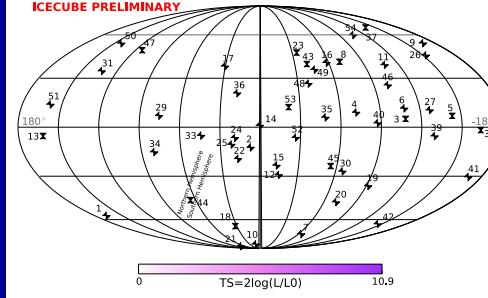
$$p_\nu = 0.0004 \text{ eV}$$

Restes de la Supernova 1987A



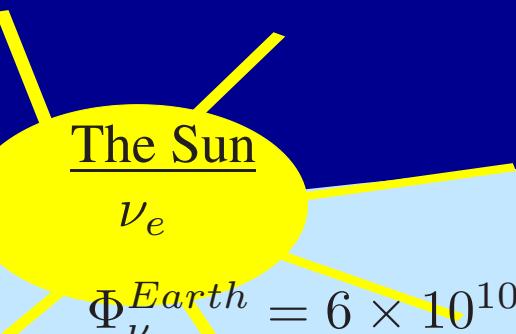
SN1987

$$E_\nu \sim \text{MeV}$$



ExtraGalactic

$$E_\nu \gtrsim 30 \text{ TeV}$$

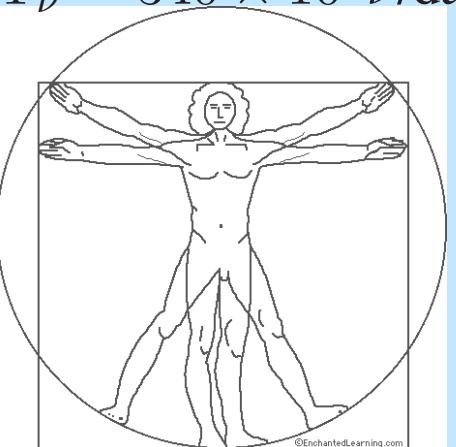


ν_e

$$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$$

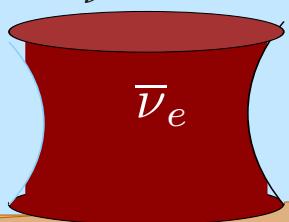
$$E_\nu \sim 0.1\text{--}20 \text{ MeV}$$

$$\frac{\text{Human Body}}{\Phi_\nu} = 340 \times 10^6 \nu/\text{day}$$



Nuclear Reactors

$$E_\nu \sim \text{few MeV}$$

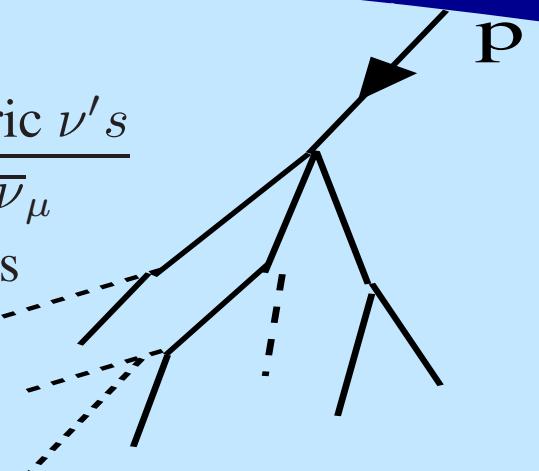


$\bar{\nu}_e$

$$\frac{\text{Earth's radioactivity}}{\Phi_\nu} \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$$

$$\frac{\text{Atmospheric } \nu' s}{\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu}$$

$$\Phi_\nu \sim 1 \nu/\text{cm}^2\text{s}$$



$$\frac{\text{Accelerators}}{E_\nu \simeq 0.3\text{--}30 \text{ GeV}}$$



Neutrino Detection

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38} \text{ cm}^2 \frac{E_\nu}{\text{GeV}}$$

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$$\Phi_\nu^{\text{ATM}} = 1 \nu / (\text{cm}^2 \text{ second}) \text{ y } \langle E_\nu \rangle = 1 \text{ GeV}$$

- How many interact?

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$$\begin{aligned}
 N_{\text{int}} &= \Phi_\nu \times \sigma^{\nu p} \times N_{\text{prot}}^{\text{human}} \times T_{\text{life}}^{\text{human}} && (M \times T \equiv \text{Exposure}) \\
 N_{\text{protons}}^{\text{human}} &= \frac{M^{\text{human}}}{g_r} \times N_A = 80 \text{ kg} \times N_A \sim 5 \times 10^{28} \text{ protons} \\
 T^{\text{human}} &= 80 \text{ years} = 2 \times 10^9 \text{ sec}
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$$N_{\text{int}} = (5 \times 10^{28}) (2 \times 10^9) \times 10^{-38} \sim 1 \text{ interaction in life}$$

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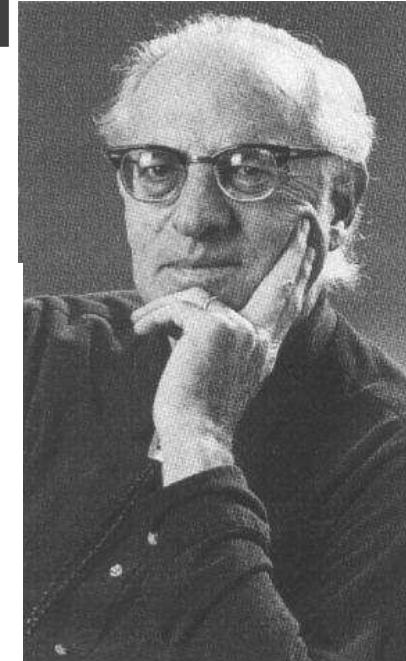
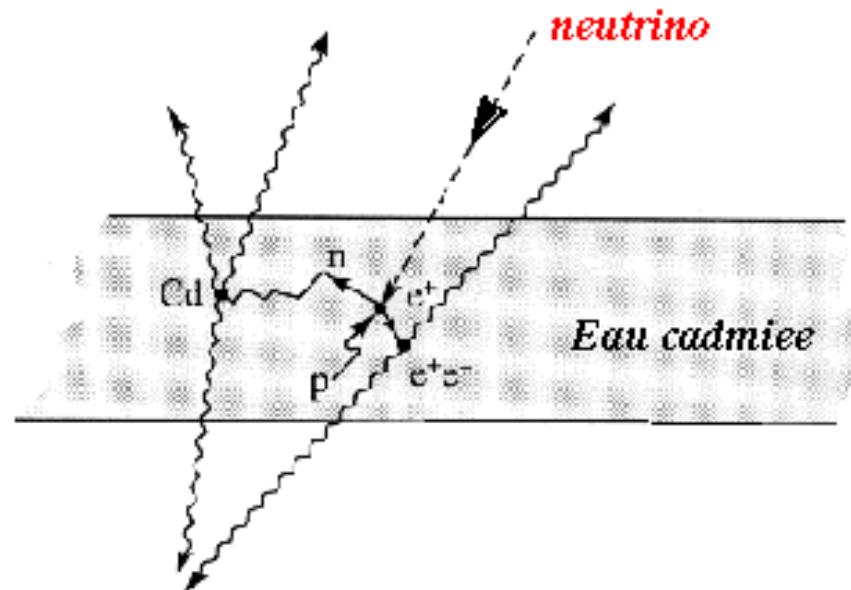
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To detect neutrinos we need very intense source and/or
a huge detector with Exposure $\sim \text{KTon} \times \text{year}$

First Neutrino Detection

In 1953 Frederick Reines and Clyde Cowan put a detector near a nuclear reactor (the most intense source available)

400 l of water
and Cadmium Chloride.



e^+ annihilates with e^- in the water and produces two γ 's simultaneously.
neutron is captured by por the cadmium and a γ 's is emitted 15 msec latter

Reines y Clyde saw clearly this signature: the first neutrino had been detected

Neutrinos = “Left-handed”

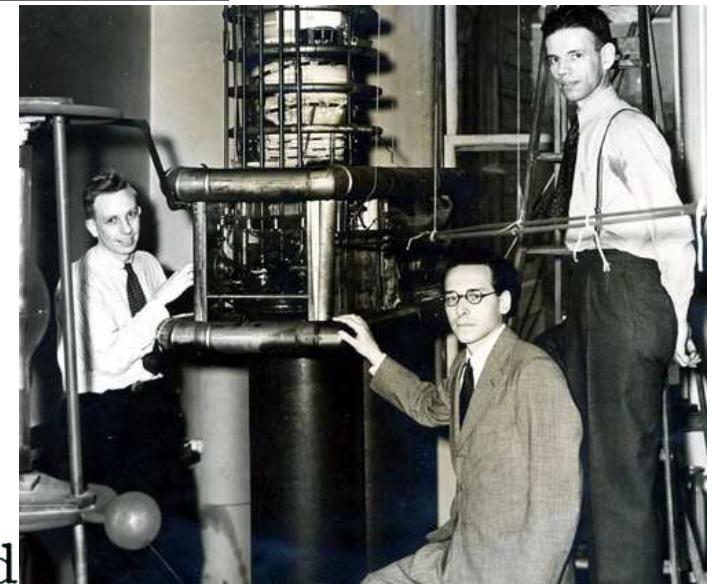
Helicity of Neutrinos*

M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR

Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m} , which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,¹ $0-$, we find that the neutrino is “left-handed,” i.e., $\sigma_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).



WARNING: Helicity versus Chirality

a Gonzalez-Garcia

- We define the chiral projections

$$P_{R,L} = \frac{1 \pm \gamma_5}{2} \quad \Rightarrow \quad \psi_L = \frac{1 - \gamma_5}{2} \psi \quad \psi_R = \frac{1 + \gamma_5}{2} \psi$$

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- The Lagrangian of a massive **free** fermion ψ is $\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial - m)\psi(x)$
- 4 independent states with (E, \vec{p}) $(\gamma \cdot p - m)u_s(\vec{p}) = 0$ $(\gamma \cdot p + m)v_s(\vec{p}) = 0$ $s = 1, 2$

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 \Rightarrow Neither Chirality nor J_i can simultaneously with E, \vec{p} characterize the fermion

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$$P_{\pm} = \frac{1}{2} \left(1 \pm 2\vec{J} \frac{\vec{p}}{|p|} \right) = \frac{1}{2} \left(1 \pm \vec{\Sigma} \frac{\vec{p}}{|p|} \right)$$

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ν in the SM

- The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times \color{red}SU(2)_L \times \color{blue}U(1)_Y \Rightarrow \color{green}SU(3)_C \times \color{magenta}U(1)_{EM}$$

- 3 Generations of Fermions:

$(1, 2, -\frac{1}{2})$	$(3, 2, \frac{1}{6})$	$(1, 1, -1)$	$(3, 1, \frac{2}{3})$	$(3, 1, -\frac{1}{3})$
L_L	Q_L^i	E_R	U_R^i	D_R^i
$\begin{pmatrix} \nu_e \\ e \\ \nu_\mu \\ \mu \\ \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \\ c^i \\ s^i \\ t^i \\ b^i \end{pmatrix}_L$	e_R	u_R^i	d_R^i
		μ_R	c_R^i	s_R^i
		τ_R	t_R^i	b_R^i

- Spin-0 particle ϕ : $(1, 2, \frac{1}{2})$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{SSB} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \color{violet}h \end{pmatrix}$$

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$$Q_{EM} = \color{red}{T_{L3}} + \color{blue}{Y}$$

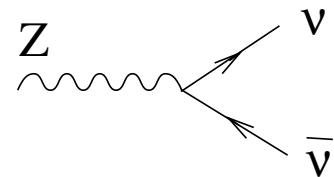
- ν's are $T_{L3} = \frac{1}{2}$ components of L_L
- ν's have no strong or EM interactions
- No ν_R (\equiv singlets of gauge group)

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Number of Neutrinos

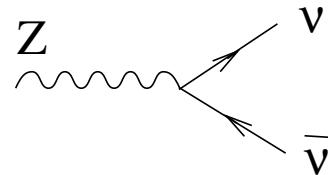
- The counting of light left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L} \gamma^\mu \nu_{\alpha L}$$

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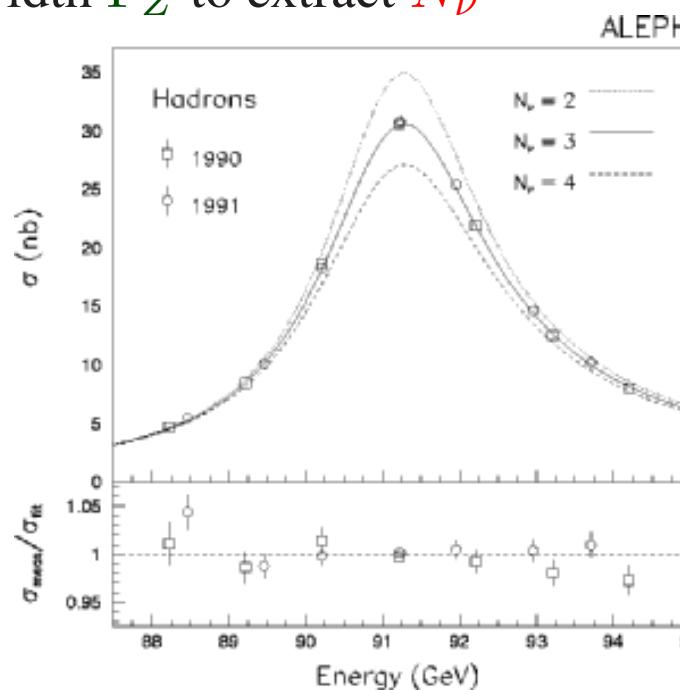
- For $m_{\nu_i} < m_Z/2$ one can use the total Z -width Γ_Z to extract N_ν

$$\begin{aligned} N_\nu &= \frac{\Gamma_{\text{inv}}}{\Gamma_\nu} \equiv \frac{1}{\Gamma_\nu} (\Gamma_Z - \Gamma_h - 3\Gamma_\ell) \\ &= \frac{\Gamma_\ell}{\Gamma_\nu} \left[\sqrt{\frac{12\pi R_{h\ell}}{\sigma_h^0 m_Z^2}} - R_{h\ell} - 3 \right] \end{aligned}$$

Γ_{inv} = the invisible width

Γ_h = the total hadronic width

Γ_ℓ = width to charged lepton



Leads $N_\nu = 2.9840 \pm 0.0082$

SM Fermion Lagrangian

$$\begin{aligned}
\mathcal{L} = & \sum_{k=1}^3 \sum_{i,j=1}^3 \overline{Q_{L,k}^i} \gamma^\mu \left(i\partial_\mu - g_s \frac{\lambda_{a,ij}}{2} G_\mu^a - g \frac{\tau_a}{2} \delta_{ij} W_\mu^a - g' \frac{1}{6} \delta_{ij} B_\mu \right) Q_{L,k}^j \\
& \sum_{k=1}^3 \sum_{i,j=1}^3 + \overline{U_{R,k}^i} \gamma^\mu \left(i\partial_\mu - g_s \frac{\lambda_{a,ij}}{2} G_\mu^a - g' \frac{2}{3} \delta_{ij} B_\mu \right) U_{R,k}^j \\
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& \sum_{k=1}^3 + \overline{L_{L,k}} \gamma^\mu \left(i\partial_\mu - g \frac{\tau_i}{2} W_\mu^i + g' \frac{1}{2} B_\mu \right) L_{L,k} + \overline{E_{R,k}} \gamma^\mu \left(i\partial_\mu + g' B_\mu \right) E_{R,k} \\
& - \sum_{k,k'=1}^3 \left(\lambda_{kk'}^u \overline{Q}_{L,k} (i\tau_2) \phi^* U_{R,k'} + \lambda_{kk'}^d \overline{Q}_{L,k} \phi D_{R,k'} + \lambda_{kk'}^l \overline{L}_{L,k} \phi E_{R,k'} + h.c. \right)
\end{aligned}$$

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\end{aligned}$$

- Invariant under global rotations

$$Q_{L,k} \rightarrow e^{i\alpha_B/3} Q_{L,k} \quad U_{R,k} \rightarrow e^{i\alpha_B/3} U_{R,k} \quad D_{R,k} \rightarrow e^{i\alpha_B/3} D_{R,k} \quad L_{L,i} \rightarrow e^{i\alpha_{L_k}/3} L_{L,k} \quad E_{R,k} \rightarrow e^{i\alpha_{L_k}/3} E_{R,k}$$

SM Fermion Lagrangian

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\Rightarrow Accidental (\equiv not imposed) global symmetry: $B \times L_e \times L_\mu \times L_\tau$

\Rightarrow Each lepton flavour, L_i , is conserved

\Rightarrow Total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

SM = massless ν' s and LFC

oncha Gonzalez-Garcia

- A **fermion mass** can be seen as at a **Left-Right transition**

$$m_f \bar{\psi} \psi = m_f \bar{\psi}_L \psi_R + h.c. \quad (\text{this is not } SU(2)_L \text{ gauge invariant})$$

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- There are no right-handed neutrinos
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In SM ν 's are *Strictly Massless* & Lepton Flavours are *Strictly Conserved*

- We have observed with high (or good) precision:
 - * Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (**SK,MINOS, ICECUBE**)
 - * Accel. ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 300/800$ Km (**K2K, T2K, MINOS, NO ν A**)
 - * Some accelerator ν_μ appear as ν_e at $L \sim 300/800$ Km (**T2K, MINOS,NO ν A**)
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All this implies that L_α are violated

and There is Physics Beyond SM

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
 - Their own antiparticle such as $\gamma, \pi^0 \dots$
 - Different from their antiparticle such as $K^0, \bar{K}^0 \dots$
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$$\Rightarrow \text{It is described by a Dirac field } \nu(x) = \sum_{s,\vec{p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-i\vec{p}x} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{i\vec{p}x} \right]$$

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\Rightarrow And the charged conjugate neutrino field \equiv the antineutrino field

$$\nu^C = C \nu C^{-1} = \sum_{s, \vec{p}} \left[b_s(\vec{p}) u_s(\vec{p}) e^{-i\vec{p}x} + a_s^\dagger(\vec{p}) v_s(\vec{p}) e^{i\vec{p}x} \right] = -C \bar{\nu}^T$$

$(C = i\gamma^2\gamma^0)$

which contain two sets of creation–annihilation operators

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\Rightarrow These two fields can rewritten in terms of 4 chiral fields

$$\nu_L, \nu_R, (\nu_L)^C, (\nu_R)^C \quad \text{with } \nu = \nu_L + \nu_R \text{ and } \nu^C = (\nu_L)^C + (\nu_R)^C$$

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The difference arises when including *a neutrino mass*

Adding ν Mass: Dirac Mass

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$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\bar{\nu}_R M_D^\nu \nu_L + h.c. \equiv -\frac{1}{2} (\bar{\nu}_R M_D^\nu \nu_L + \overline{(\nu_L)^c} M_D^\nu{}^T (\nu_R)^c) + h.c. \equiv -\sum_k m_k \bar{\nu}_k^D \nu_k^D$$

$$M_D^\nu = \frac{1}{\sqrt{2}} \lambda^\nu v \quad v = \text{Dirac mass for neutrinos}$$

$$V_R^{\nu\dagger} M_D V^\nu = \text{diag}(m_1, m_2, m_3)$$

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$$V_R^{\nu\dagger} M_D V^\nu = \text{diag}(m_1, m_2, m_3)$$

\Rightarrow The eigenstates of M_D^ν are Dirac fermions (same as quarks and charged leptons)

$$\nu^D = V^{\nu\dagger} \nu_L + V_R^{\nu\dagger} \nu_R$$

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$$\mathcal{L}_Y^{(\nu)} = -\lambda_{ij}^\nu \bar{\nu}_{Ri} L_{Lj} \tilde{\phi}^\dagger + h.c. \quad (\tilde{\phi} = i\tau_2 \phi^*)$$

- Under spontaneous symmetry-breaking $\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$

$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\bar{\nu}_R M_D^\nu \nu_L + h.c. \equiv -\frac{1}{2} (\bar{\nu}_R M_D^\nu \nu_L + \overline{(\nu_L)^c} M_D^\nu{}^T (\nu_R)^c) + h.c. \equiv -\sum_k m_k \bar{\nu}_k^D \nu_k^D$$

$$M_D^\nu = \frac{1}{\sqrt{2}} \lambda^\nu v \quad v = \text{Dirac mass for neutrinos}$$

$$V_R^{\nu\dagger} M_D V^\nu = \text{diag}(m_1, m_2, m_3)$$

\Rightarrow The eigenstates of M_D^ν are Dirac fermions (same as quarks and charged leptons)

$$\nu^D = V^{\nu\dagger} \nu_L + V_R^{\nu\dagger} \nu_R$$

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\Rightarrow Total Lepton number is conserved by construction (not accidentally):

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Adding ν Mass: Majorana Mass

oncha Gonzalez-Garcia

- One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

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M_M^ν =Majorana mass for ν 's is symmetric

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- in SM $B - L$ is non anomalous ⇒ $\mathcal{L}_{\text{mass}}^{(\text{Maj})}$ not generated non-perturbatively in SM

ν Mass \Rightarrow Lepton Mixing

- CC and mass for 3 charged leptons ℓ_i and N neutrinos in weak basis $\nu^W \equiv \begin{pmatrix} \nu_{L,e} \\ \nu_{L,\mu} \\ \nu_{L,\tau} \\ (\nu_{R,1})^C \\ \vdots \\ \vdots \end{pmatrix}$
- $$\mathcal{L}_{CC} + \mathcal{L}_M = -\frac{g}{\sqrt{2}} \sum_{i=1}^3 \overline{\ell_{L,i}^W} \gamma^\mu \nu_i^W W_\mu^+ - \sum_{i,j=1}^3 \overline{\ell_{L,i}^W} M_{\ell ij} \ell_{R,j}^W - \frac{1}{2} \sum_{i,j=1}^N \overline{\nu_i^{cW}} M_{\nu ij} \nu_j^W + \text{h.c.}$$

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- For example for 3 Dirac ν 's : 3 Mixing angles + 1 Dirac Phase

$$U_{\text{LEP}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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The New Minimal Standard Model

- Minimal Extension to allow for LFV \Rightarrow give Mass to the Neutrino
 - * Introduce ν_R AND impose L conservation \Rightarrow Dirac $\nu \neq \nu^c$:

$$\mathcal{L} = \mathcal{L}_{SM} - M_\nu \overline{\nu_L} \nu_R + h.c.$$

- * NOT impose L conservation \Rightarrow Majorana $\nu = \nu^c$

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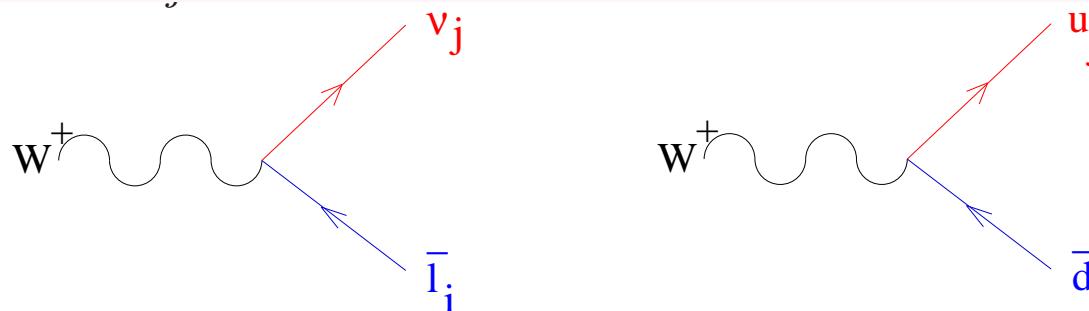
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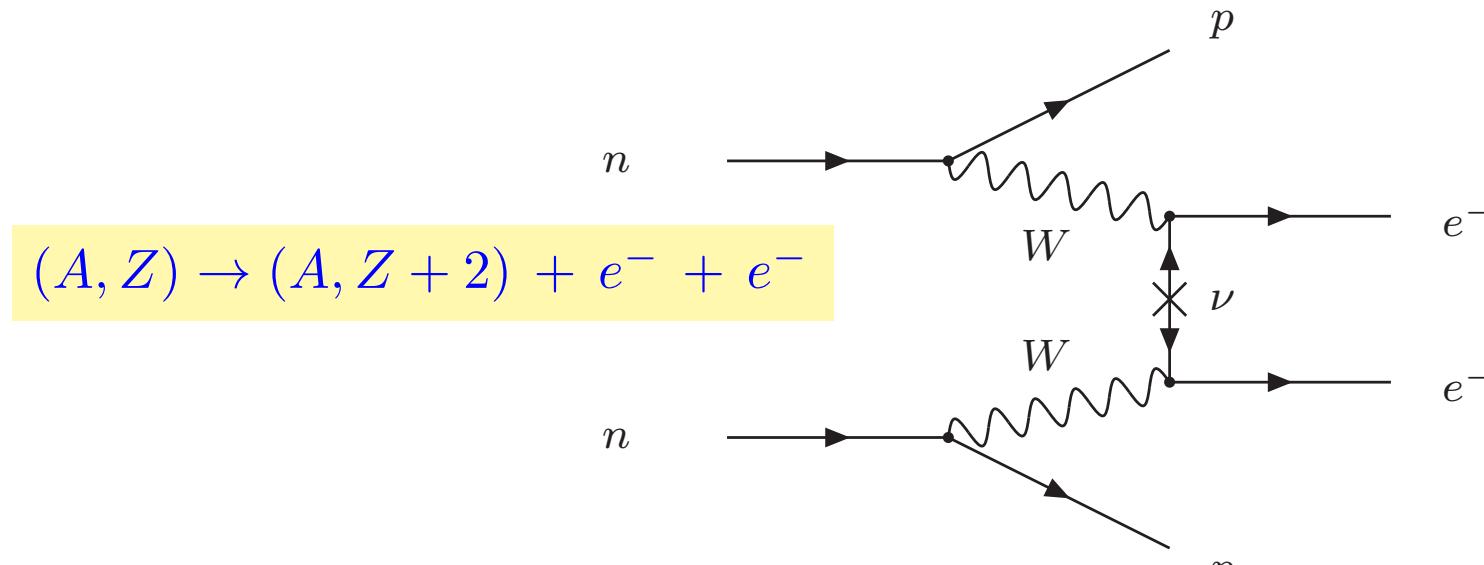
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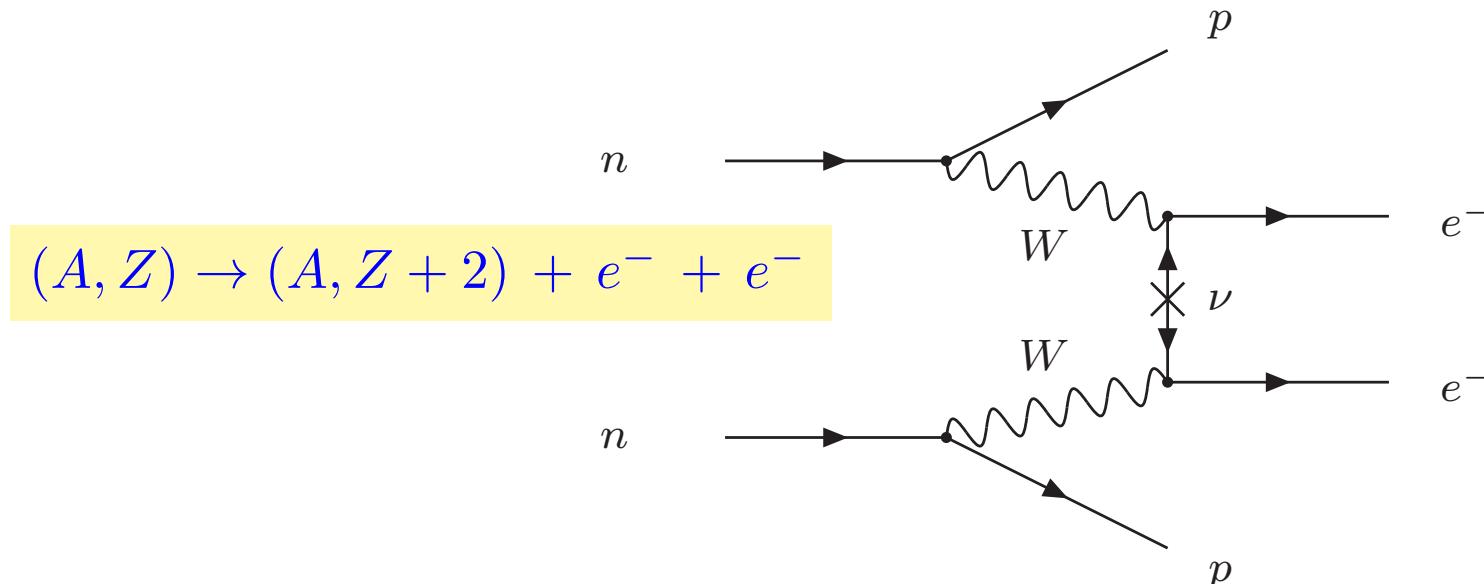
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Dirac or Majorana? ν -less Double- β Decay



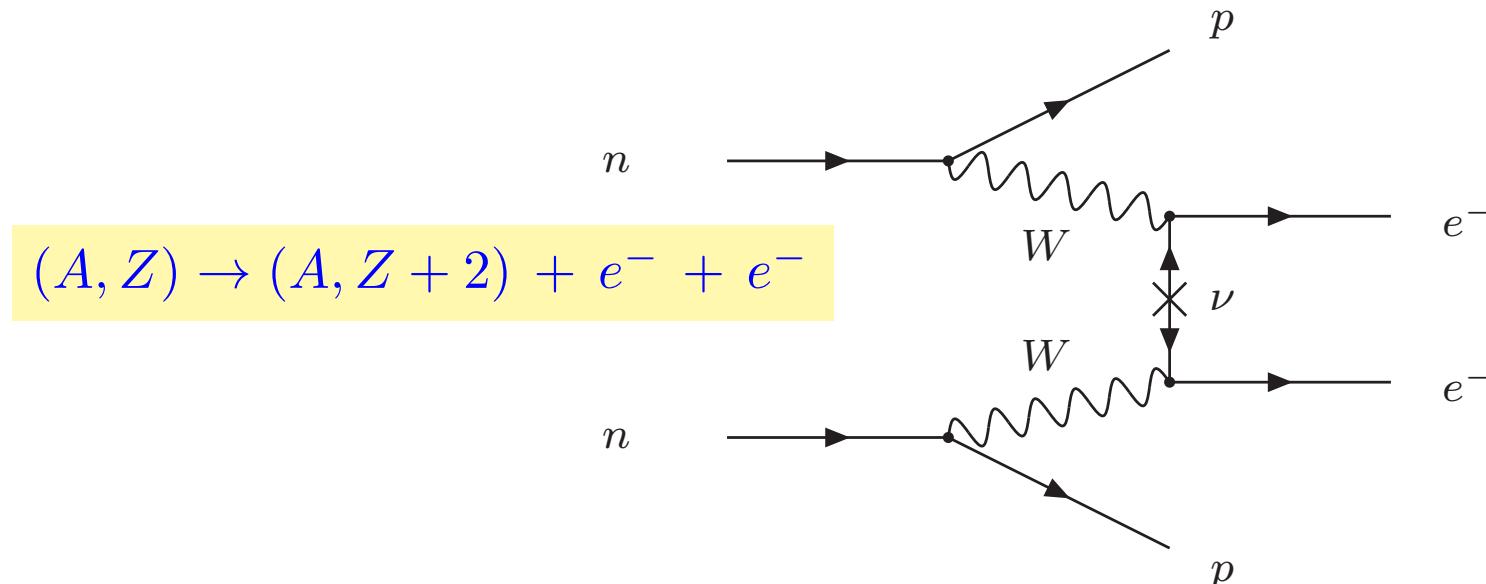
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 \Rightarrow no same state \Rightarrow Amplitude = 0
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- If Majorana m_ν only source of L -violation
 \Rightarrow Amplitude of ν -less- $\beta\beta$ decay is proportional to $\langle m_{\beta\beta} \rangle = \sum U_{ej}^2 m_j$

Neutrino Mass Scale: Tritium β Decay

Gonzalez-Garcia

- Fermi proposed a kinematic search of ν_e mass from beta spectra in 3H beta decay



- For “allowed” nuclear transitions, the electron spectrum is given by phase space alone

$$K(T) \equiv \sqrt{\frac{dN}{dT} \frac{1}{C_p E F(E)}} \propto \sqrt{(Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}}$$

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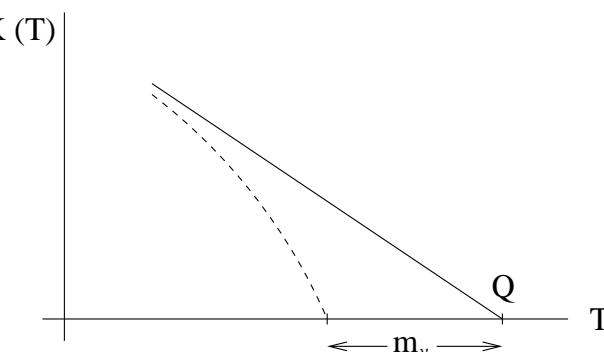
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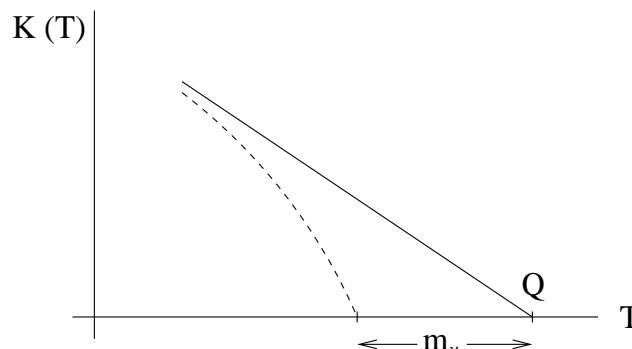
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– At present only a bound: $m_{\nu_e}^{\text{eff}} < 0.8$ eV (at 90 % CL) (Katrin)

– Katrin operating can improve present sensitivity to $m_{\nu_e}^{\text{eff}} \sim 0.3$ eV

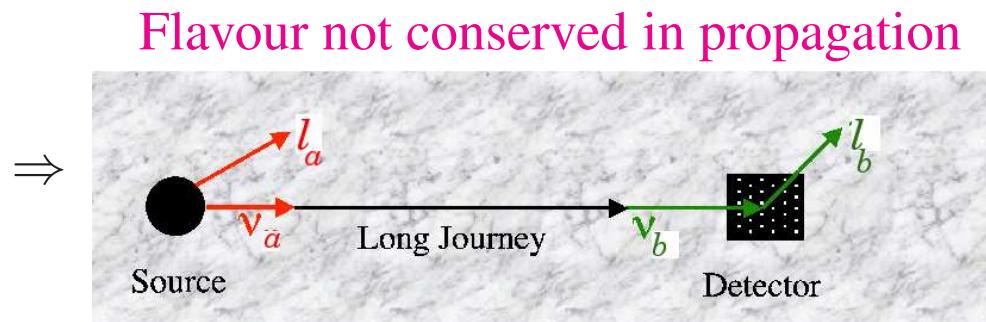
Effects of ν Mass: Flavour Transitions

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- Mass basis (free propagation in space-time): ν_1 , ν_2 and $\nu_3 \dots$

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Flavour not conserved in propagation

The diagram shows a "Source" (a black circle) emitting a neutrino in state l_a . The neutrino undergoes a "Long Journey" (represented by a horizontal arrow) and is detected at a "Detector" (a black square with a grid). At the detector, the neutrino is in state l_b . This visualizes the concept of "Flavour not conserved in propagation".

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
 - Misalignment between interaction and propagation states ($\equiv U$)
 - Difference between propagation eigenvalues
 - Propagation distance

Mass Induced Flavour Oscillations in Vacuum

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \rightarrow \nu_\alpha + N'$ is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

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Flavour Oscillations in Vacuum

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- We call E_i the neutrino energy and m_i the neutrino mass
- Under the approximations:

(1) $|\nu\rangle$ is a *plane wave* $\Rightarrow |\nu_i(t)\rangle = e^{-iE_i t} |\nu_i(0)\rangle$ and using $\langle \nu_j | \nu_i \rangle = \delta_{ij}$

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(2) relativistic ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

(3) Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = 1.27 \frac{m_i^2 - m_j^2}{\text{eV}^2} \frac{L/E}{\text{Km/GeV}}$$

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 - If $\alpha = \beta \Rightarrow \text{Im}[U_{\alpha i} U_{\alpha i}^* U_{\alpha j}^* U_{\alpha j}] = \text{Im}[|U_{\alpha i}^*|^2 |U_{\alpha j}|^2] = 0$
- \Rightarrow CP violation observable only for $\beta \neq \alpha$

Flavour Oscillations in Vacuum

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 - $\Delta m_{ij}^2 = m_i^2 - m_j^2$ The mass differences
 - $U_{\alpha j}$ The mixing angles (and Dirac phases)
- E The neutrino energy
- L Distance ν source to detector

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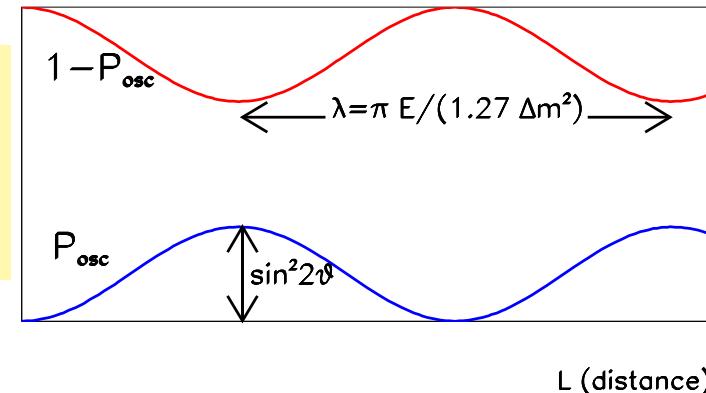
2- ν Oscillations

- When oscillations between 2- ν dominate:

$$P_{osc} = \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \text{ Appear}$$

$$P_{\alpha\alpha} = 1 - P_{osc} \quad \text{Disappear}$$

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

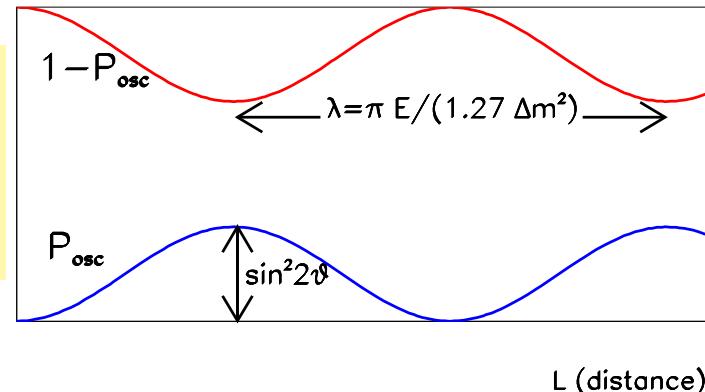


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- P_{osc} is symmetric *independently* under $\Delta m^2 \rightarrow -\Delta m^2$ or $\theta \rightarrow -\theta + \frac{\pi}{2}$
 \Rightarrow No information on ordering ($\equiv \text{sign} \Delta m^2$) nor octant of θ
- U is real \Rightarrow no CP violation

This only happens for 2ν vacuum oscillations

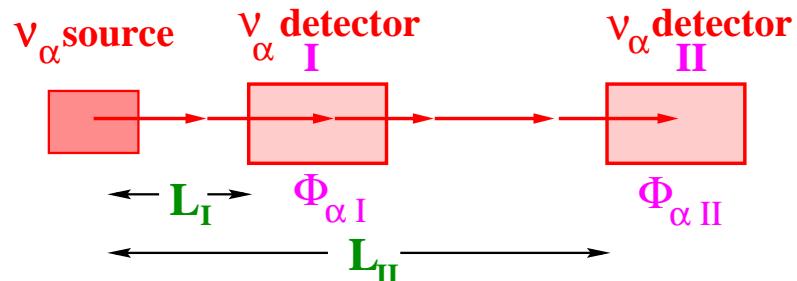
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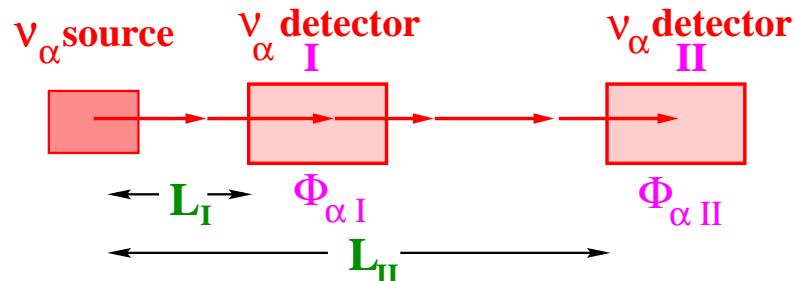


Compares $\Phi_{\alpha I}$ and $\Phi_{\alpha II}$ to look for loss

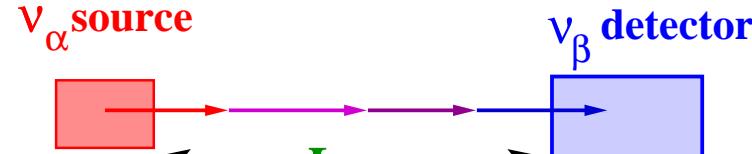
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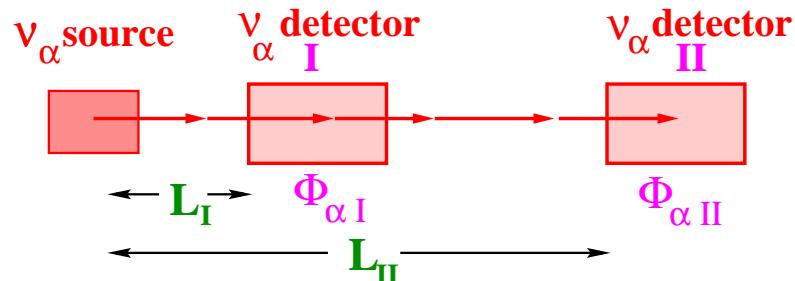
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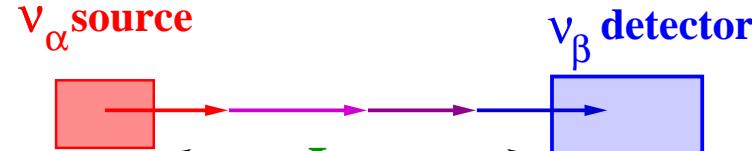
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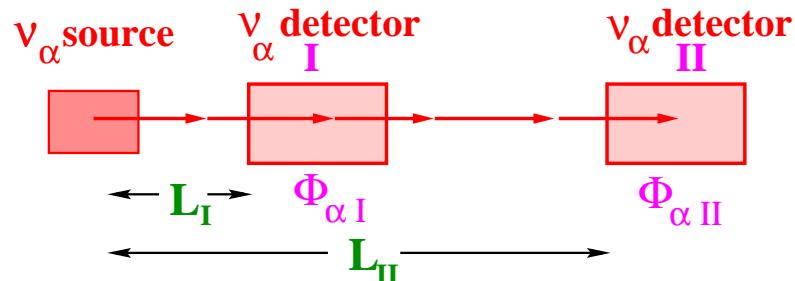
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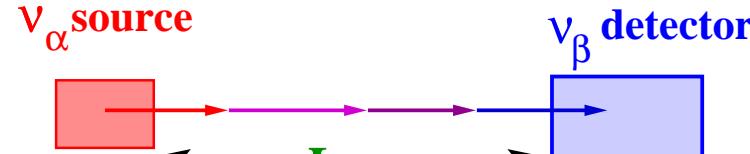
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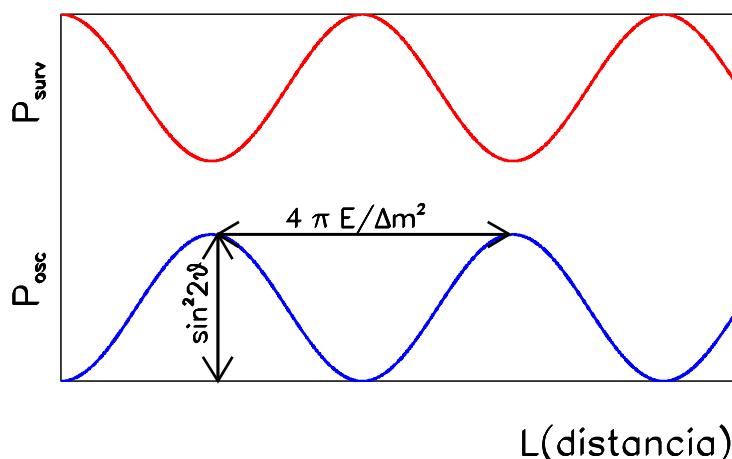


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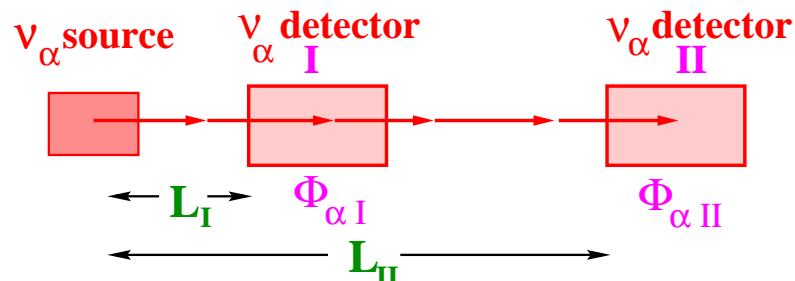
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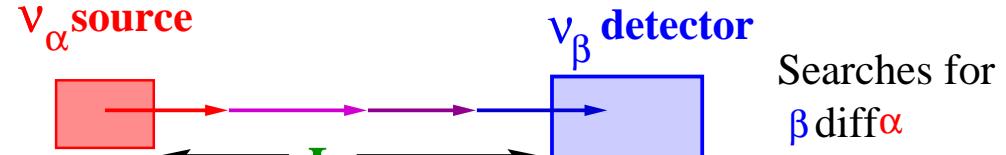
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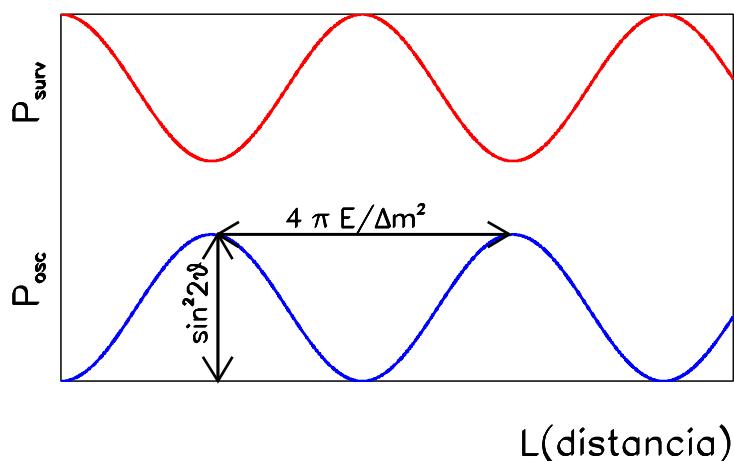


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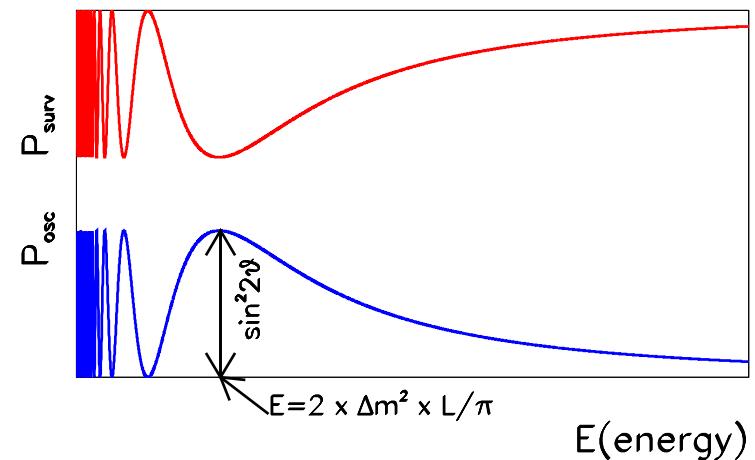
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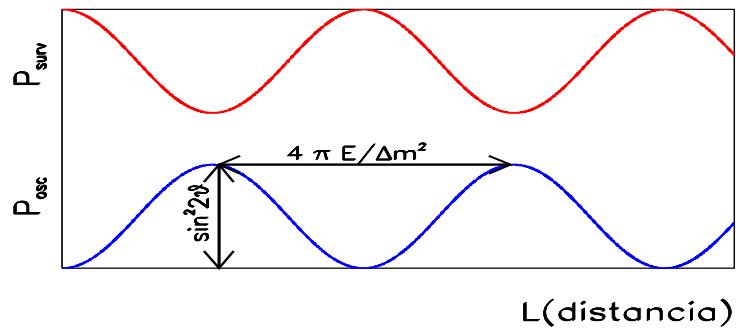
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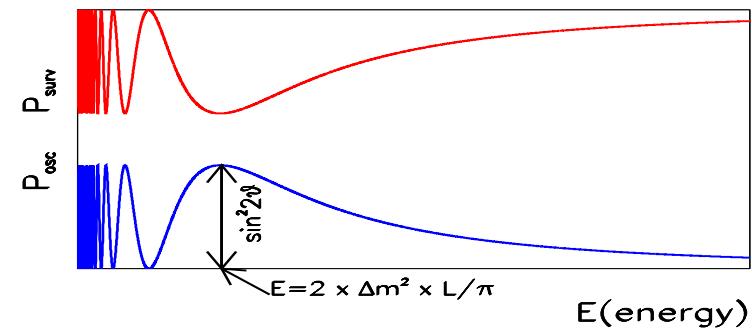
As function of the neutrino Energy



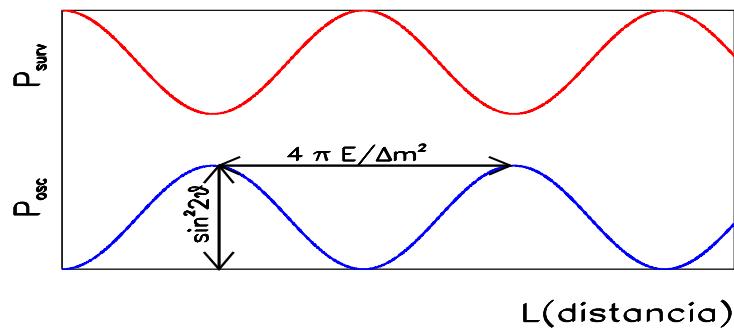
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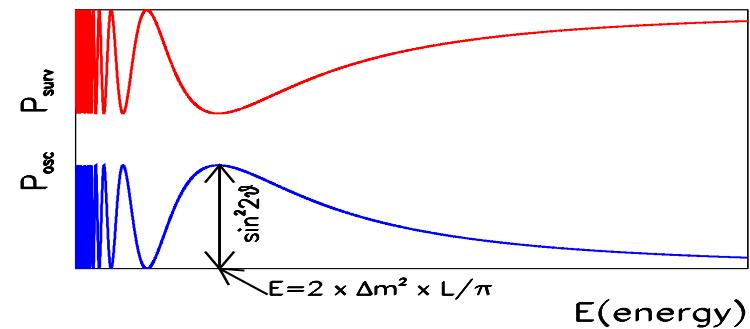
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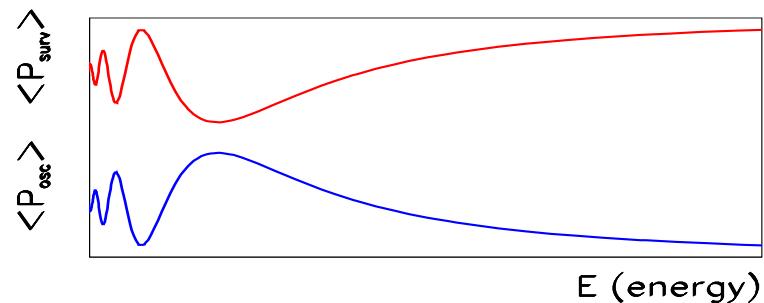
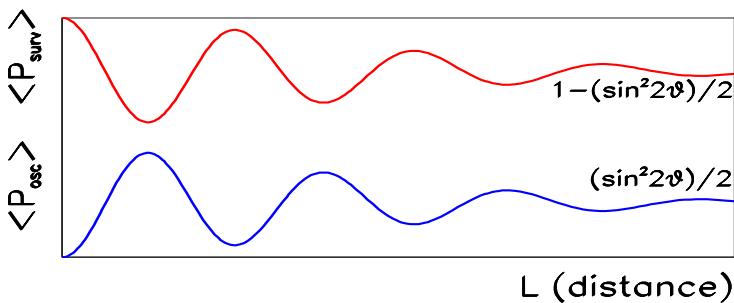
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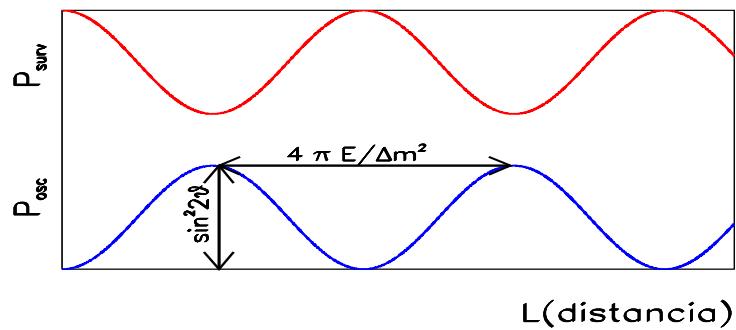
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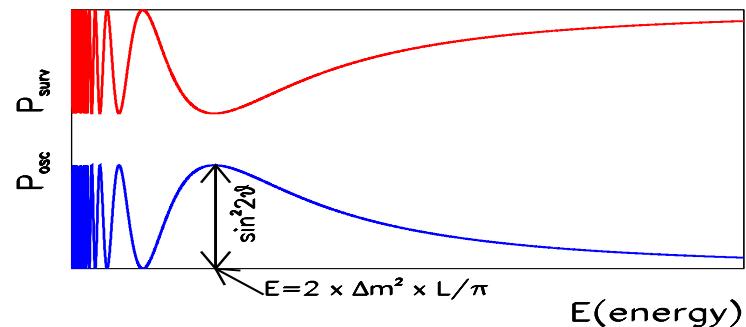
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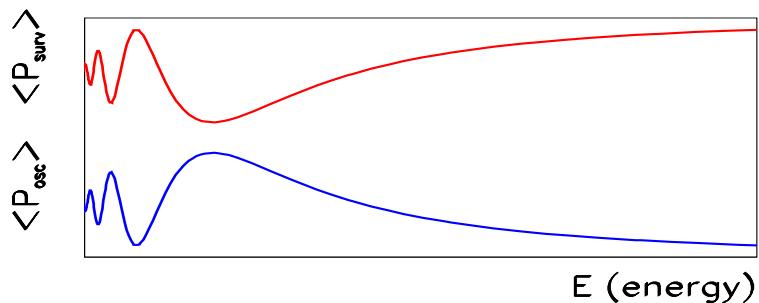
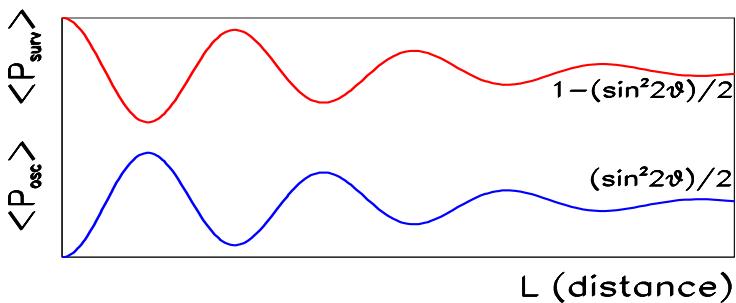
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- Maximal sensitivity for $\Delta m^2 \sim E/L$

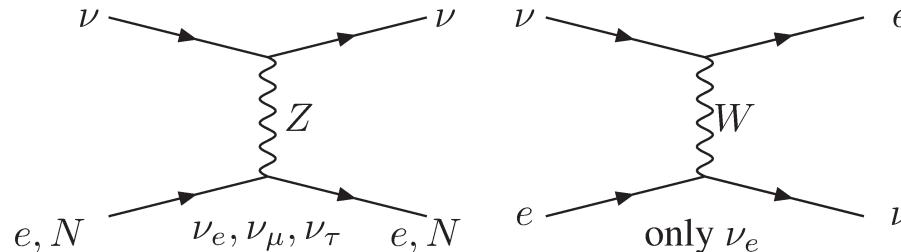
$-\Delta m^2 \ll E/L \Rightarrow \langle \sin^2(\Delta m^2 L / 4E) \rangle \approx 0 \rightarrow \langle P_{\alpha \neq \beta} \rangle \approx 0 \& \langle P_{\alpha\alpha} \rangle \approx 1$

$-\Delta m^2 \gg E/L \Rightarrow \langle \sin^2(\Delta m^2 L / 4E) \rangle \approx \frac{1}{2} \rightarrow \langle P_{\alpha \neq \beta} \rangle \approx \frac{\sin^2(2\theta)}{2} \leq \frac{1}{2} \& \langle P_{\alpha\alpha} \rangle \geq \frac{1}{2}$

Matter Effects

- If ν cross matter regions (Sun, Earth...) it interacts *coherently* with the matter fermions

– But Different flavours
have different interactions :



\Rightarrow Effective potential in ν evolution : $V_e \neq V_{\mu, \tau} \Rightarrow \Delta V^\nu = -\Delta V^{\bar{\nu}} = \sqrt{2}G_F N_e$

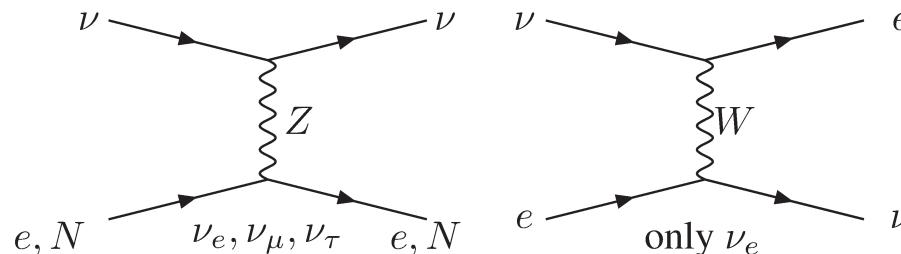
$$-i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = \left[\begin{pmatrix} V_e - V_X - \frac{\Delta m^2}{4E} \cos 2\theta & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}$$

\Rightarrow Modification of mixing angle and oscillation wavelength (MSW)

Matter Effects

- If ν cross matter regions (Sun, Earth...) it interacts *coherently* with the matter fermions

– But Different flavours
have different interactions :



⇒ Effective potential in ν evolution : $V_e \neq V_{\mu, \tau} \Rightarrow \Delta V^\nu = -\Delta V^{\bar{\nu}} = \sqrt{2}G_F N_e$

$$-i \frac{\partial}{\partial x} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = \left[\begin{pmatrix} V_e - V_X - \frac{\Delta m^2}{4E} \cos 2\theta & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}$$

⇒ *Modification of mixing angle and oscillation wavelength (MSW)*

⇒ For solar ν' s in adiabatic regime

$$P_{ee} = \frac{1}{2} [1 + \cos(2\theta_m) \cos(2\theta)]$$

Dependence on θ octant

⇒ In LBL terrestrial experiments

Dependence on sign of Δm^2
and θ octant

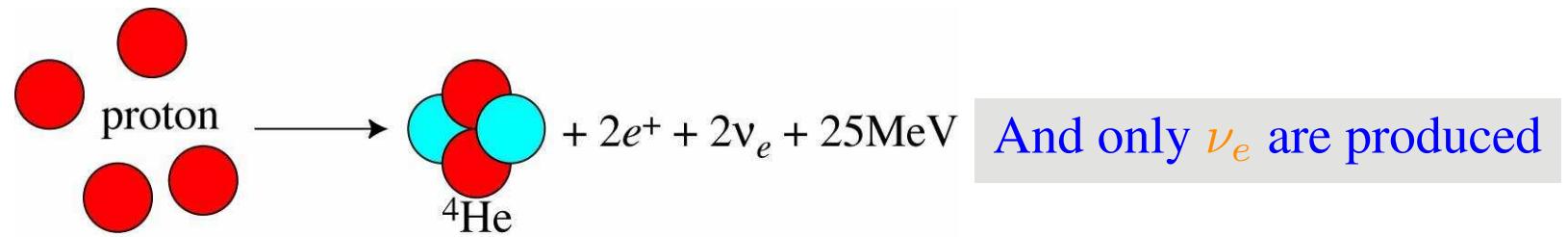
- Mass difference and mixing in matter:

$$\Delta m_{mat}^2 = \sqrt{(\Delta m^2 \cos 2\theta - 2E\Delta V)^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\sin(2\theta_{mat}) = \frac{\Delta m^2 \sin(2\theta)}{\Delta m_{mat}^2}$$

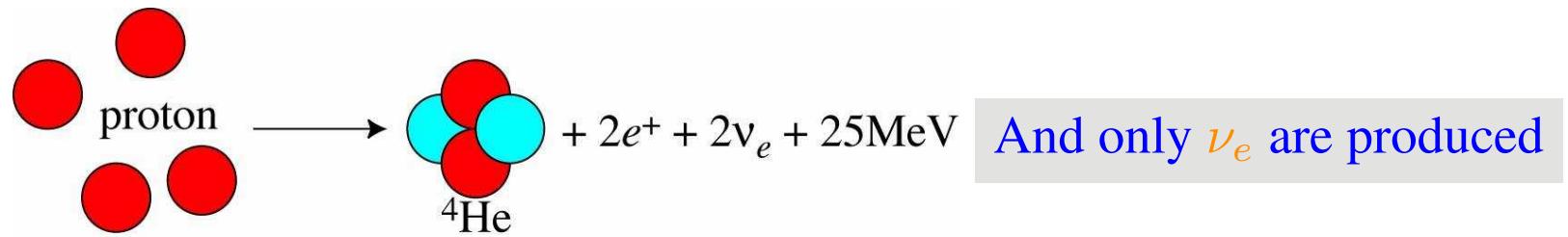
Solar Neutrinos

- Sun shines by nuclear fusion of protons into He



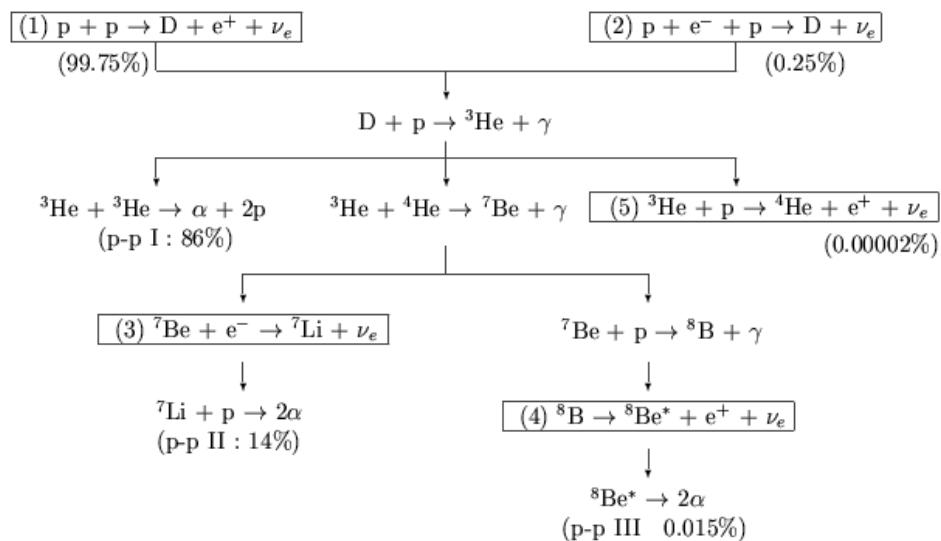
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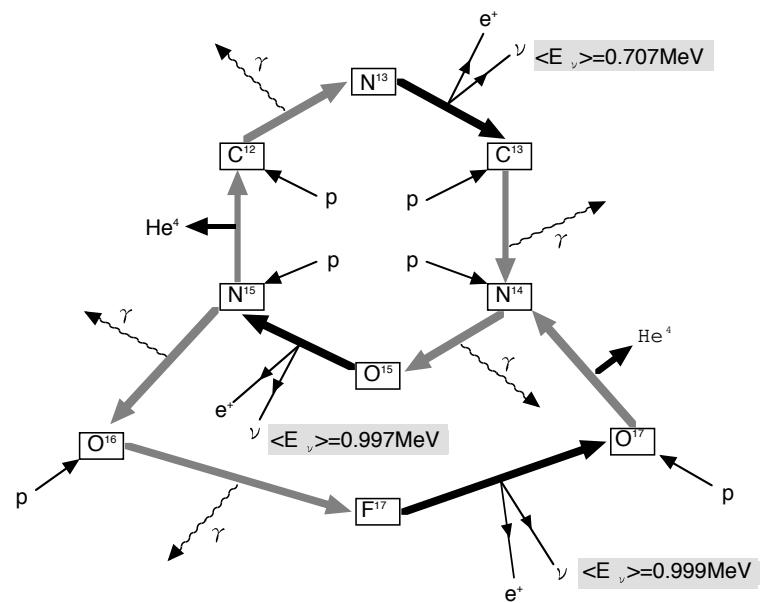


- Two main chains of nuclear reactions

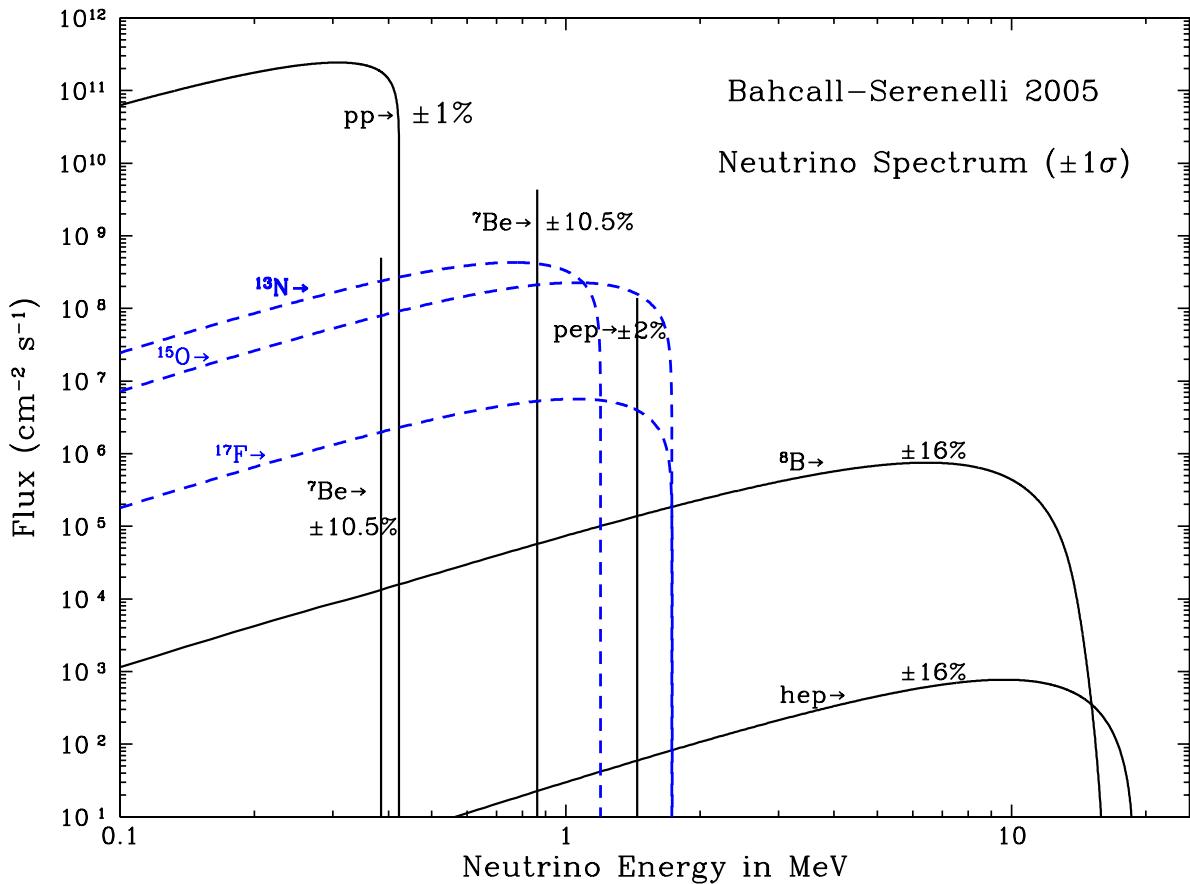
pp Chain :



CNO cycle:



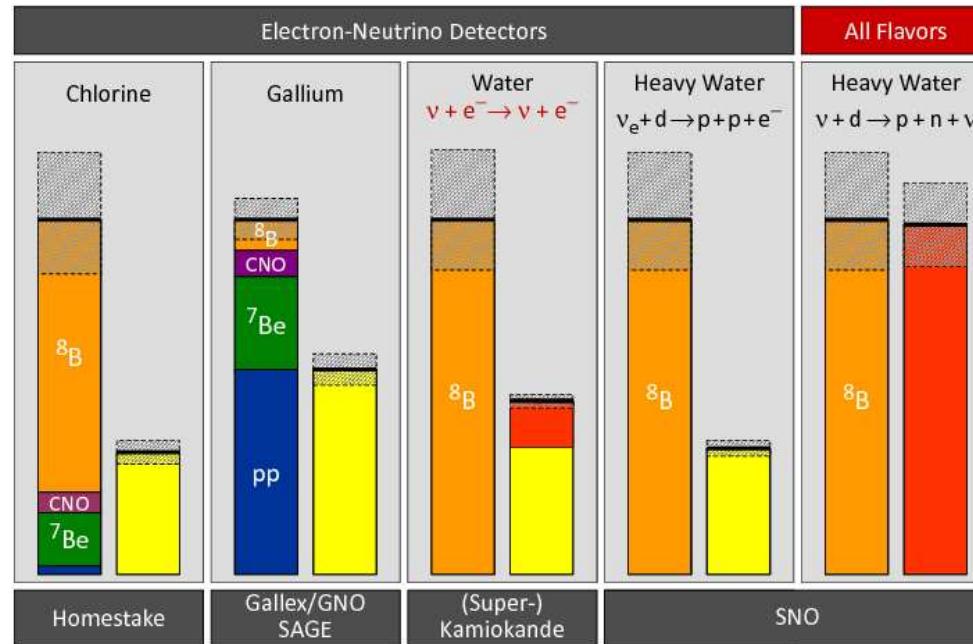
Solar Neutrinos: Fluxes



PP CHAIN	E_ν (MeV)
(pp)	≤ 0.42
$p + p \rightarrow ^2 H + e^+ + \nu_e$	
(pep)	1.552
$p + e^- + p \rightarrow ^2 H + \nu_e$	
(^7Be)	
$^7 Be + e^- \rightarrow ^7 Li + \nu_e$	0.862 (90%)
	0.384 (10%)
(hep)	
$^2 He + p \rightarrow ^4 He + e^+ + \nu_e$	≤ 18.77
(^8B)	
$^8 B \rightarrow ^8 Be^* + e^+ + \nu_e$	≤ 15
CNO CHAIN	E_ν (MeV)
(^{13}N)	≤ 1.199
$^{13} N \rightarrow ^{13} C + e^+ + \nu_e$	
(^{15}O)	≤ 1.732
$^{15} O \rightarrow ^{15} N + e^+ + \nu_e$	
(^{17}F)	≤ 1.74
$^{17} F \rightarrow ^{17} O + e^+ + \nu_e$	

Solar Neutrinos: Results

Concha Gonzalez-Garcia



Experiments measuring ν_e observe a deficit

Deficit disappears in NC

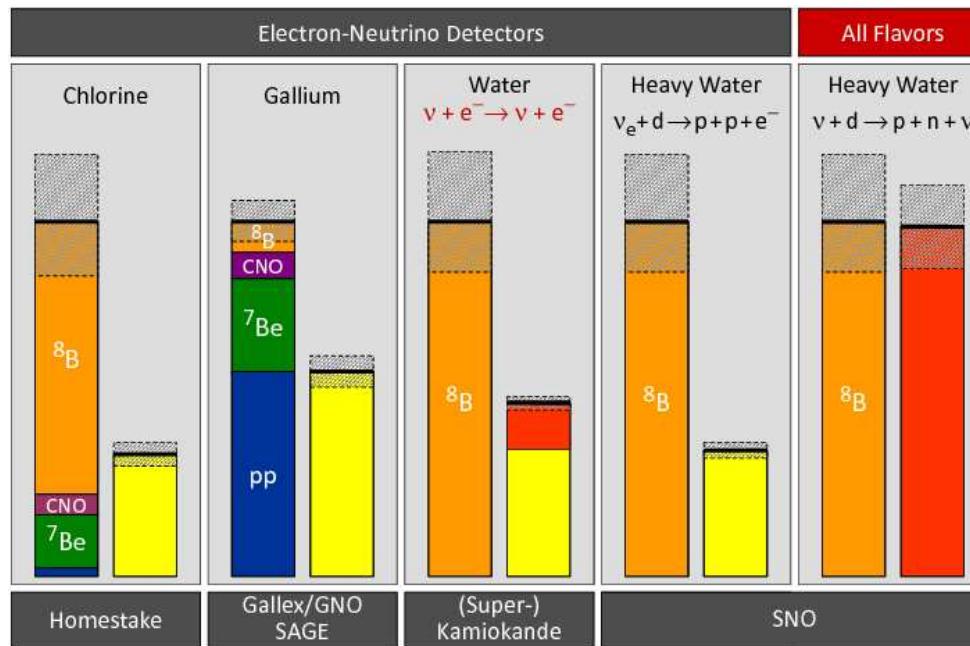
⇒ Solar Model Independent Effect

Deficit is energy dependent

Deficit ⇒ $P_{ee} \sim 30\% (< 0.5)$ for $E_\nu \gtrsim 0.8 \text{ MeV}$

Solar Neutrinos: Results

Concha Gonzalez-Garcia



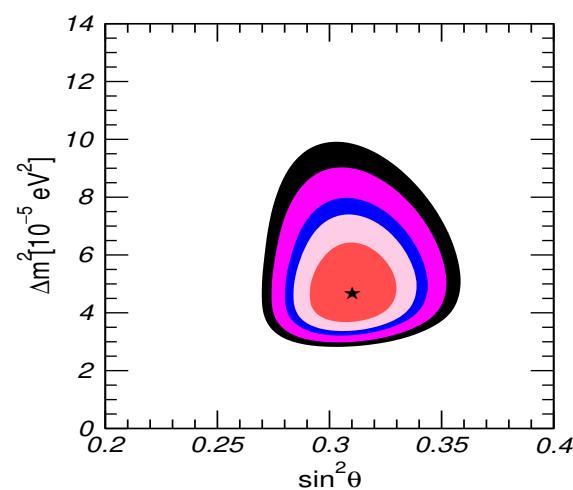
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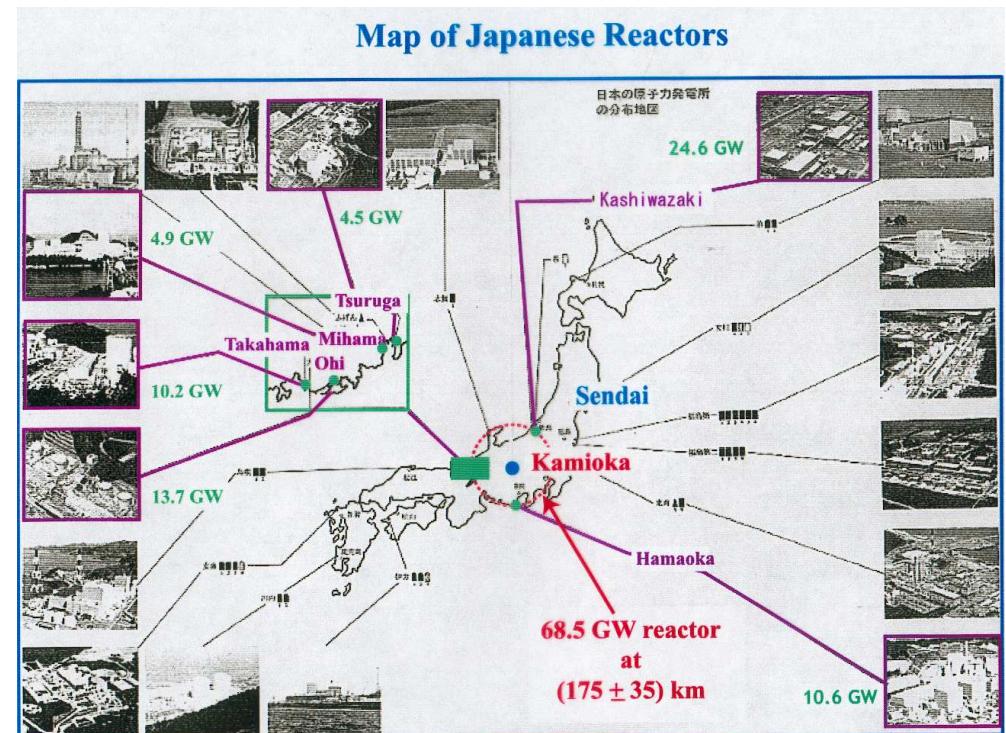
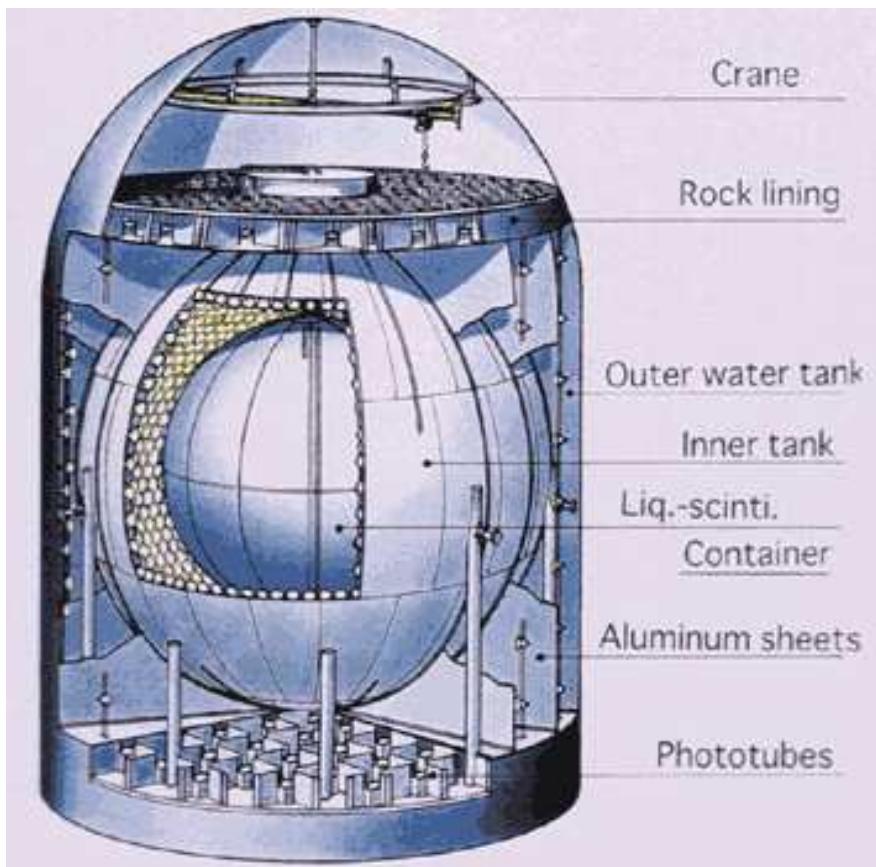
Deficit $\Rightarrow P_{ee} \sim 30\% (< 0.5)$ for $E_\nu \gtrsim 0.8$ MeV



$$\Delta m^2 \sim 5 \times 10^{-5} \text{ eV}^2, \theta \sim \frac{\pi}{6}$$

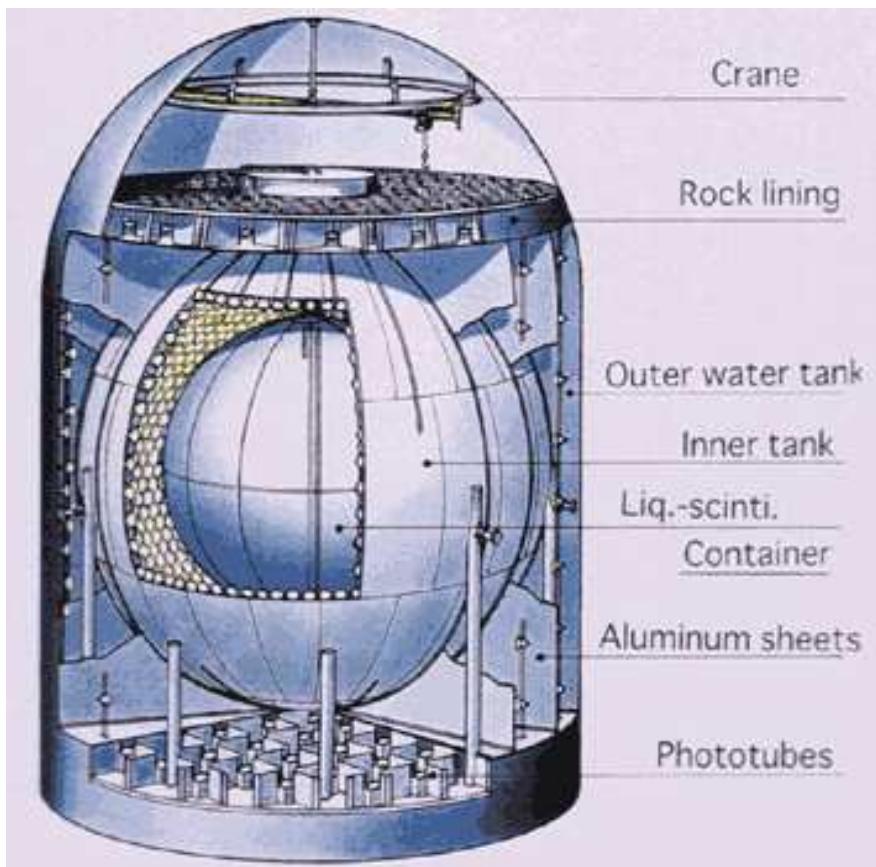
Terrestrial test: KamLAND

KamLAND: Detector of $\bar{\nu}_e$ produced in nuclear reactors in Japan at an average distance of 180 Km



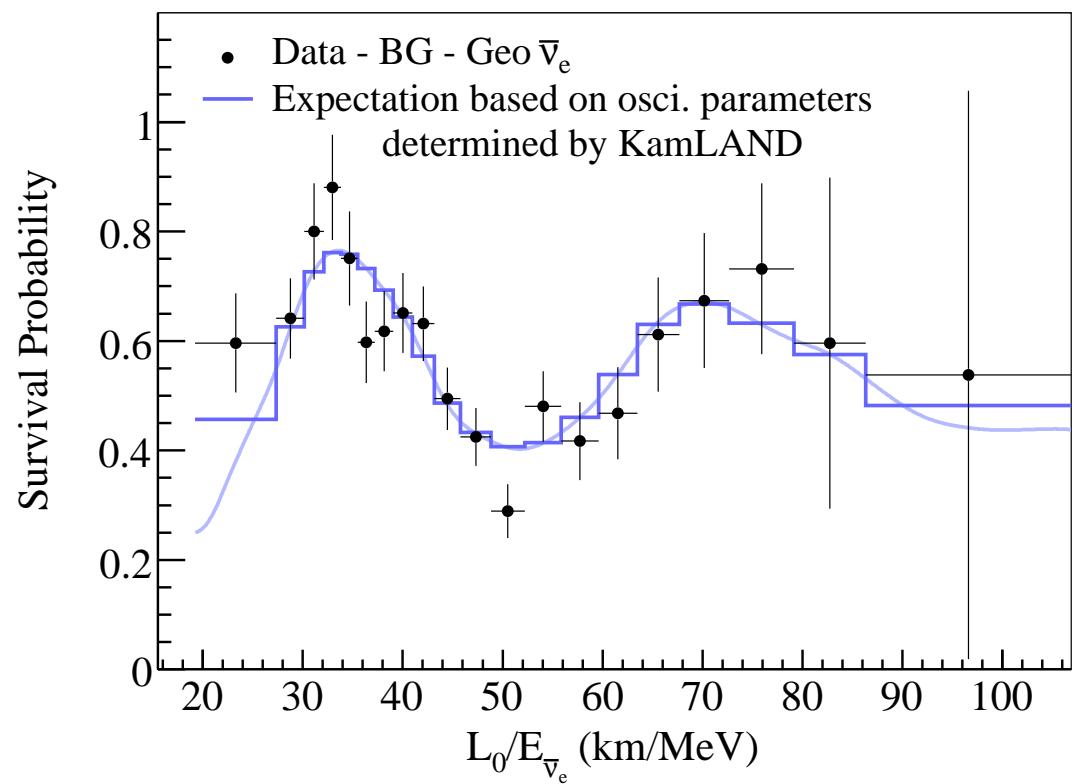
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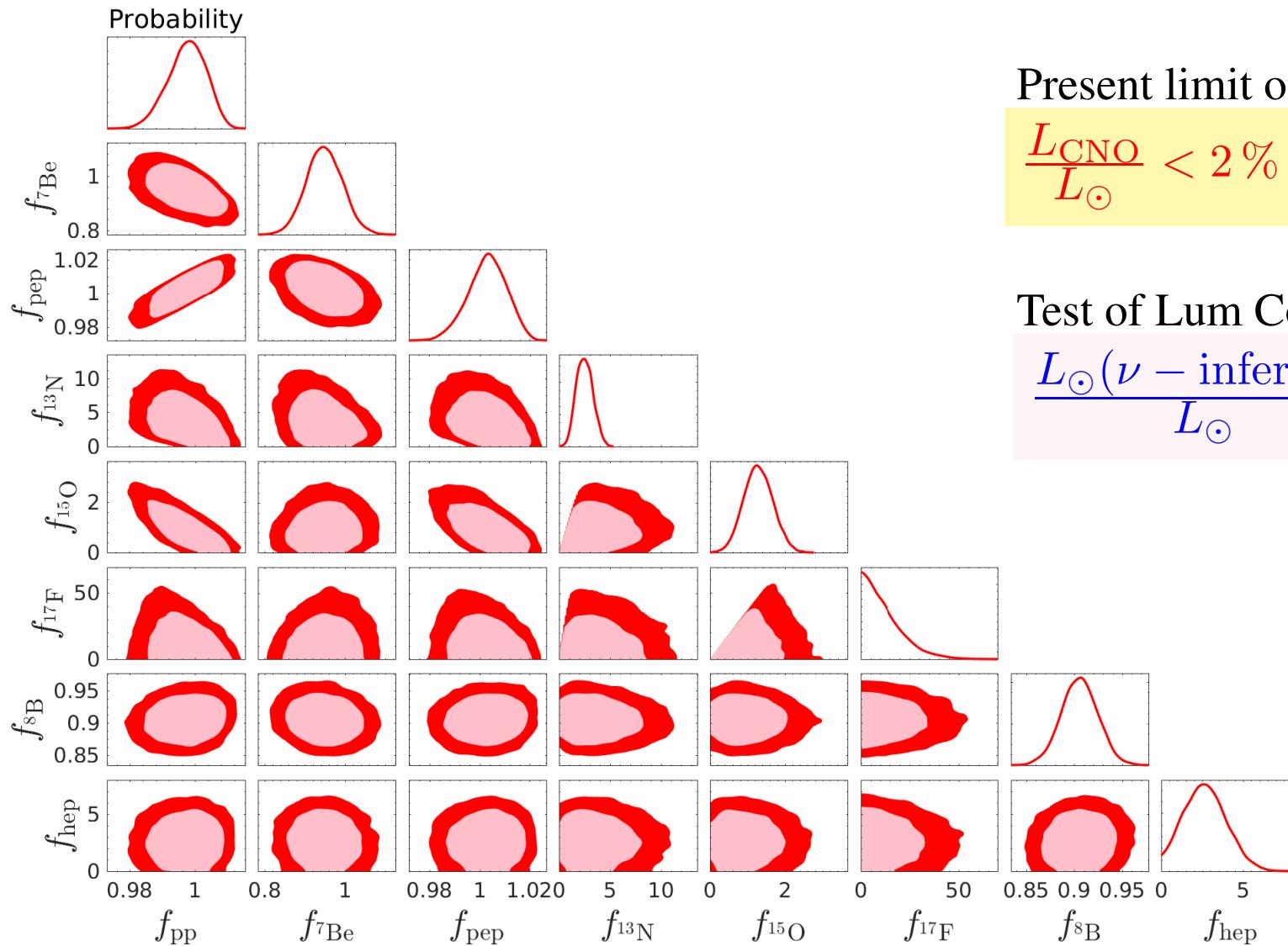
Results of KamLAND
compared with P_{ee} for

$$\theta = 35^\circ \text{ y } \Delta m^2 = 7.5 \times 10^{-5} (\text{eV}/c^2)^2$$



Byproduct: Testing How the Sun Shines with ν' s

Fitting together Δm^2 , θ and normalization of ν -producing reactions: $f_i = \frac{\Phi_i}{\Phi_{SSM}^{SSM}}$
 \Rightarrow Constraint on solar energy produced by nuclear



Present limit on CNO:

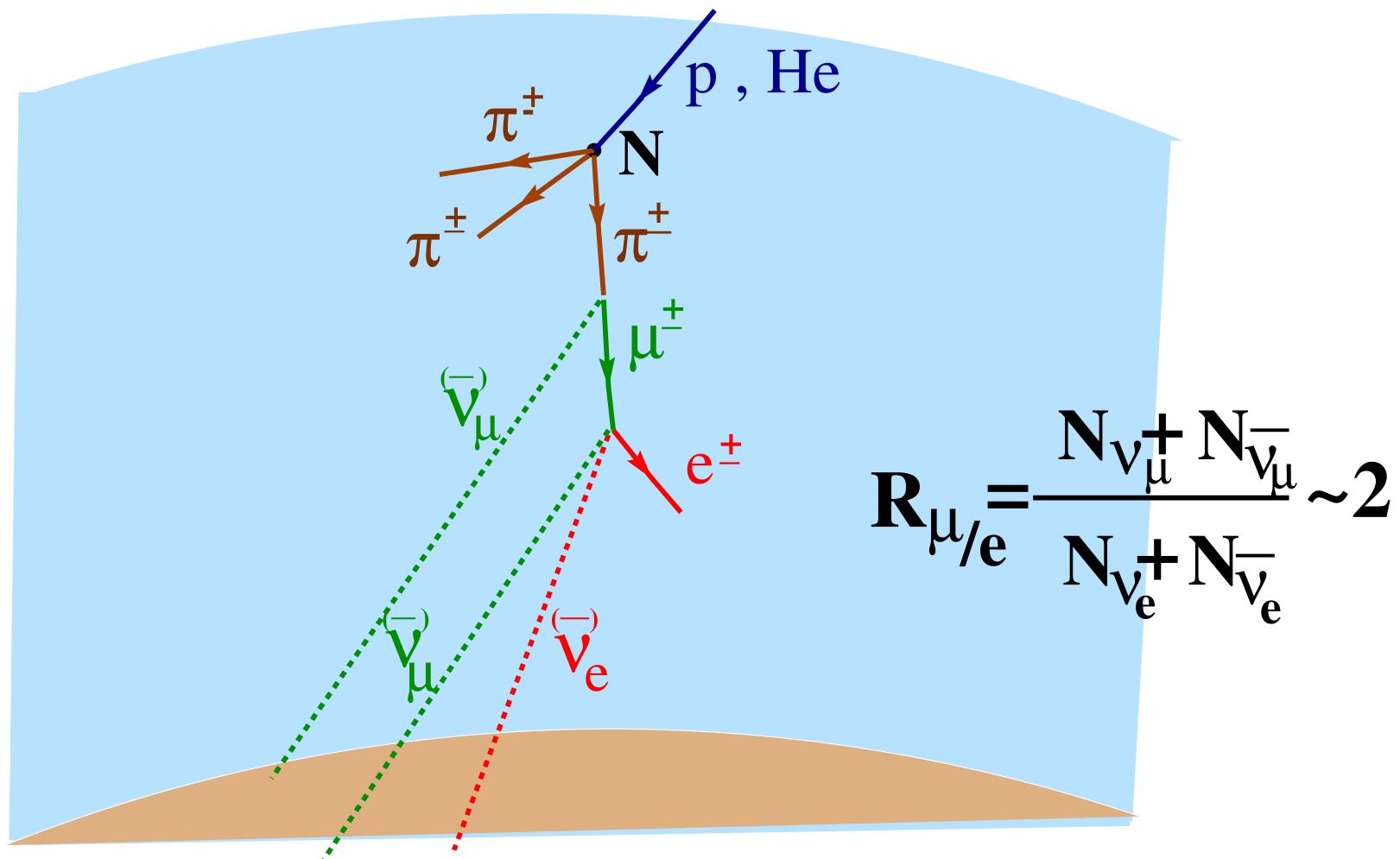
$$\frac{L_{\text{CNO}}}{L_{\odot}} < 2\% \text{ (3}\sigma\text{)}$$

Test of Lum Constraint:

$$\frac{L_{\odot}(\nu - \text{inferred})}{L_{\odot}} = 1.04 \pm 0.07$$

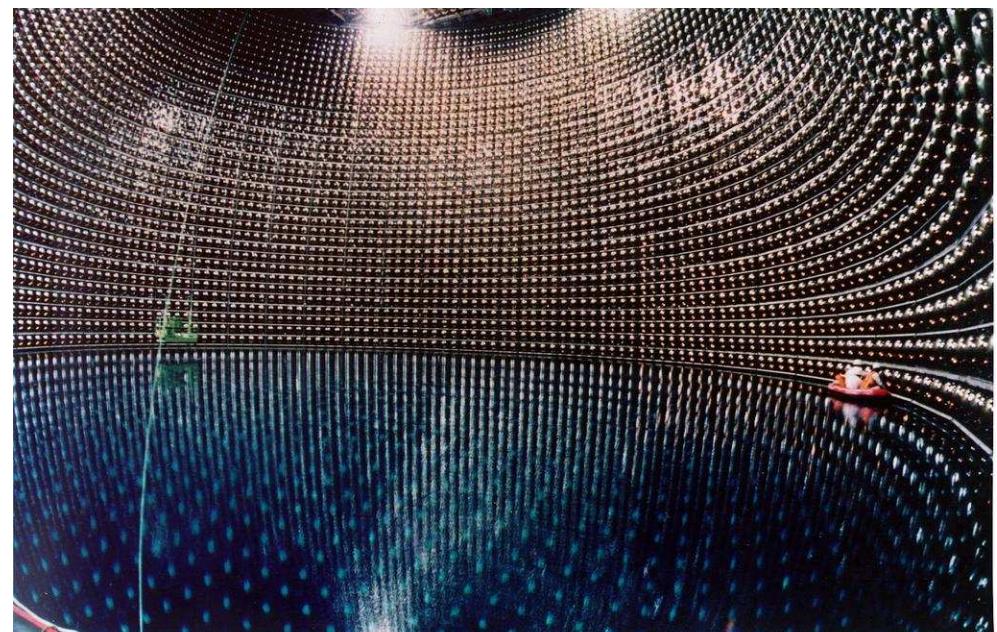
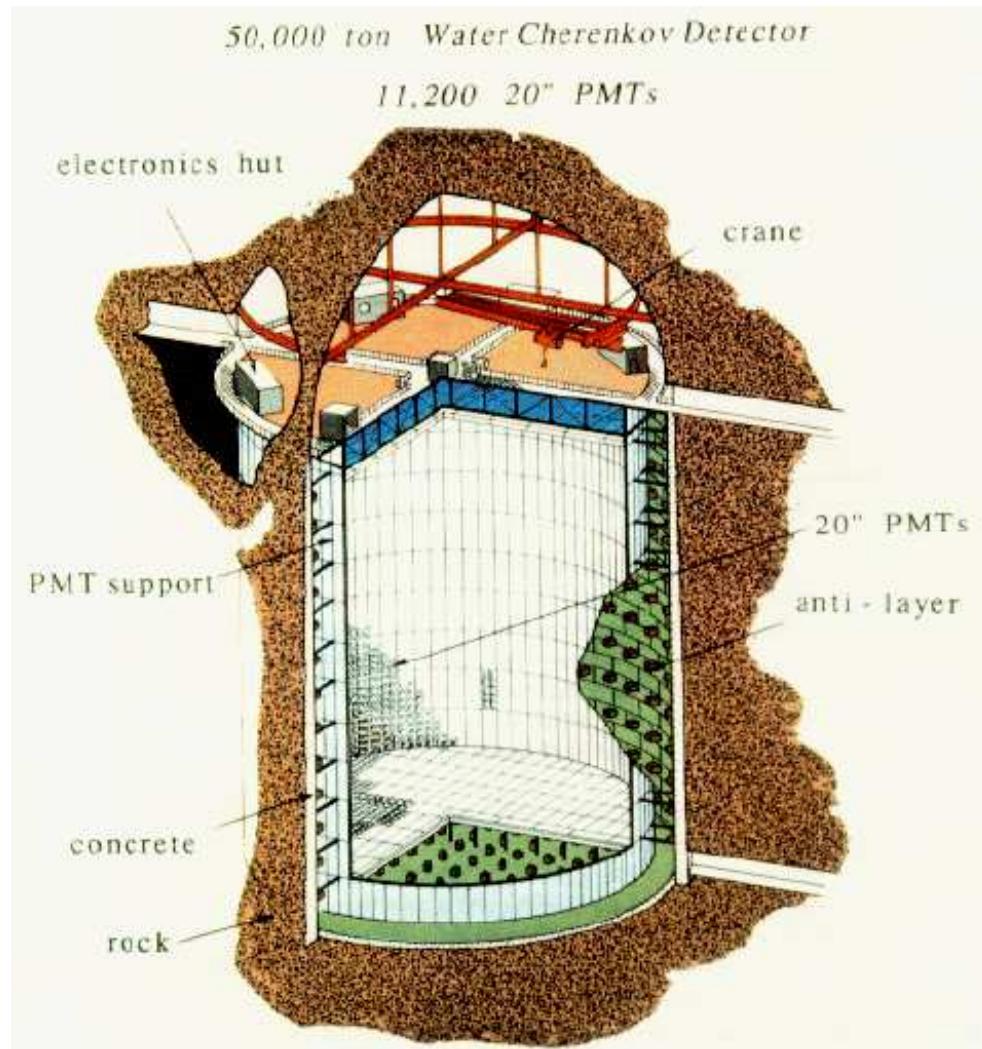
Atmospheric Neutrinos

Atmospheric $\nu_{e,\mu}$ are produced by the interaction of cosmic rays (p, He ...) with the atmosphere



Detection of Atmospheric Neutrinos: SuperKamiokande

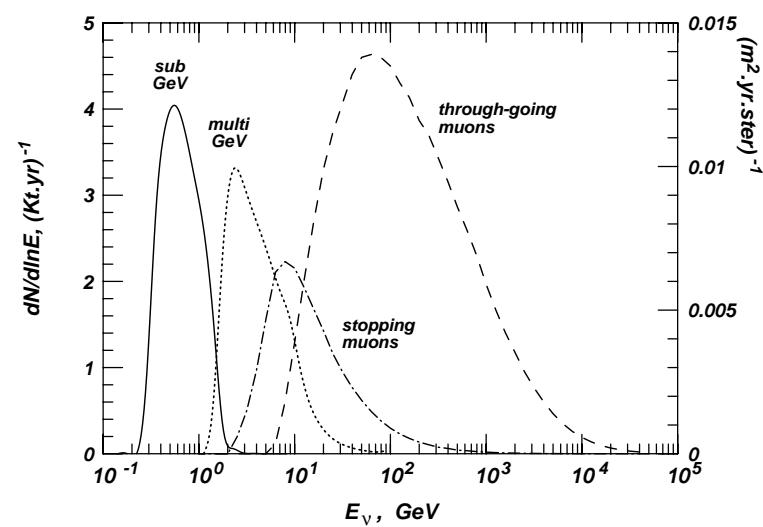
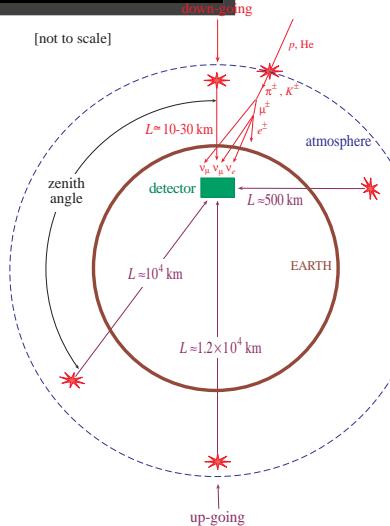
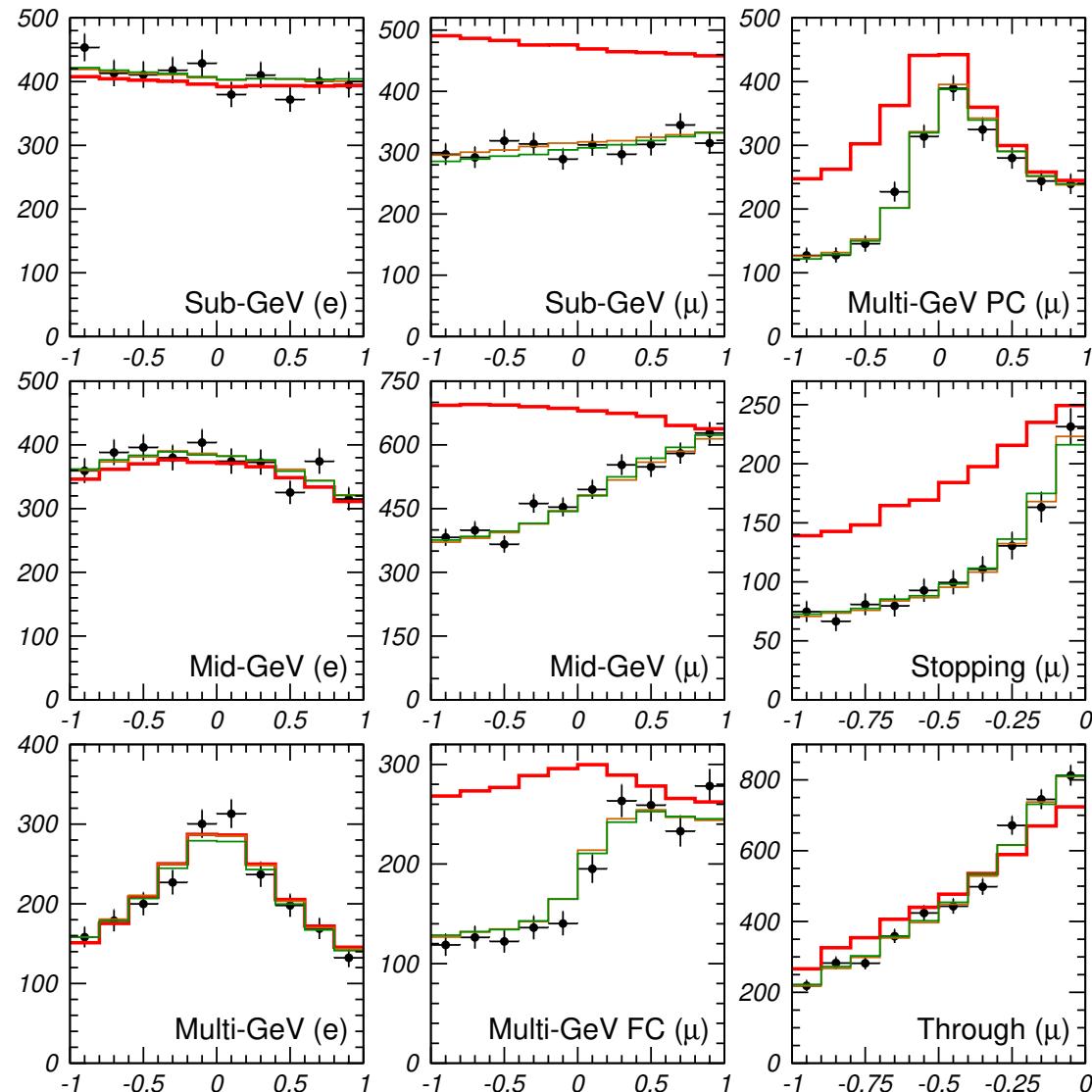
Located in the Kamiokande mine in the center of Japan at $\sim 1\text{ Km}$ deep
50 Kton of water surrounded by ~ 12000 photomultipliers



Atmospheric Neutrinos: Results

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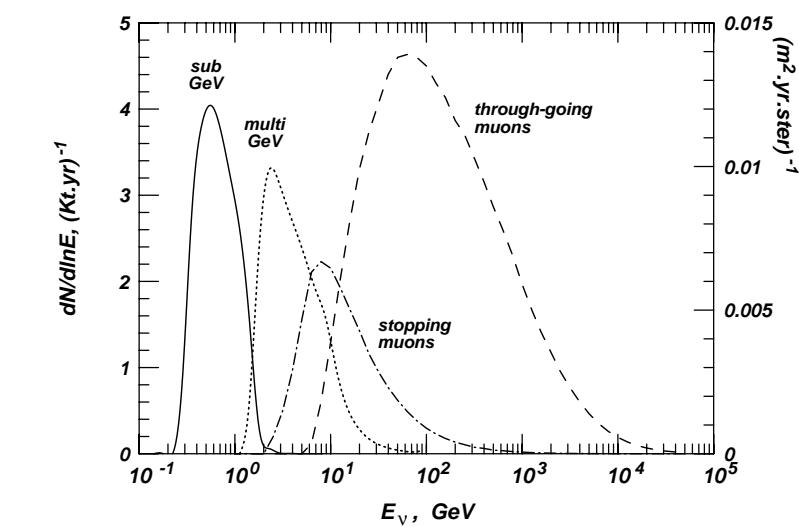
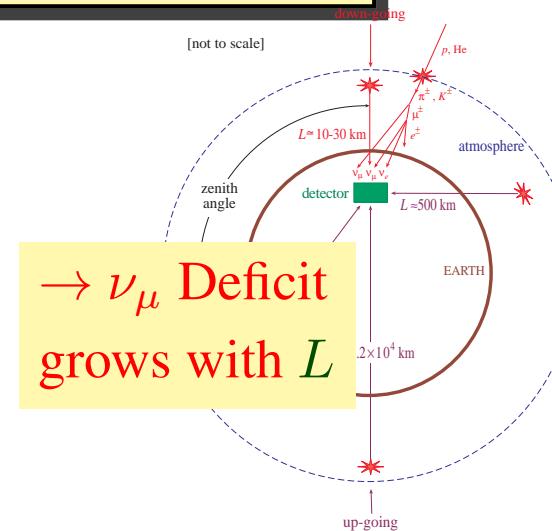
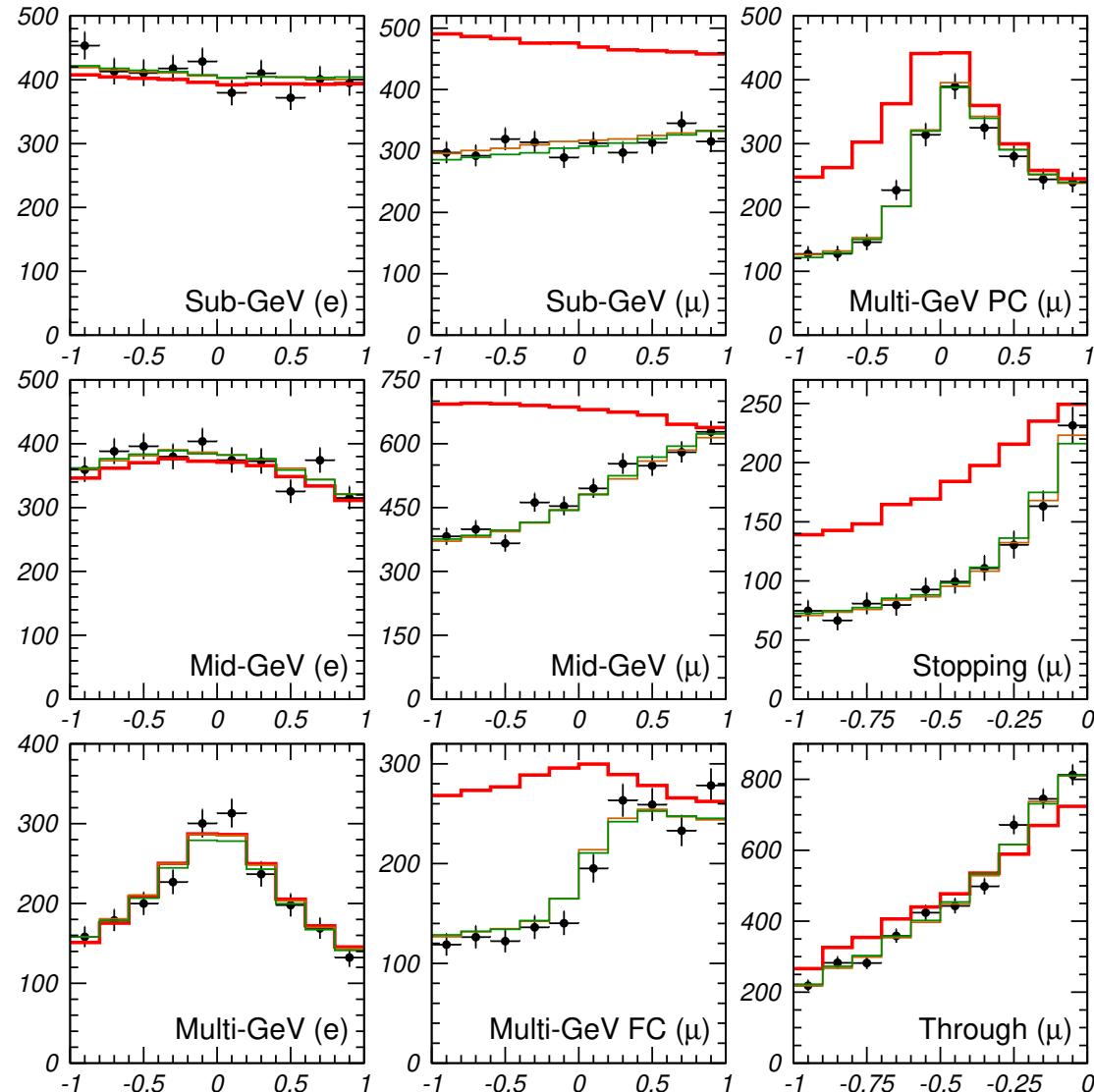
- SKI+II+III+IV data:



Atmospheric Neutrinos: Results

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- SKI+II+III+IV data:

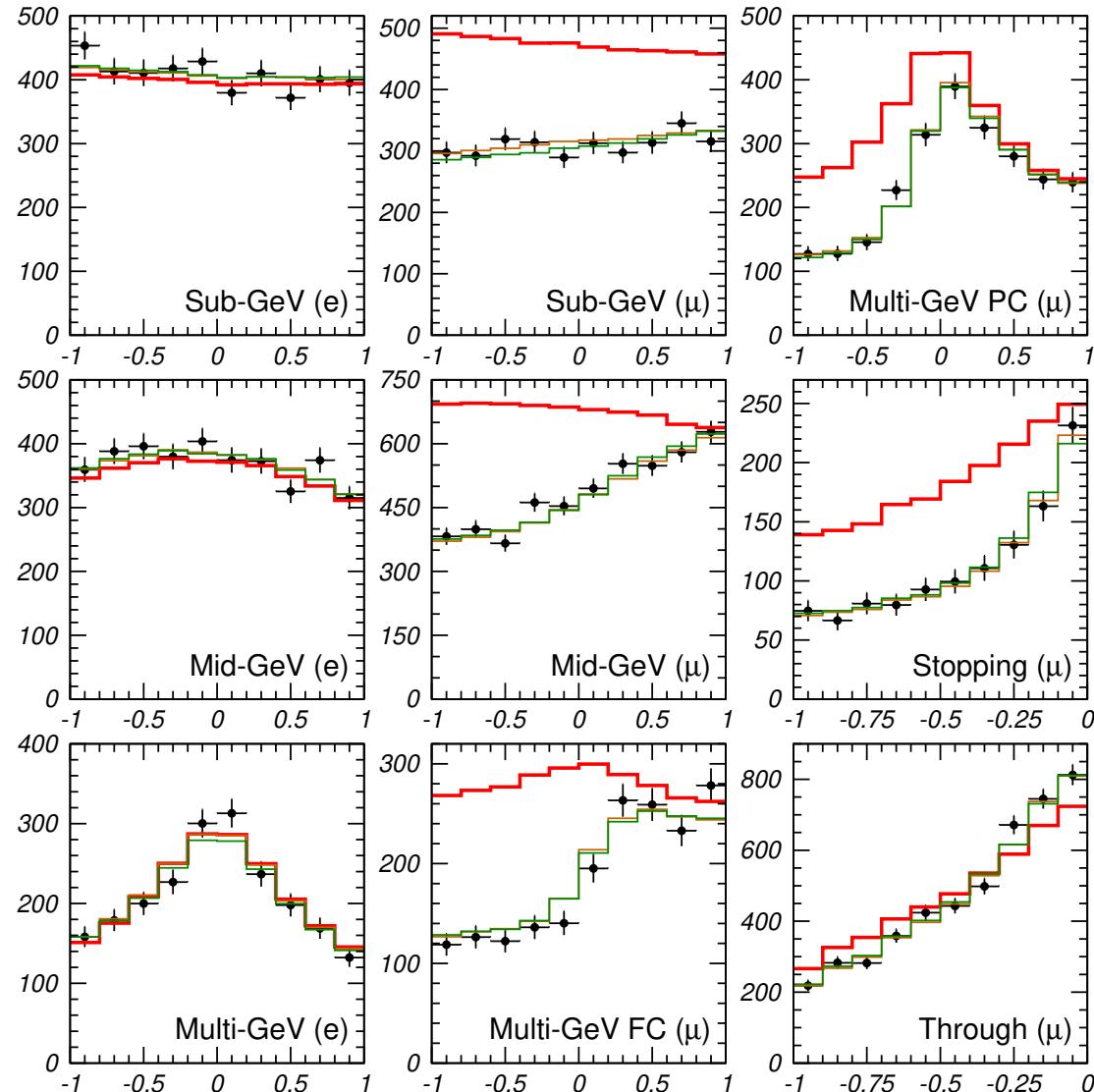


→ ν_μ Deficit decreases with E

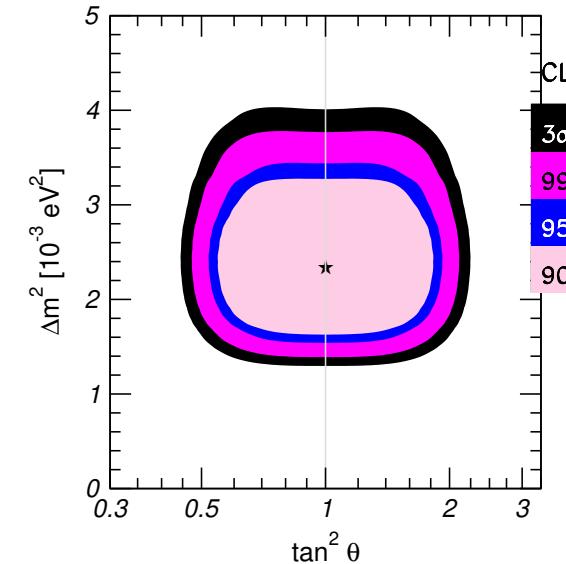
Atmospheric Neutrinos: Results

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- SKI+II+III+IV data:



Best explained by $\nu_\mu \rightarrow \nu_\tau$



$$\Delta m^2 \sim 2.4 \times 10^{-3} \text{ eV}^2$$

$$\tan^2 \theta \sim 1 \Rightarrow \theta \sim \frac{\pi}{4}$$

Alternative Oscillation Mechanisms

- Oscillations are due to:
 - Misalignment between CC-int and propagation states: Mixing \Rightarrow Amplitude
 - Difference phases of propagation states \Rightarrow Wavelength. For Δm^2 -OSC $\lambda = \frac{4\pi E}{\Delta m^2}$

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 - Misalignment between CC-int and propagation states: Mixing \Rightarrow Amplitude
 - Difference phases of propagation states \Rightarrow Wavelength. For Δm^2 -OSC $\lambda = \frac{4\pi E}{\Delta m^2}$

- ν masses are not the only mechanism for oscillations

Violation of Equivalence Principle (VEP): Gasperini 88, Halprin,Leung 01

Non universal coupling of neutrinos $\gamma_1 \neq \gamma_2$ to gravitational potential ϕ

Violation of Lorentz Invariance (VLI): Coleman, Glashow 97

Non universal asymptotic velocity of neutrinos $c_1 \neq c_2 \Rightarrow E_i = \frac{m_i^2}{2p} + c_i p$

Interactions with space-time torsion: Sabbata, Gasperini 81

Non universal couplings of neutrinos $k_1 \neq k_2$ to torsion strength Q

Violation of Lorentz Invariance (VLI) Colladay, Kostelecky 97; Coleman, Glashow 99

due to CPT violating terms: $\bar{\nu}_L^\alpha b_\mu^{\alpha\beta} \gamma_\mu \nu_L^\beta \Rightarrow E_i = \frac{m_i^2}{2p} \pm b_i$

$$\lambda = \frac{\pi}{E|\phi|\delta\gamma}$$

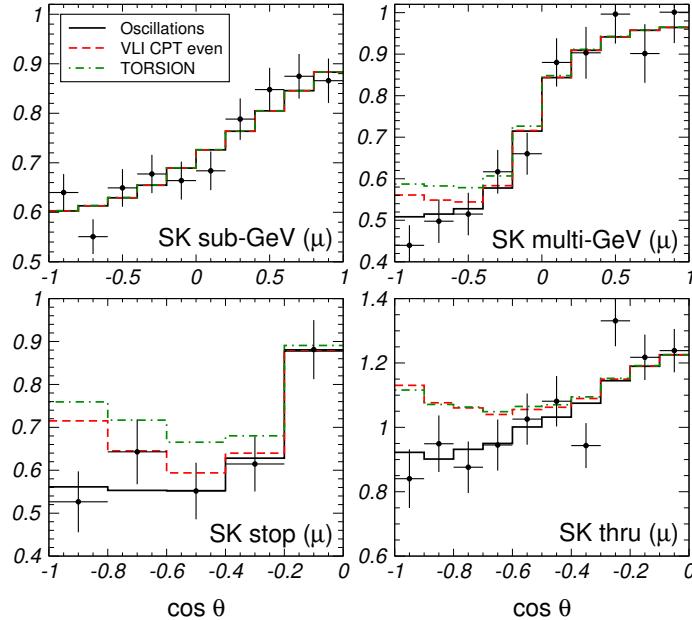
$$\lambda = \frac{2\pi}{E\Delta c}$$

$$\lambda = \frac{2\pi}{Q\Delta k}$$

$$\lambda = \pm \frac{2\pi}{\Delta b}$$

Alternative Mechanisms vs ATM ν 's

- Strongly constrained with ATM data (MCG-G, M. Maltoni PRD 04,07)



$$\frac{|\Delta c|}{c} \leq 1.2 \times 10^{-24}$$

$$|\phi \Delta \gamma| \leq 5.9 \times 10^{-25}$$

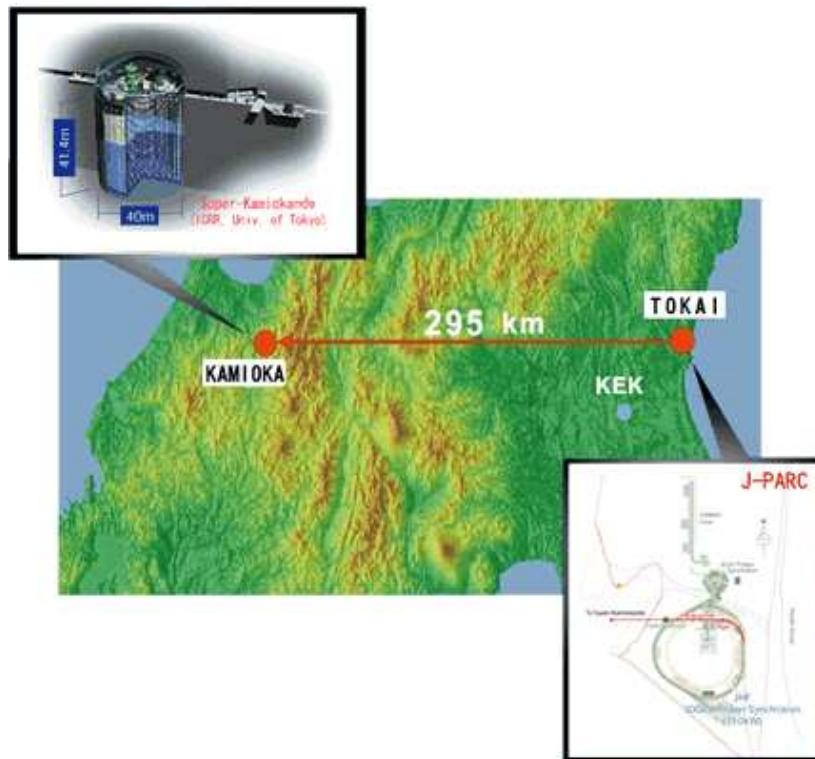
At 90% CL: $|Q \Delta k| \leq 4.8 \times 10^{-23} \text{ GeV}$

$$|\Delta b| \leq 3.0 \times 10^{-23} \text{ GeV}$$

ν_μ Disappearance in Accelerator ν Fluxes

T2K:

ν_μ produced in Tokai (Japan)
detected in SK at ~ 250 Km



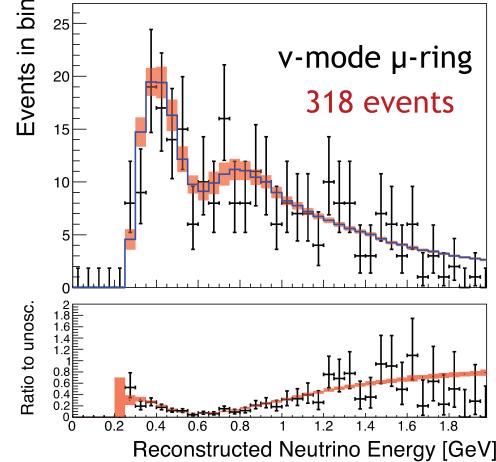
MINOS, NO ν A

ν_μ produced en Fermilab (Illinois)
detected in Minnesota at ~ 800 Km

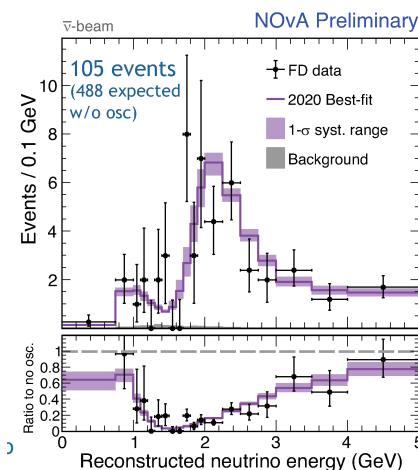
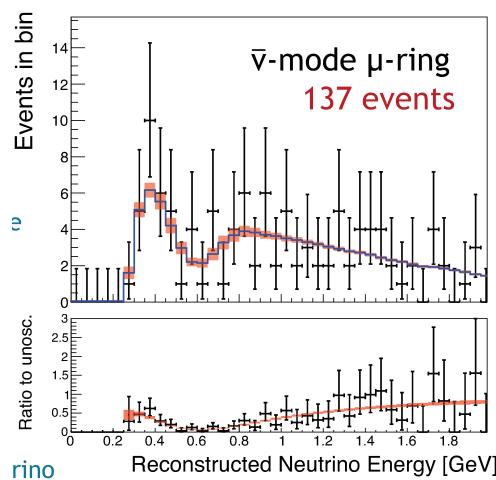
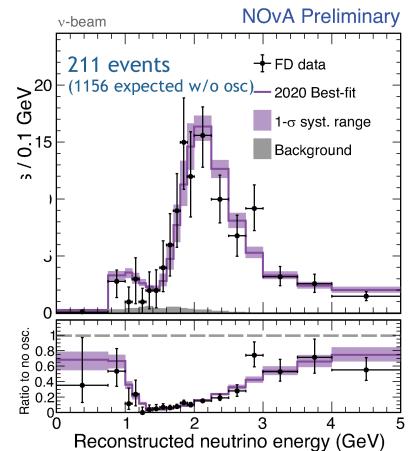


Long Baseline Experiments: ν_μ Disappearance

K2K/T2K 2004–:



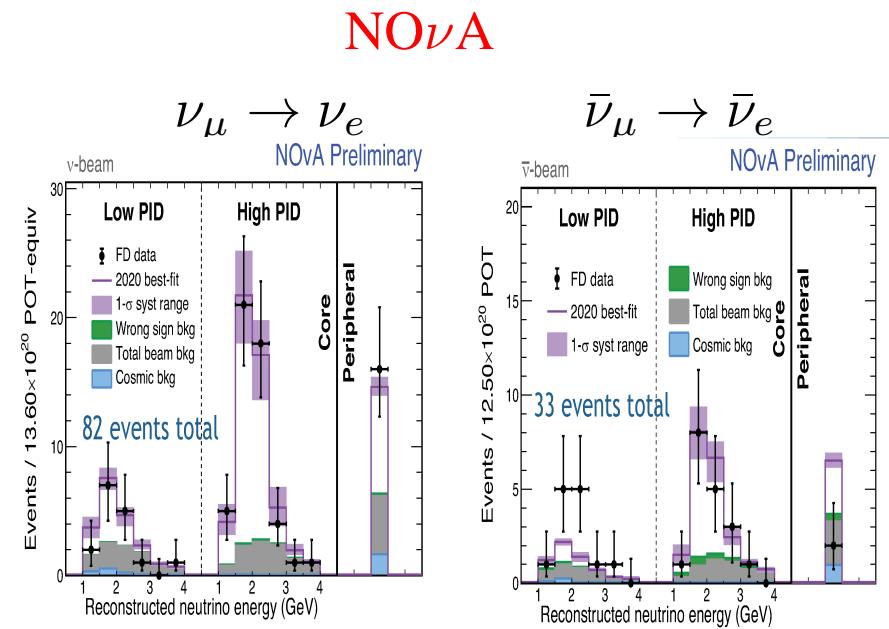
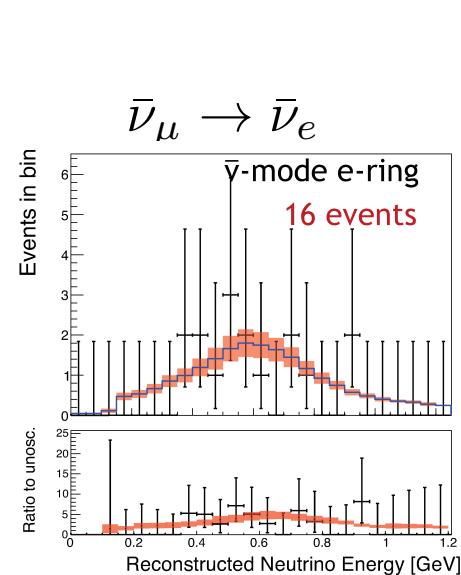
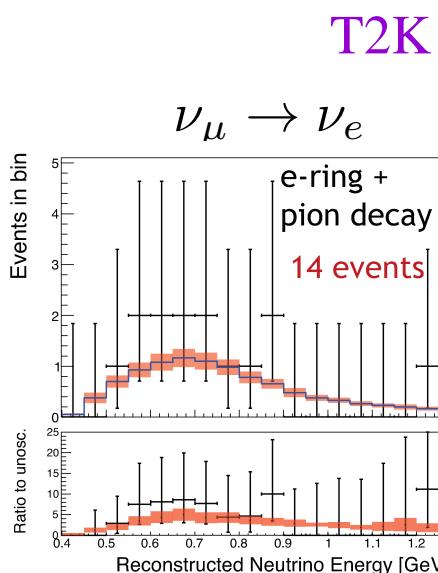
NO ν A: 2015–



ν_μ oscillations with $\Delta m^2 \sim 2.5 \times 10^{-3}$ eV² and mixing compatible with $\frac{\pi}{4}$

Long Baseline Experiments: ν_e Appearance

- Observation of $\nu_\mu \rightarrow \nu_e$ transitions with $E/L \sim 10^{-3}$ eV²



- Test of $P(\nu_\mu \rightarrow \nu_e)$ vs $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ \Rightarrow Leptonic CP violation

Medium Baseline Reactor Experiments

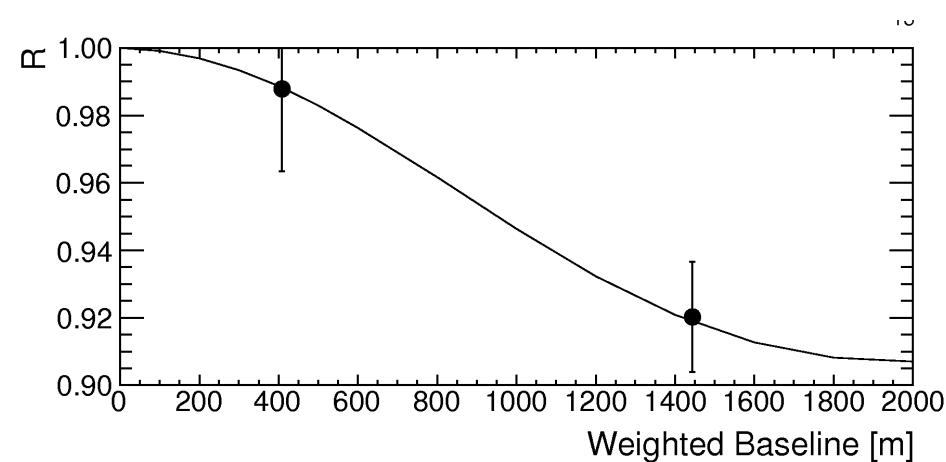
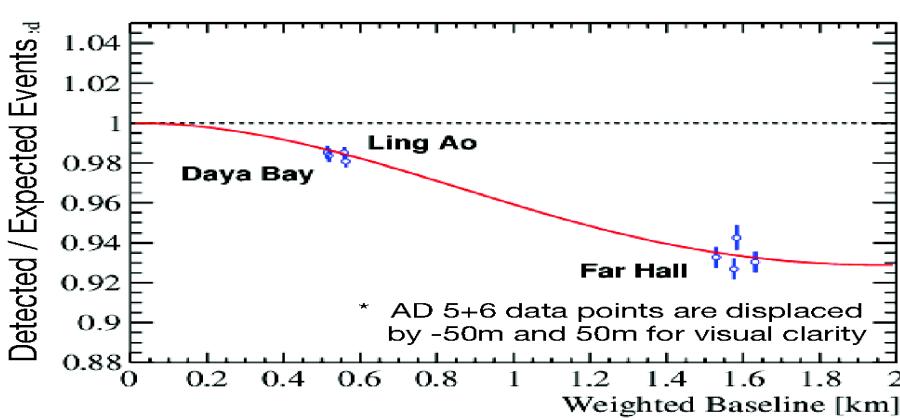
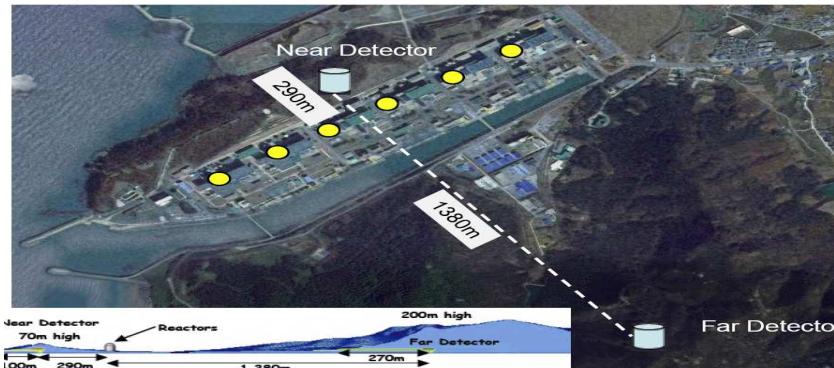
Gonzalez-Garcia

- Searches for $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance at $L \sim \text{Km}$ ($E/L \sim 10^{-3} \text{ eV}^2$)
- Relative measurement: near and far detectors

Daya-Bay



Reno



Medium Baseline Reactor Experiments

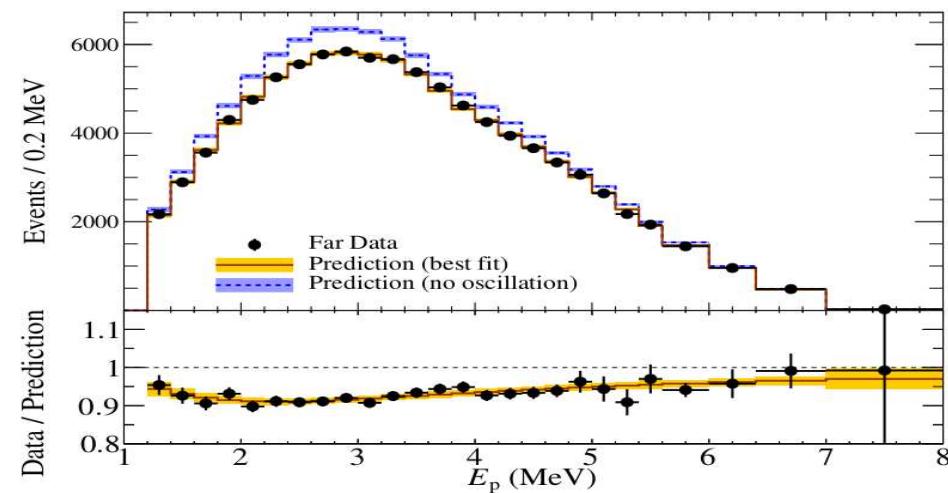
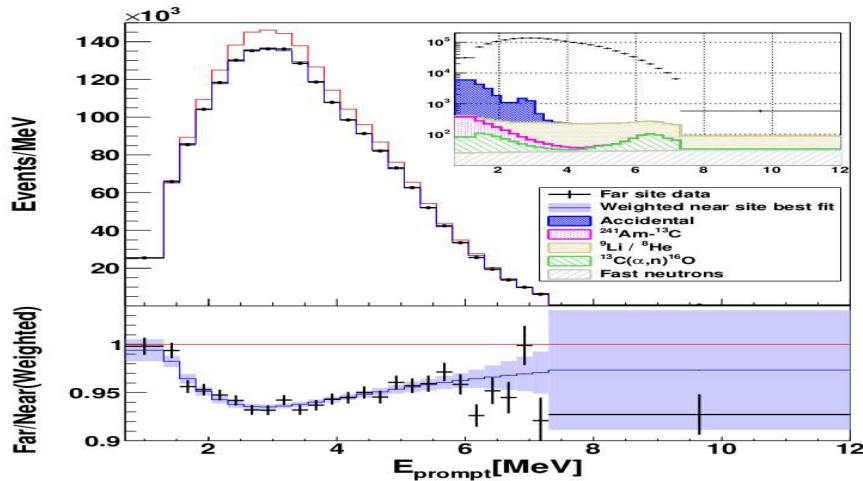
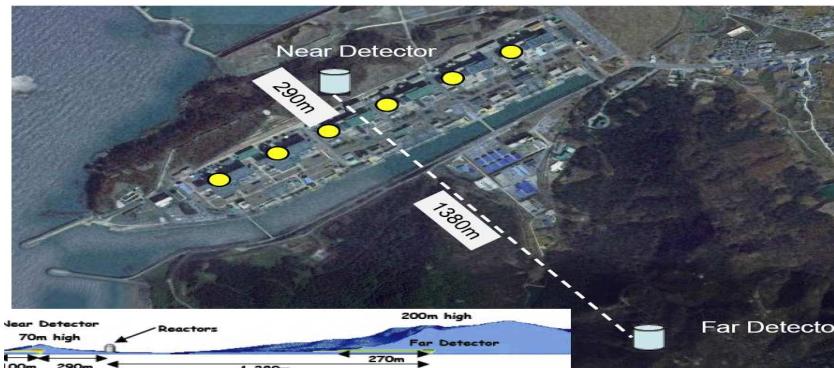
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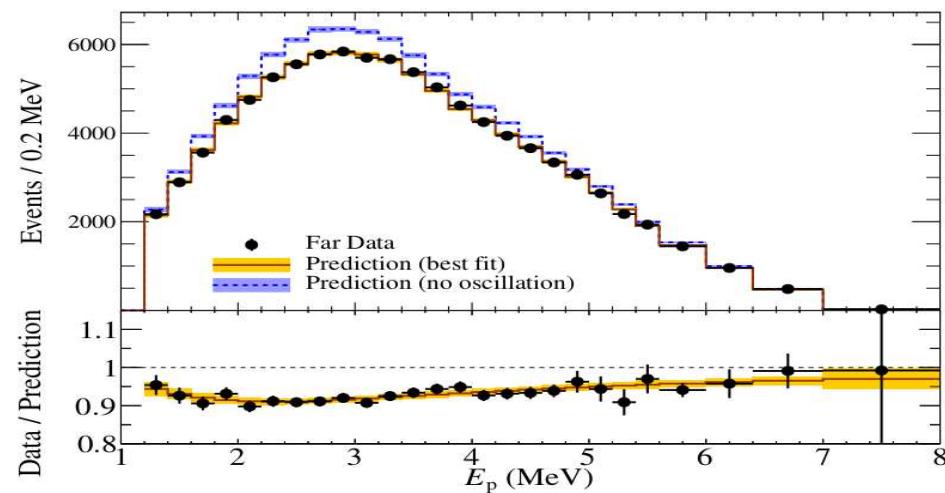
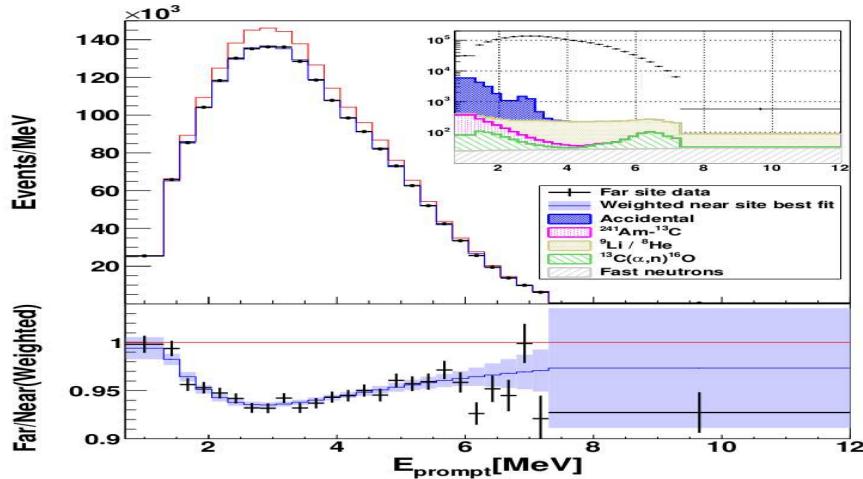
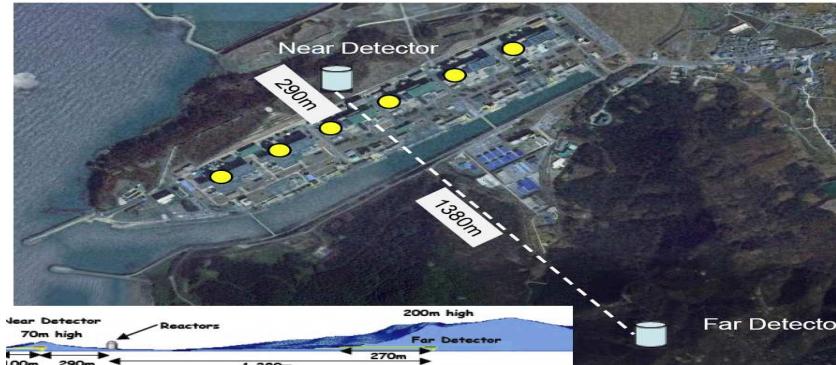
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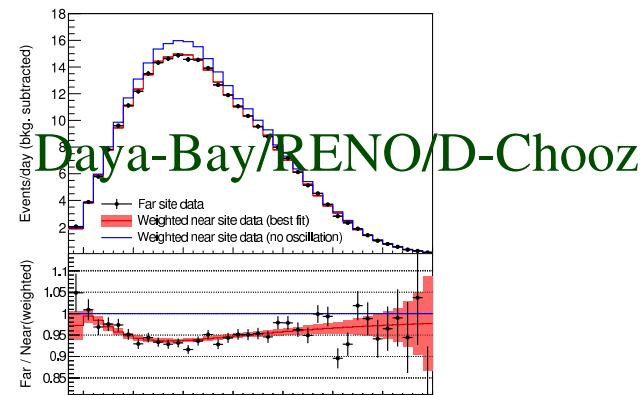
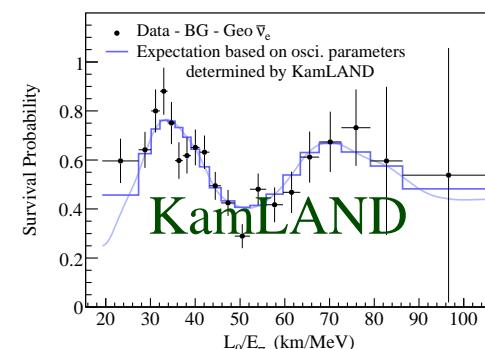
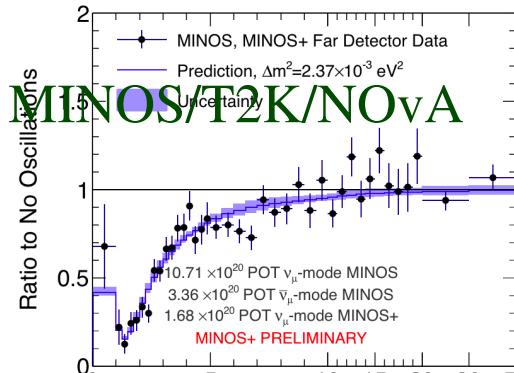
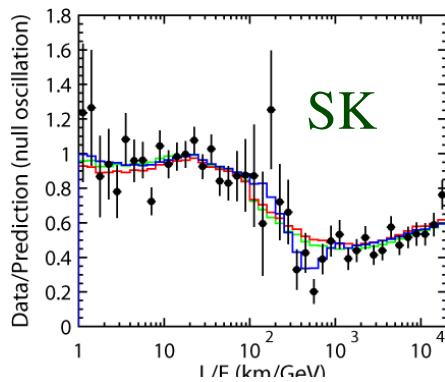


Described with $\Delta m^2 \sim 2.5 \times 10^{-3} \text{ eV}^2$ (as ν_μ ATM and LBL acc but for ν_e) and $\theta \sim 9^\circ$

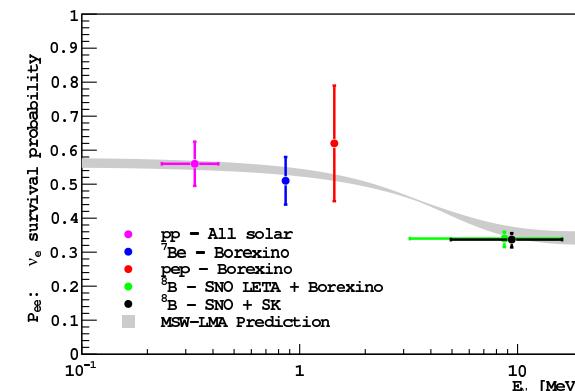
- We have observed with high (or good) precision:

- * Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (**SK, MINOS, ICECUBE**) $\frac{\Delta m^2}{eV^2} \sim 2 \cdot 10^{-3}$
- * Accel. ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 300/800$ Km (**K2K, T2K, MINOS, NO ν A**) $\theta \sim 45^\circ$
- * Some accelerator ν_μ appear as ν_e at $L \sim 300/800$ Km (**T2K, MINOS, NO ν A**) $\theta \sim 8^\circ$
- * Solar ν_e convert to ν_μ/ν_τ (**Cl, Ga, SK, SNO, Borexino**) $\frac{\Delta m^2}{eV^2} \sim 10^{-5}, \theta \sim 30^\circ$
- * Reactor $\bar{\nu}_e$ disappear at $L \sim 200$ Km (**KamLAND**)
- * Reactor $\bar{\nu}_e$ disappear at $L \sim 1$ Km (**D-Chooz, Daya Bay, Reno**) $\frac{\Delta m^2}{eV^2} \sim 2 \cdot 10^{-3}, \theta \sim 8^\circ$

- Confirmed Vacuum oscillation L/E pattern with 2 frequencies



MSW conversion in Sun



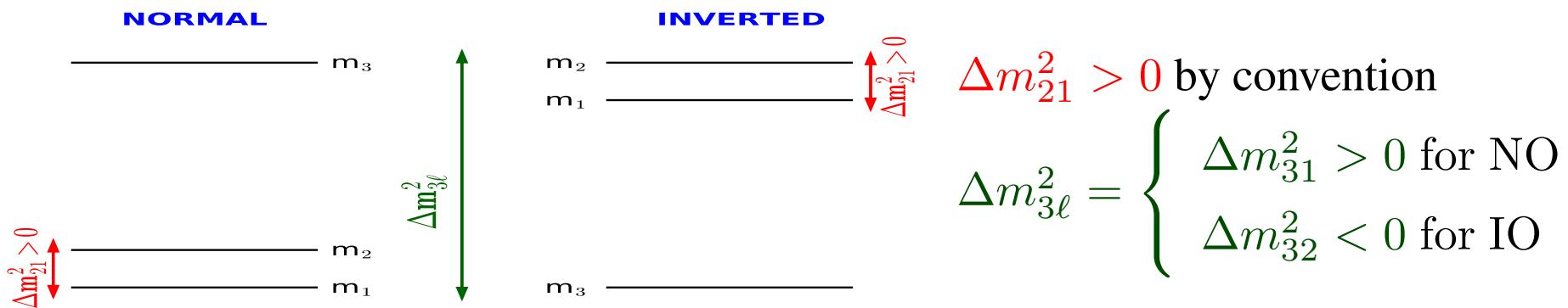
3 ν Flavour Parameters

Concha Gonzalez-Garcia

- For 3 ν 's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\text{LEP}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\text{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Convention: $0 \leq \theta_{ij} \leq 90^\circ$ $0 \leq \delta \leq 360^\circ \Rightarrow$ 2 Orderings

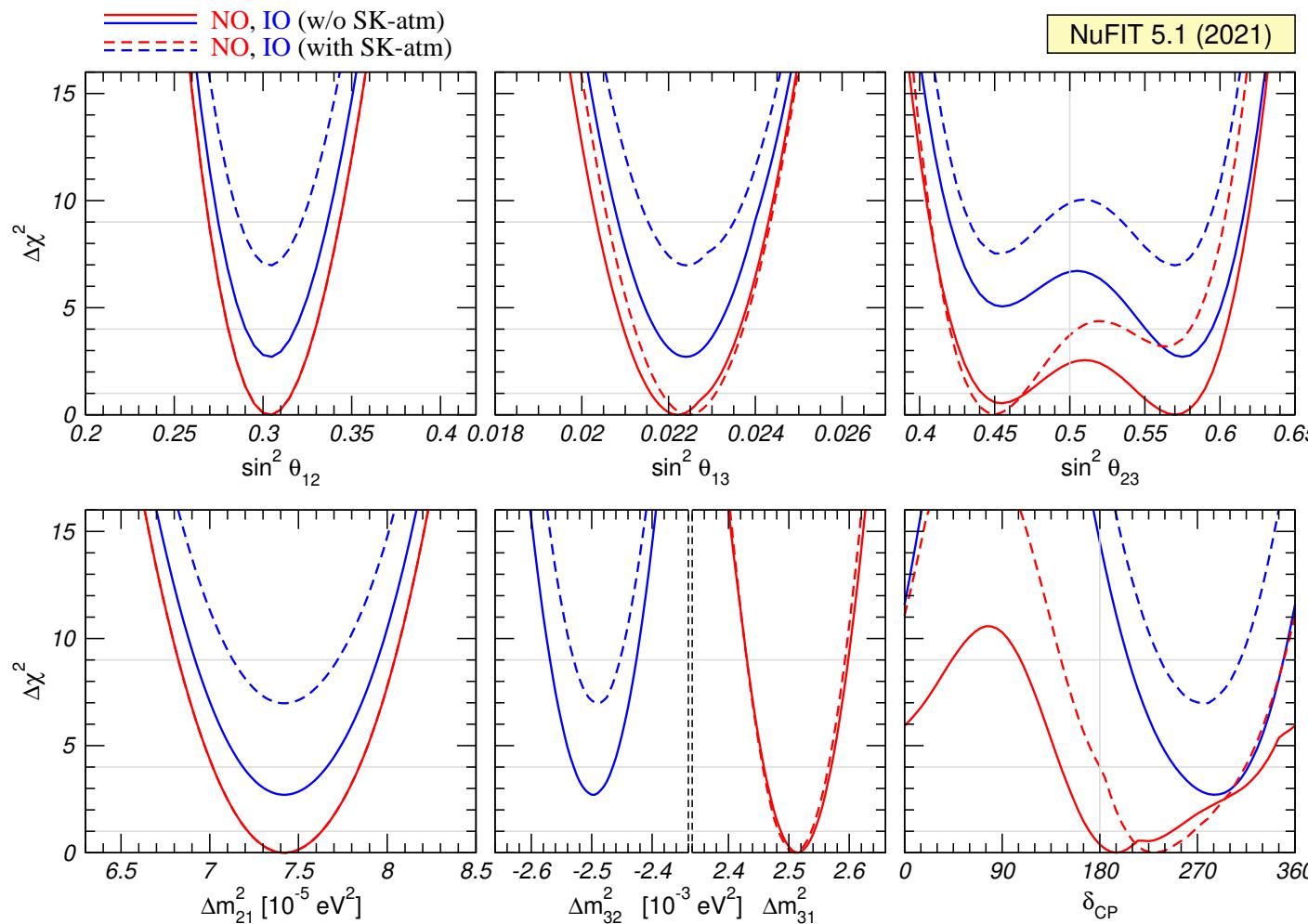


Experiment	Dominant Dependence	Important Dependence
Solar Experiments	θ_{12}	$\Delta m_{21}^2, \theta_{13}$
Reactor LBL (KamLAND)	Δm_{21}^2	θ_{12}, θ_{13}
Reactor MBL (Daya Bay, Reno, D-Chooz)	$\theta_{13} \Delta m_{3\ell}^2$	
Atmospheric Experiments (SK, IC)		$\theta_{23}, \Delta m_{3\ell}^2, \theta_{13}, \delta_{\text{CP}}$
Acc LBL ν_μ Disapp (Minos, T2K, NOvA)	$\Delta m_{3\ell}^2 \theta_{23}$	
Acc LBL ν_e App (Minos, T2K, NOvA)	δ_{CP}	θ_{13}, θ_{23}

Summary: Global 3 ν Flavour Parameters

Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]

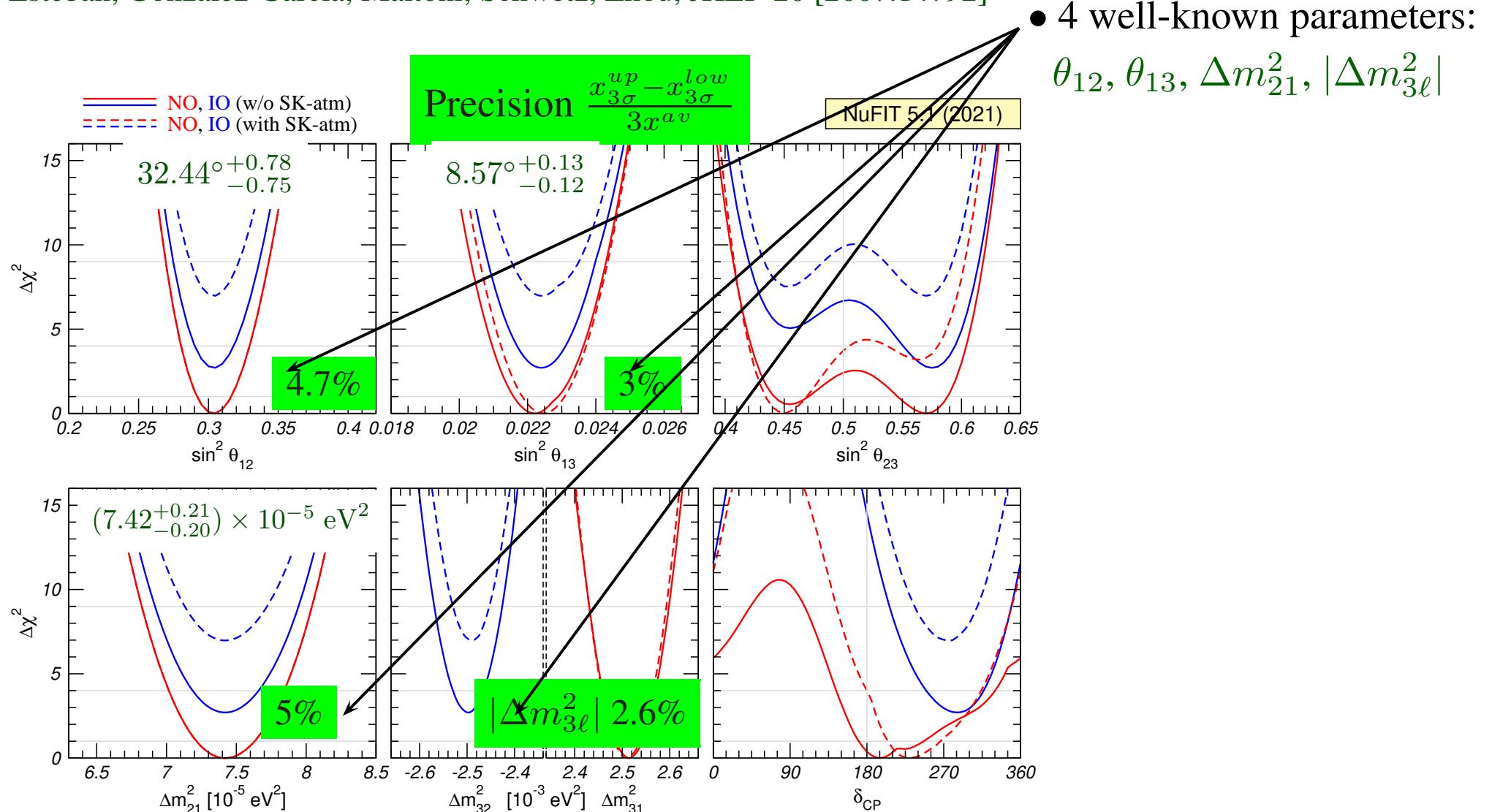


$\text{SK-atm} \equiv \chi^2$ table from
SK1-4 for 372 kton-years

Summary: Global 3 ν Flavour Parameters

Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]



Summary: Global 3 ν Flavour Parameters

Evolution of global 3 flavour fit

Gonzalez-Garcia, Maltoni, TS [arXiv:2111.03086]

	2012 NuFIT 1.0	2014 NuFIT 2.0	2016 NuFIT 3.0	2018 NuFIT 4.0	2021 NuFIT 5.1	
θ_{12}	15%	14%	14%	14%	14%	1.07
θ_{13}	30%	15%	11%	8.9%	9.0%	3.3
θ_{23}	43%	32%	32%	27%	27%	1.6
Δm_{21}^2	14%	14%	14%	16%	16%	0.88
$ \Delta m_{3\ell}^2 $	17%	11%	9%	7.8%	6.7% [6.5%]	2.5
δ_{CP}	100%	100%	100%	100% [92%]	100% [83%]	1 [1.2]
$\Delta\chi^2_{IO-NO}$	± 0.5	-0.97	+0.83	+4.7 [+9.3]	+2.6 [+7.0]	

w/o [w] SK atm data

↑

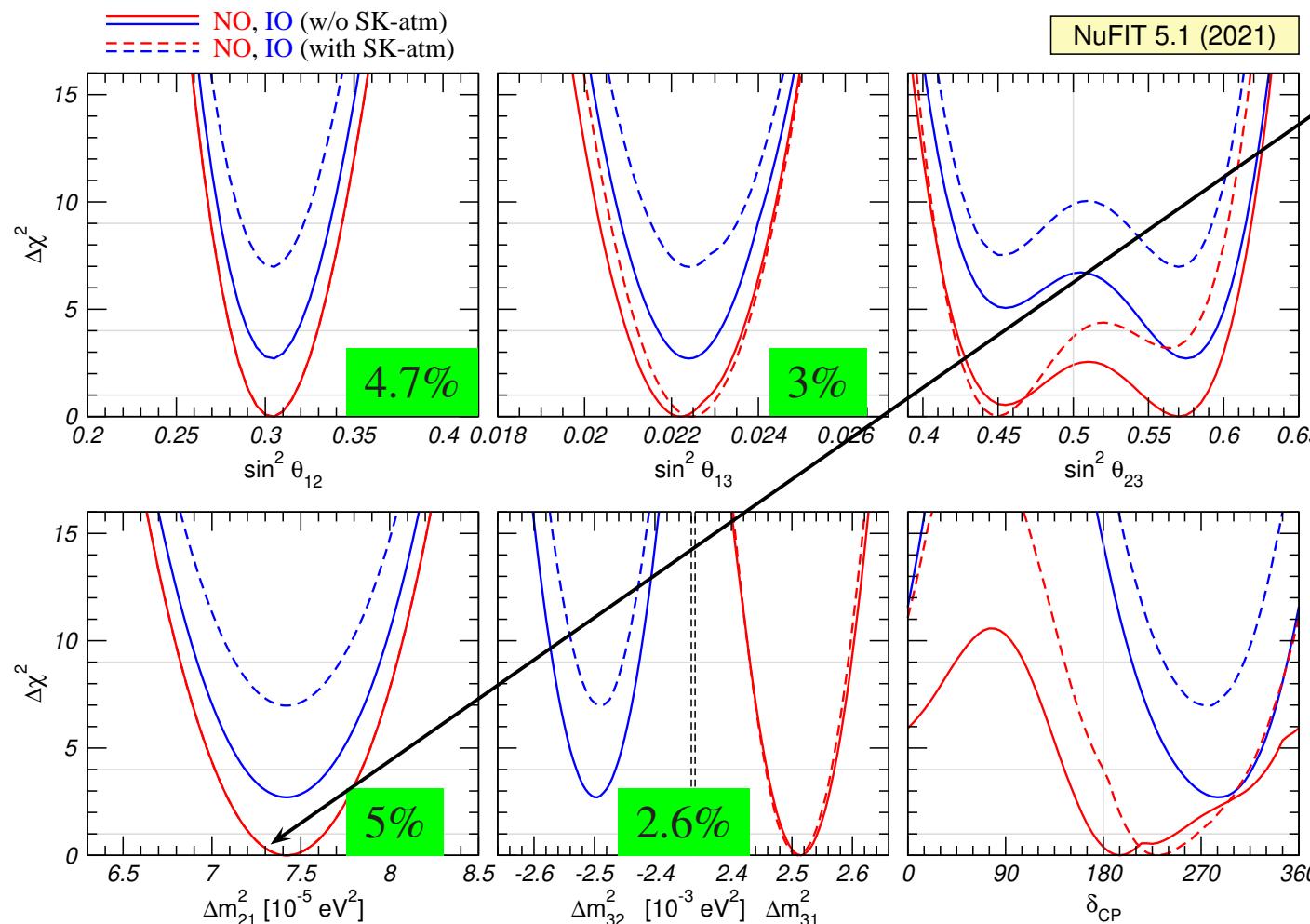
relat. precision at 3σ :
$$\frac{2(x^+ - x^-)}{(x^+ + x^-)}$$

improvement factor from 2012 to 2021

Summary: Global 3 ν Flavour Parameters

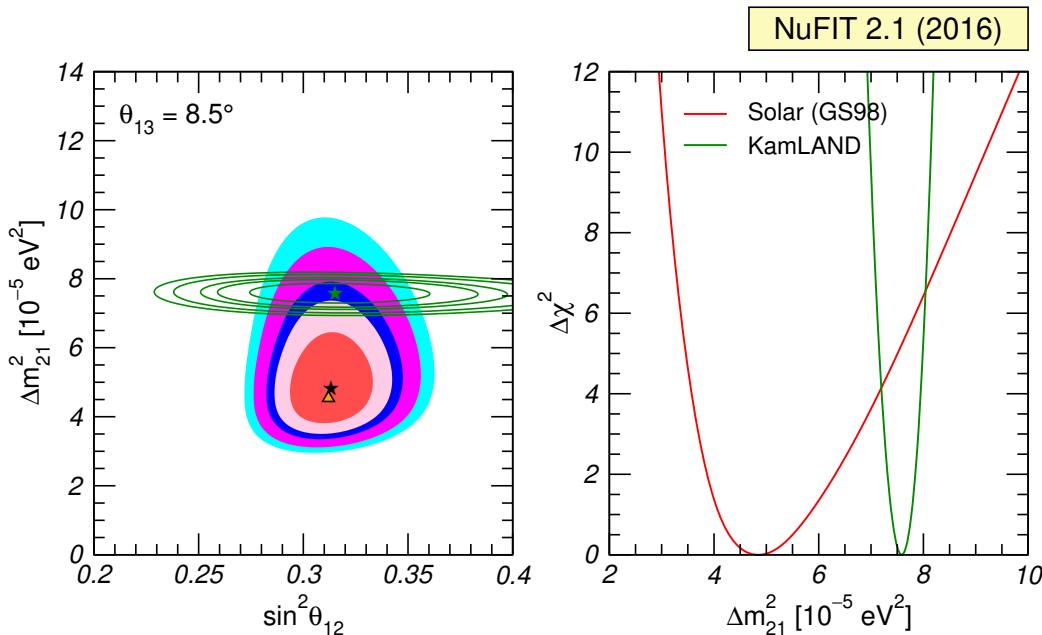
Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]



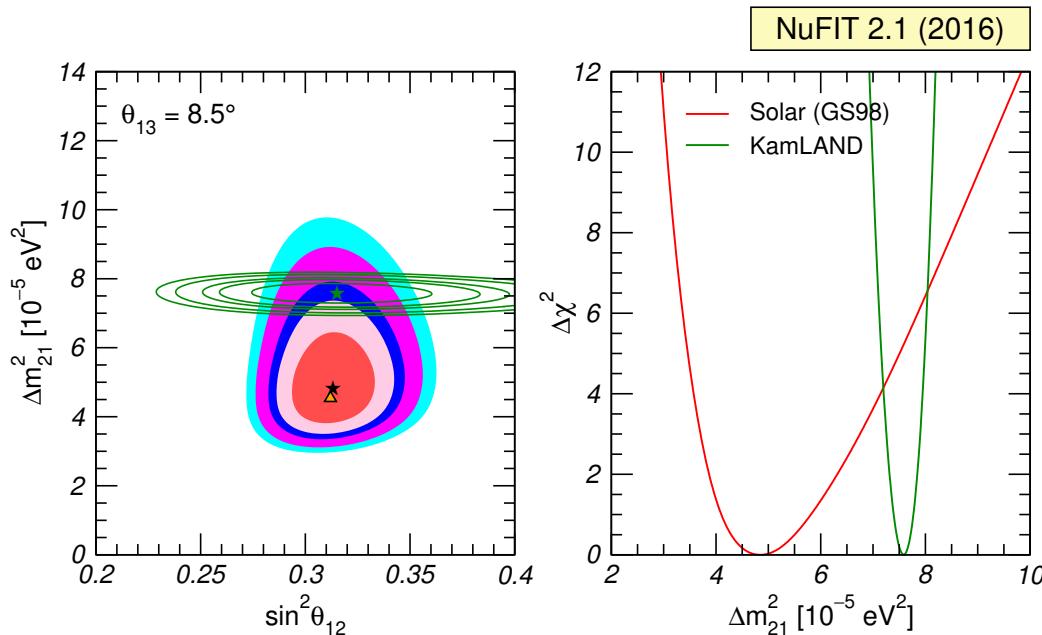
- 4 well-known parameters:
 $\theta_{12}, \theta_{13}, \Delta m_{21}^2, |\Delta m_{3\ell}^2|$
 Δm_{21}^2 Solar vs KLAND
Tension Resolved

- Last decade: after including $\theta_{13} \simeq 9^\circ$ the comparison of KamLAND vs Solar



θ_{12} better than 1σ agreement
But $\sim 2\sigma$ tension on Δm_{12}^2

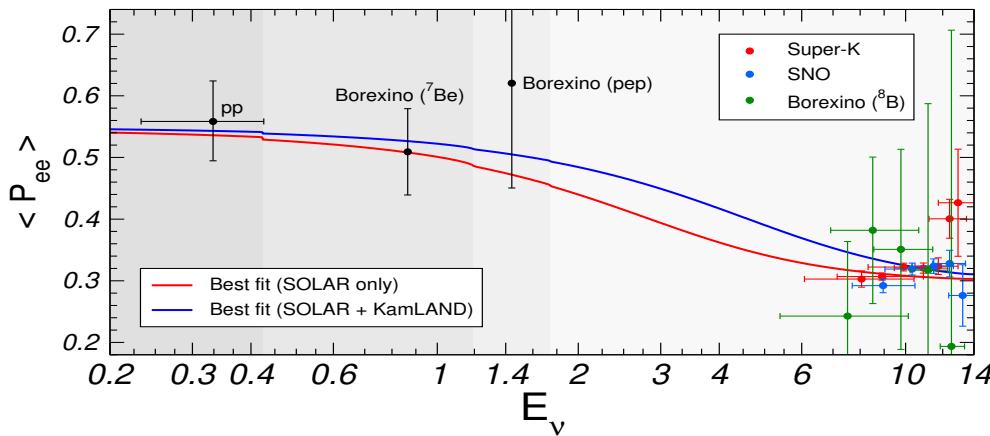
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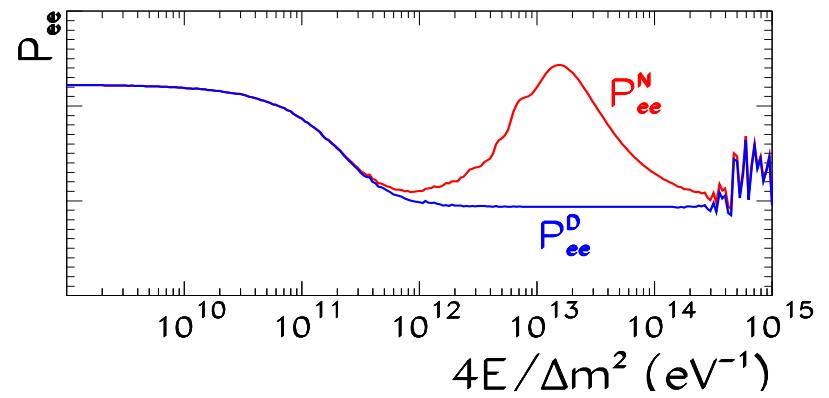
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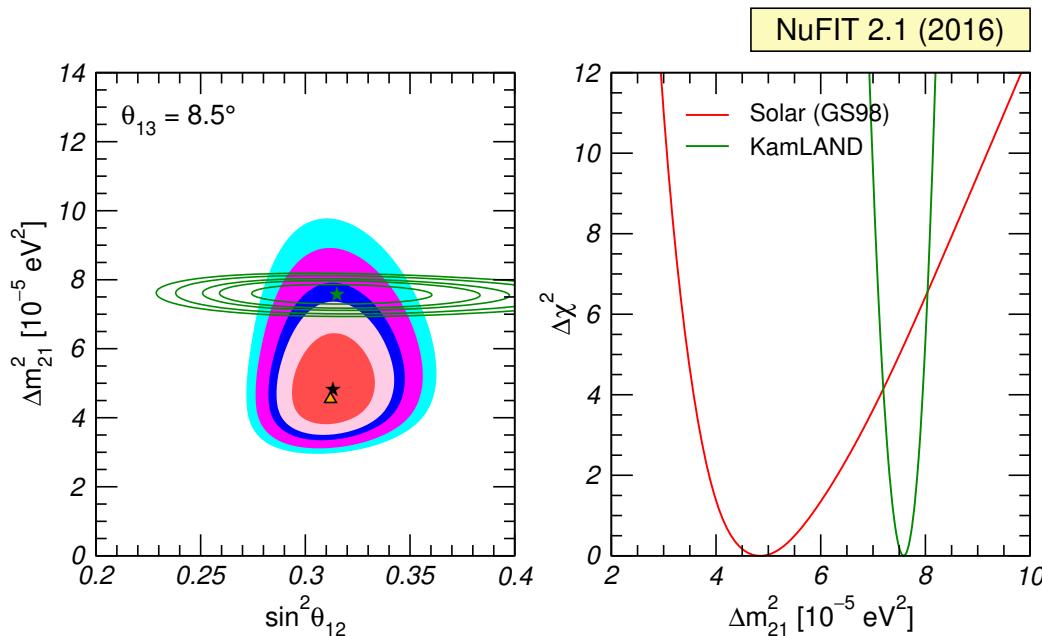
Smaller-than-expected MSW low-E turn-up
in SK/SNO spectrum at global b.f.



“too large” of Day/Night at SK
 $A_{D/N, SK4-2055} = [-3.1 \pm 1.6(\text{stat.}) \pm 1.4(\text{sys.})]\%$



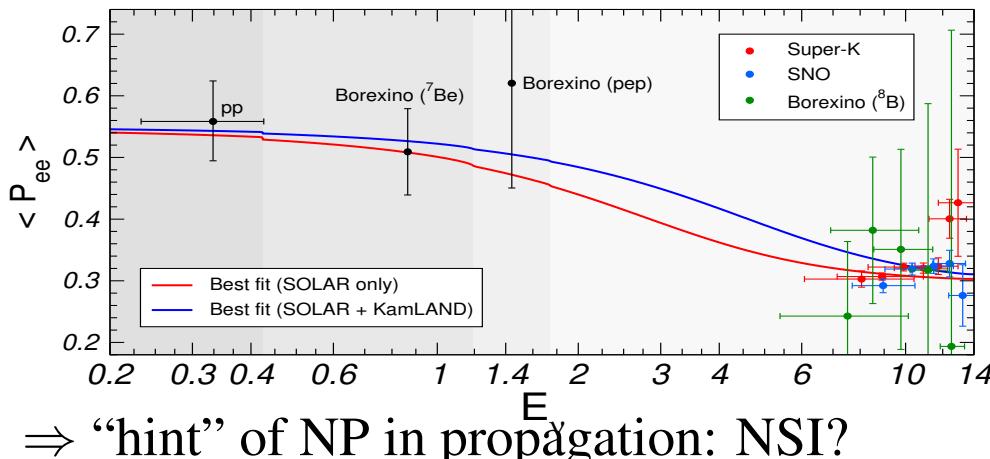
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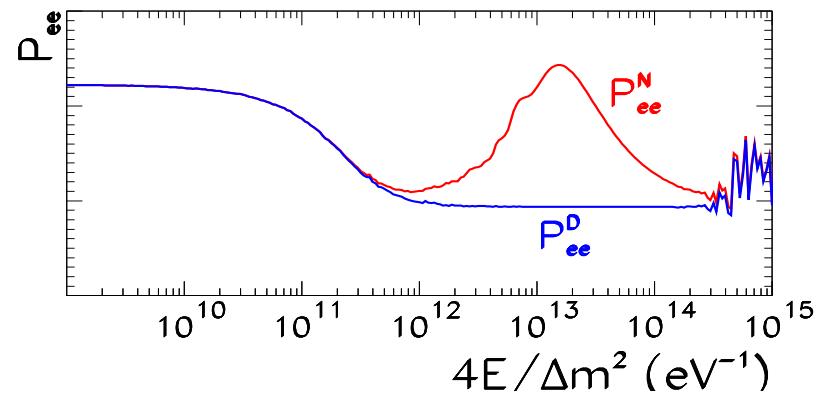
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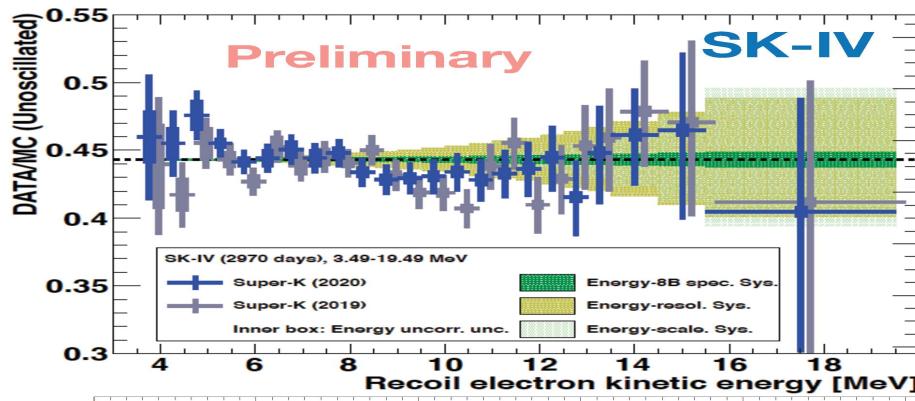
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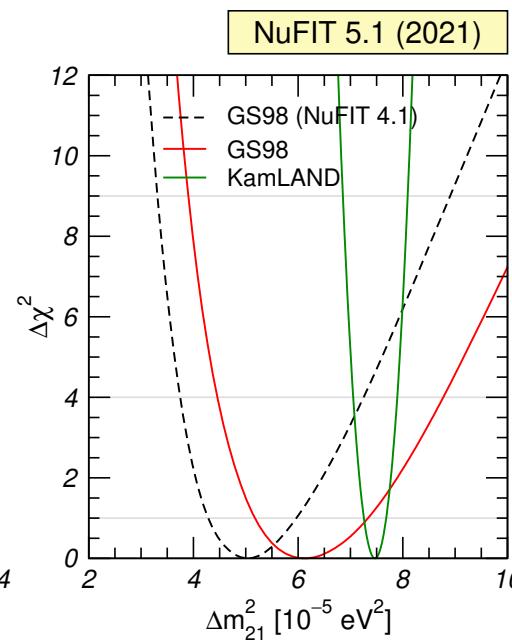
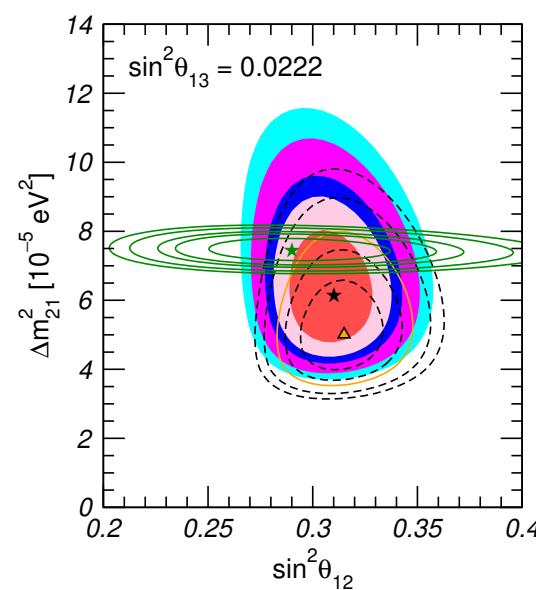
⇒ “hint” of NP in propagation: NSI?

- AFTER NU2020: With SK4 2970 days data

Slightly more pronounced low-E turn-up



- In NuFIT 5.1



\Rightarrow Agreement of Δm_{21}^2 between solar and KamLAND at 1 σ

Smaller of Day/Night at

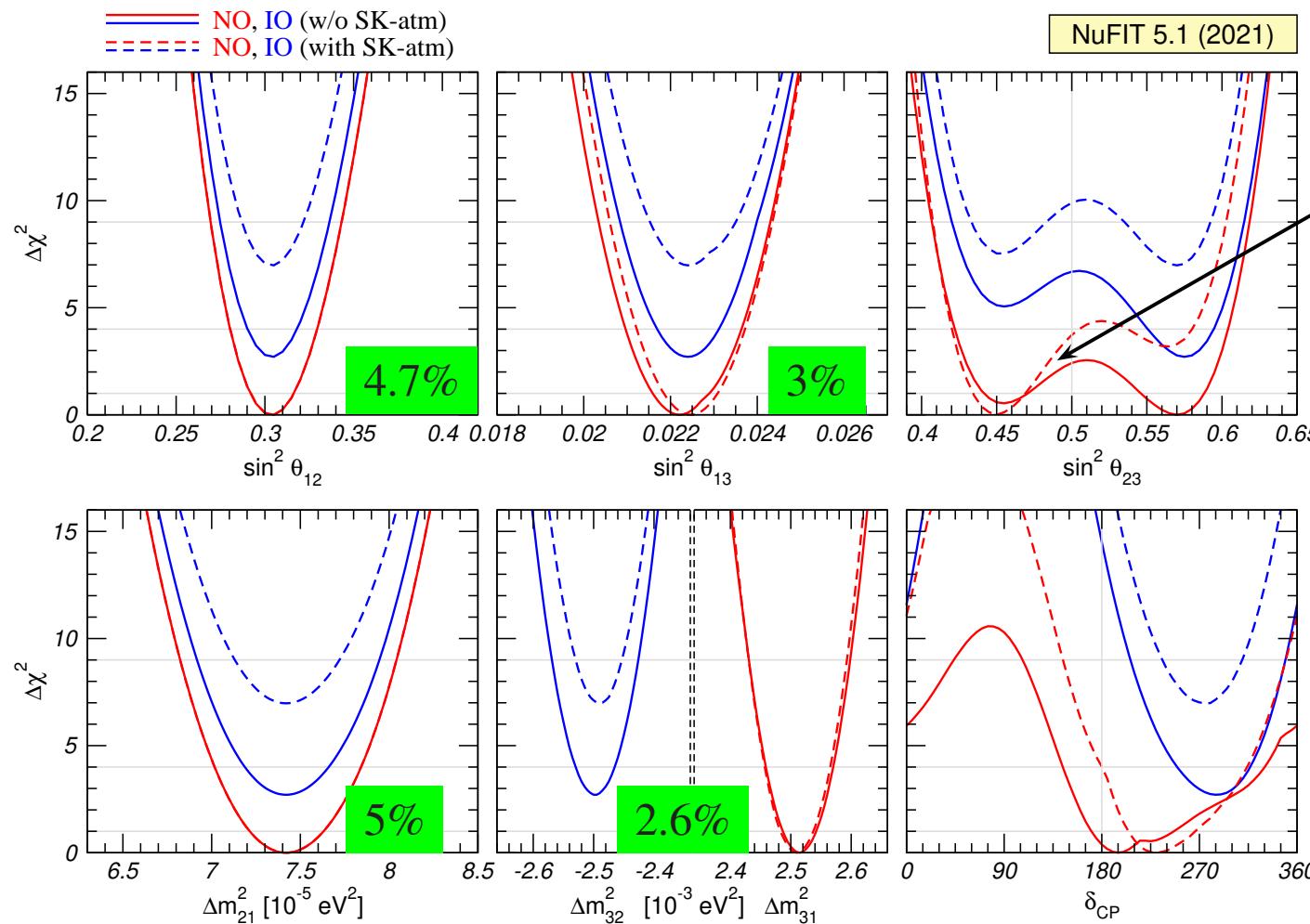
$$A_{D/N, SK4-2055} = [-3.1 \pm 1.6(\text{stat.}) \pm 1.4(\text{sys.})]\%$$

$$A_{D/N, SK4-2970} = [-2.1 \pm 1.1]\%$$

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Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]

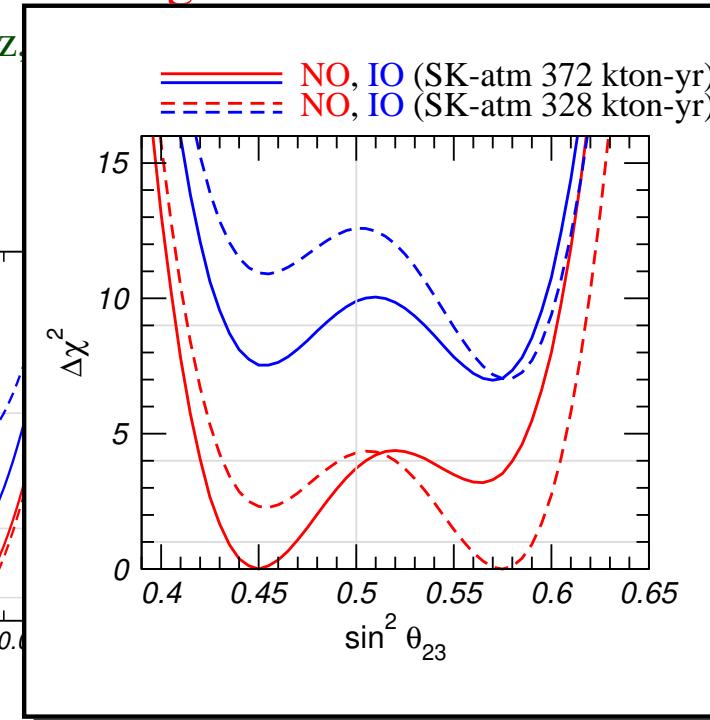
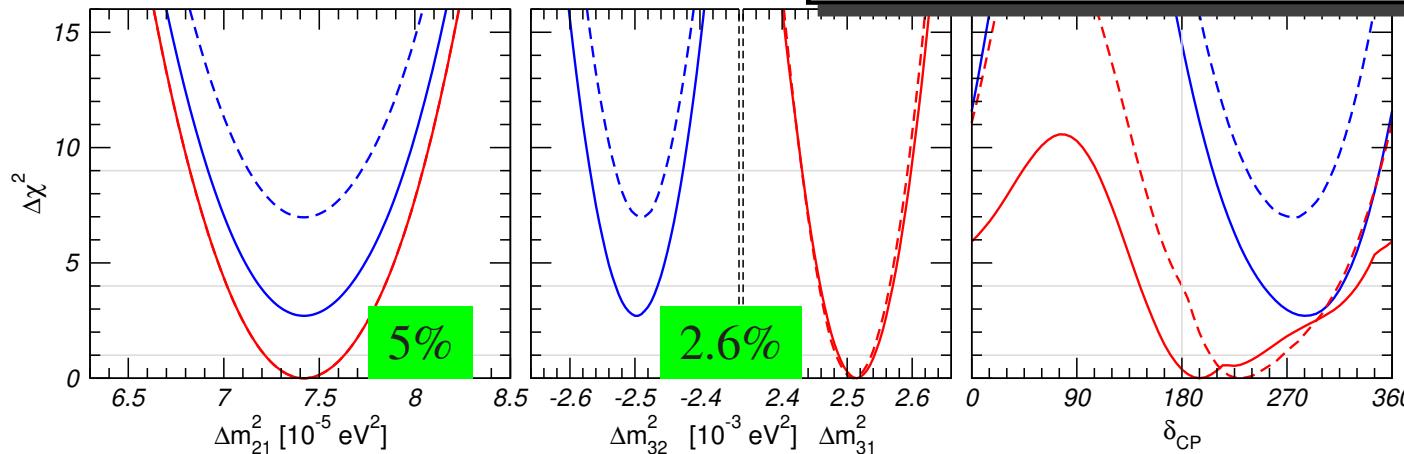
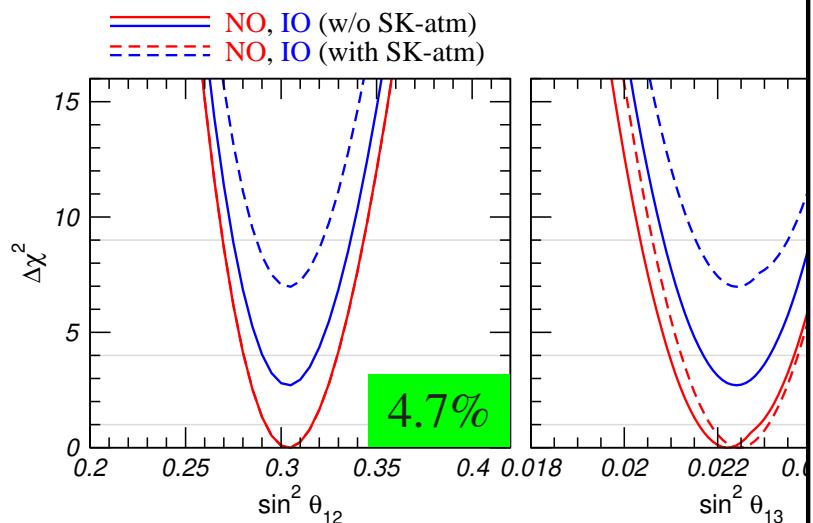


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Tension Resolved
- θ_{23} : Least known angle
 Maximal? Octant?
 non-robust wrt ATM

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Esteban, Gonzalez-Garcia, Maltoni, Schwetz,



l-known parameters:
 $\sin^2 \theta_{13}$, Δm^2_{21} , $|\Delta m^2_{3\ell}|$
 Solar vs KLAND
 on Resolved
 Least known angle
 Normal? Octant?
 Robust wrt ATM

Flavour Parameters: Mixing Matrix

- We have the three leptonic mixing angles determined (at $\pm 3\sigma/6$)

$$|U|_{3\sigma} = \begin{pmatrix} 0.801 \rightarrow 0.844 & 0.513 \rightarrow 0.579 & 0.143 \rightarrow 0.156 \\ 0.233 \rightarrow 0.507 & 0.461 \rightarrow 0.694 & 0.639 \rightarrow 0.778 \\ 0.261 \rightarrow 0.526 & 0.471 \rightarrow 0.701 & 0.611 \rightarrow 0.761 \end{pmatrix}$$

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- Good progress but still precision very far from:

$$|V|_{\text{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.0065 & (3.51 \pm 0.15) \times 10^{-3} \\ 0.2252 \pm 0.00065 & 0.97344 \pm 0.00016 & (41.2^{+1.1}_{-5}) \times 10^{-3} \\ (8.67^{+0.29}_{-0.31}) \times 10^{-3} & (40.4^{+1.1}_{-0.5}) \times 10^{-3} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

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- Also very different flavour mixing of leptons vs quarks

Leptonic CPV in 3ν Mixing: Jarlskog Invariant

- Leptonic CP $\Rightarrow P_{\nu_\alpha \rightarrow \nu_\beta} \neq P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}$
- In 3ν always

$$P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} \propto J \quad \text{with} \quad J = \text{Im}(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 2} U_{\beta 1}^*) = J_{\text{LEP,CP}}^{\max} \sin \delta_{\text{CP}}$$

$$J_{\text{LEP,CP}}^{\max} = \frac{1}{8} c_{13} \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2 2\theta_{12}$$

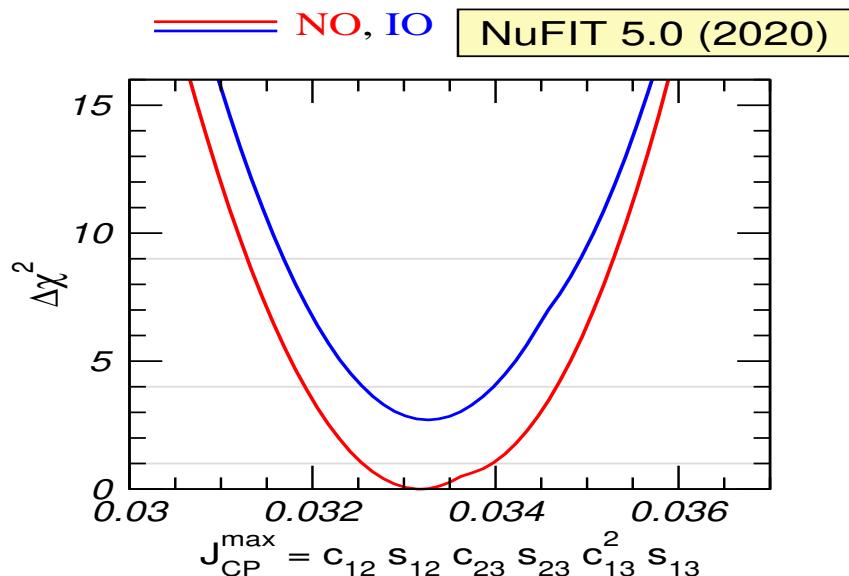
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- Maximum Allowed Leptonic CPV:



$$J_{\text{LEP,CP}}^{\max} = (3.29 \pm 0.07) \times 10^{-2}$$

to compare with

$$J_{\text{CKM,CP}} = (3.04 \pm 0.21) \times 10^{-5}$$

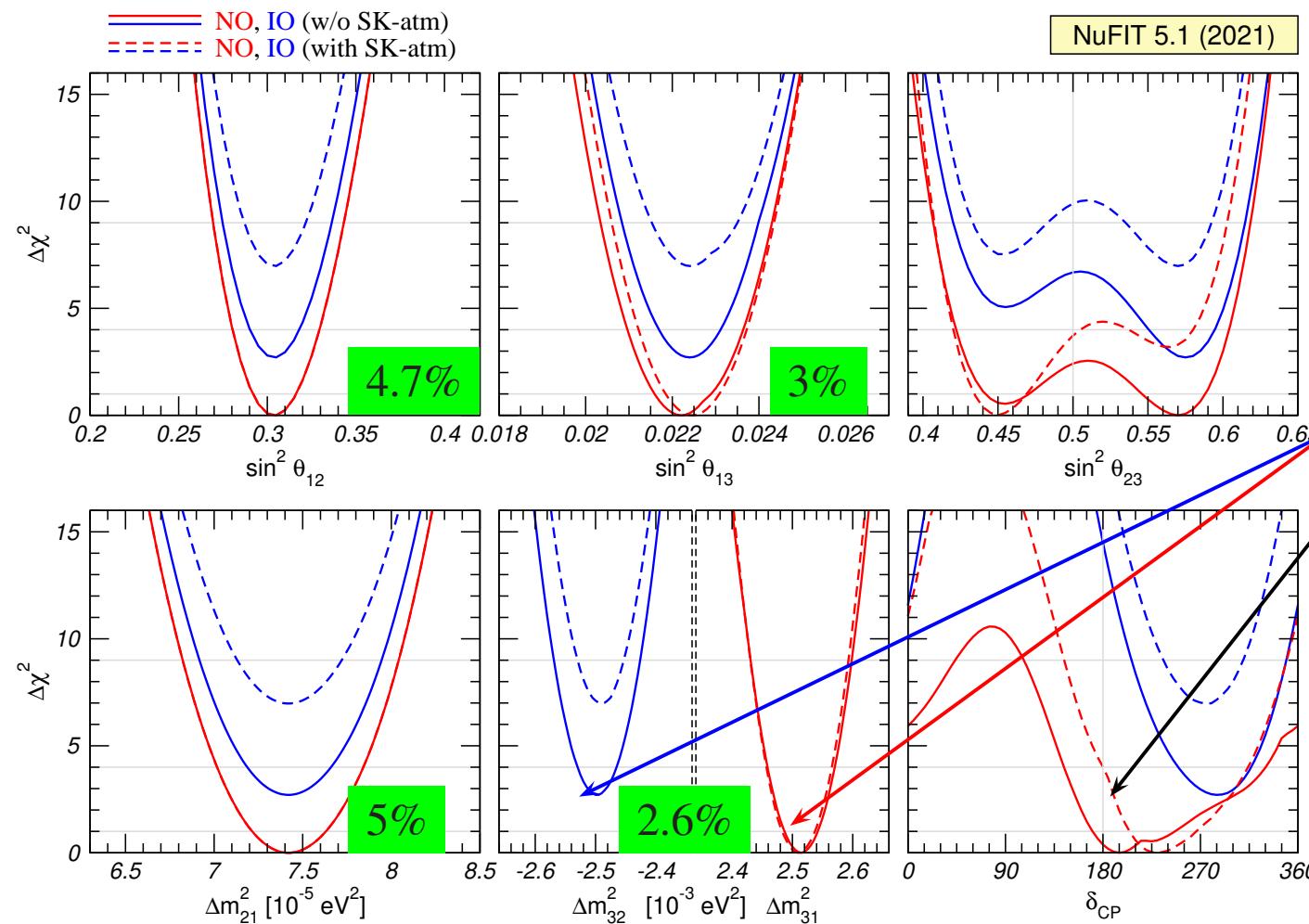
\Rightarrow Leptonic CPV may be largest CPV
in New Minimal SM

if $\sin \delta_{\text{CP}}$ not too small

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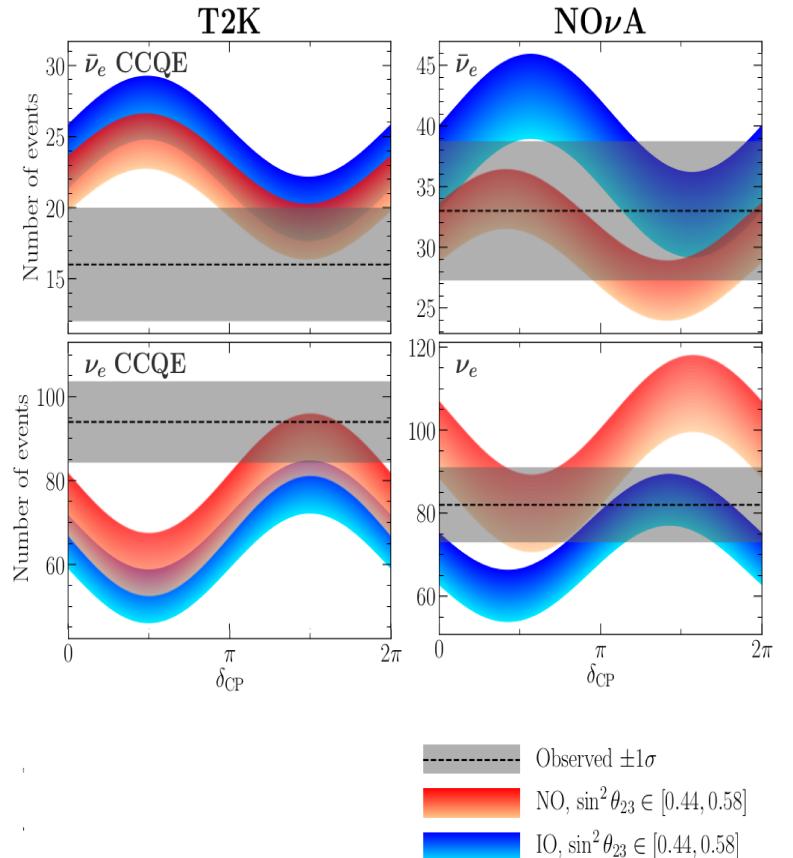
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- Δm_{21}^2 Solar vs KLAND Tension Resolved
- θ_{23} : Least known angle Maximal? Octant? non-robust wrt ATM
- Ordering NO or IO?

CPV?:

CPV and Ordering in LBL: ν_e appearance

Gonzalez-Garcia

ν_e and $\bar{\nu}_e$ appearance events

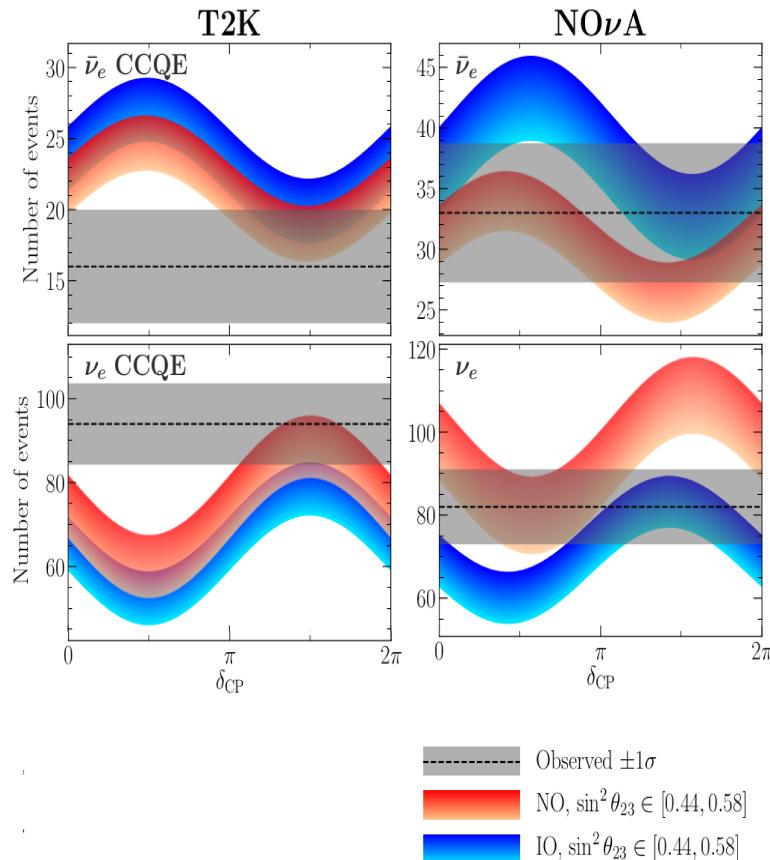


⇒ Each T2K and NO ν A favour NO

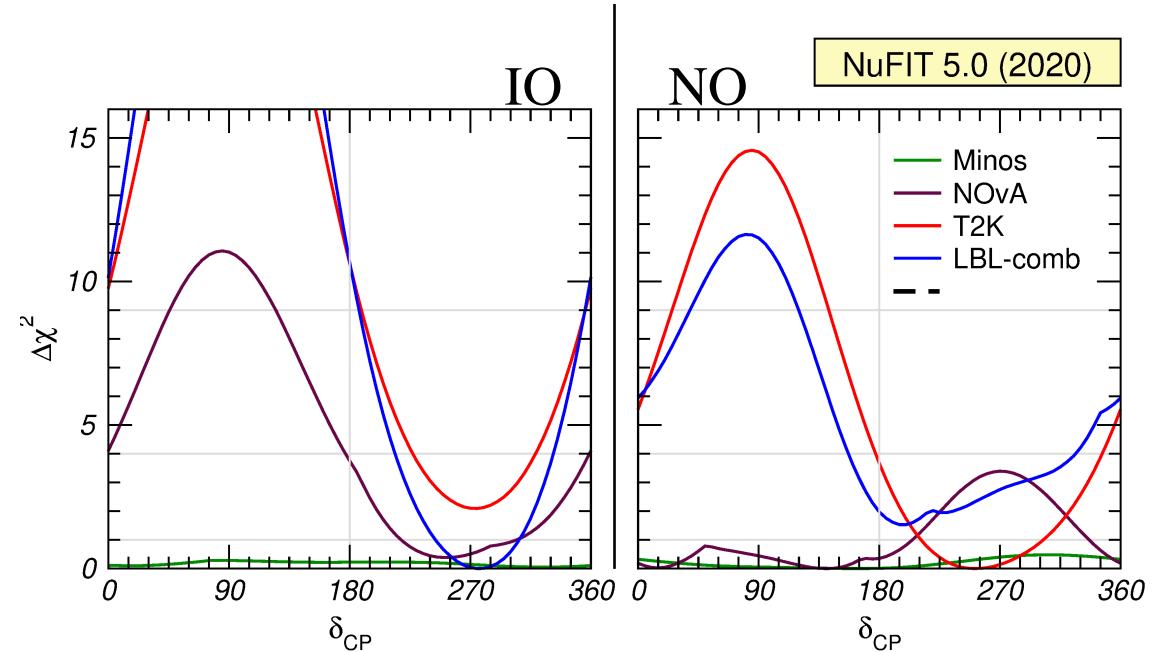
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But tension in favoured values of δ_{CP} in NO



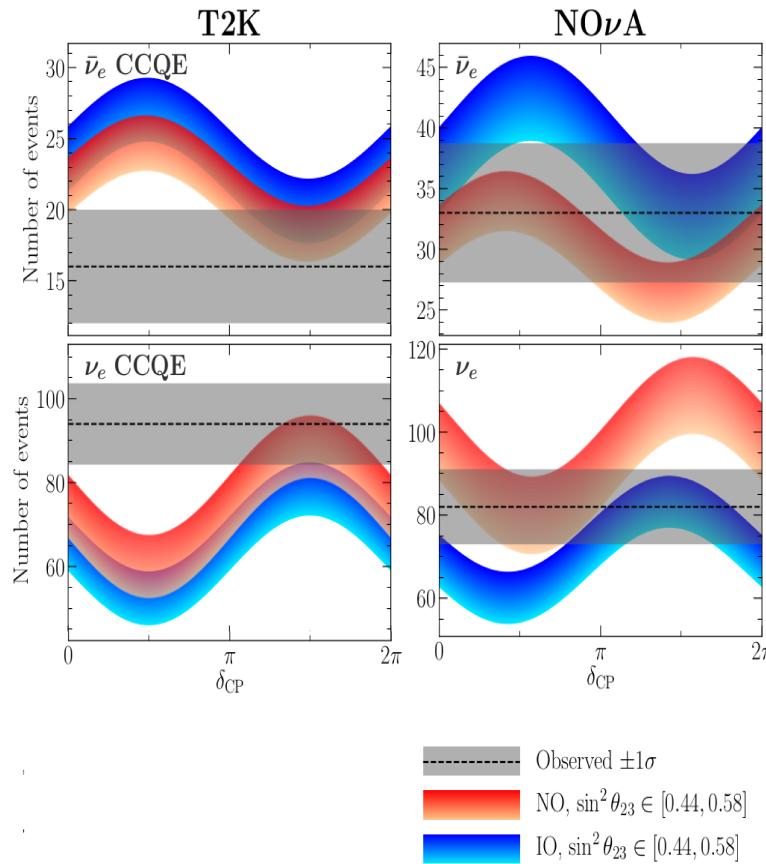
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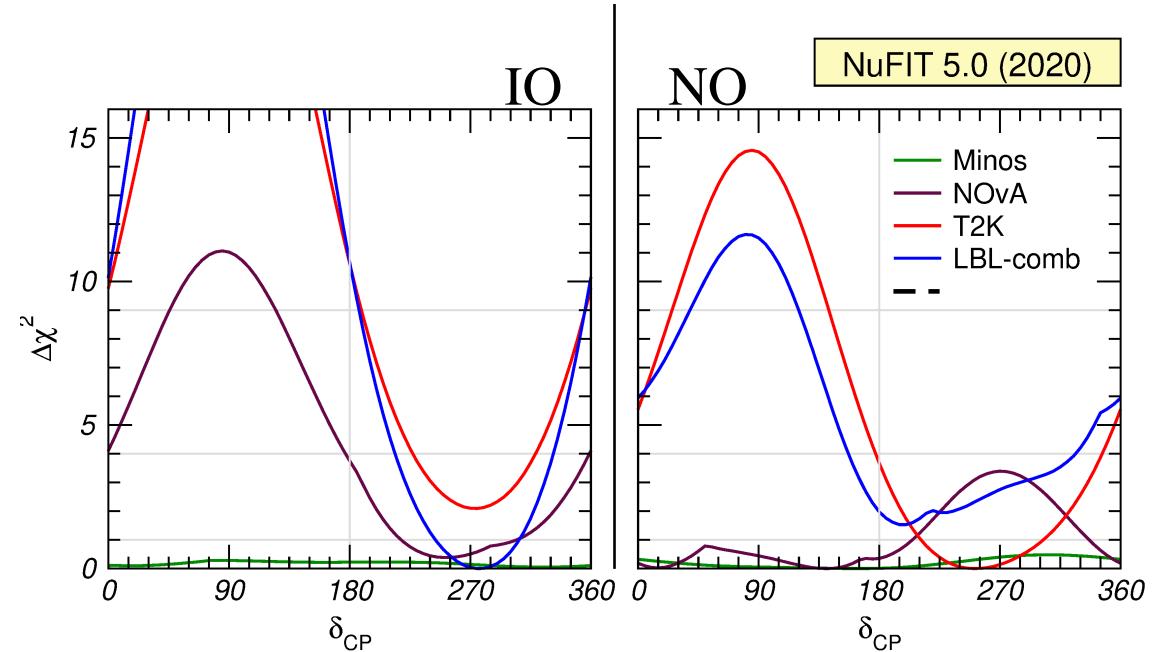
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- Parameter goodness-of-fit (PG) test:

	normal ordering			inverted ordering		
	χ^2_{PG}/n	p-value	# σ	χ^2_{PG}/n	p-value	# σ
T2K vs NOvA (θ_{13} free)	6.7/4	0.15	1.4 σ	3.6/4	0.46	0.7 σ
T2K vs NOvA (θ_{13} fix)	6.5/3	0.088	1.7 σ	2.8/3	0.42	0.8 σ

No significant incompatibility

Δm_{3l}^2 in LBL & Reactors

- At LBL determined in ν_μ and $\bar{\nu}_\mu$ disappearance spectrum

$$\Delta m_{\mu\mu}^2 \simeq \Delta m_{3l}^2 + \frac{c_{12}^2 \Delta m_{21}^2}{s_{12}^2 \Delta m_{21}^2} \begin{matrix} \text{NO} \\ \text{IO} \end{matrix} + \dots$$

- At MBL Reactors (Daya-Bay, Reno, D-Chooz) determined in $\bar{\nu}_e$ disapp spectrum

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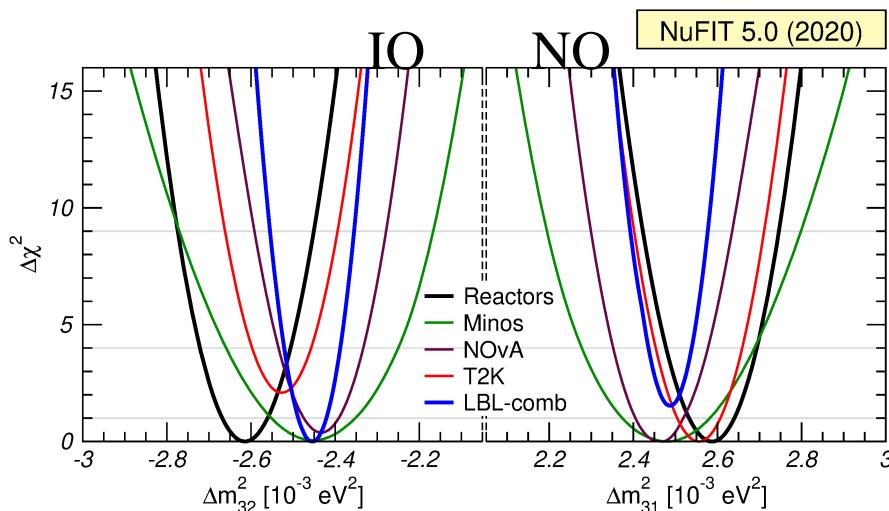
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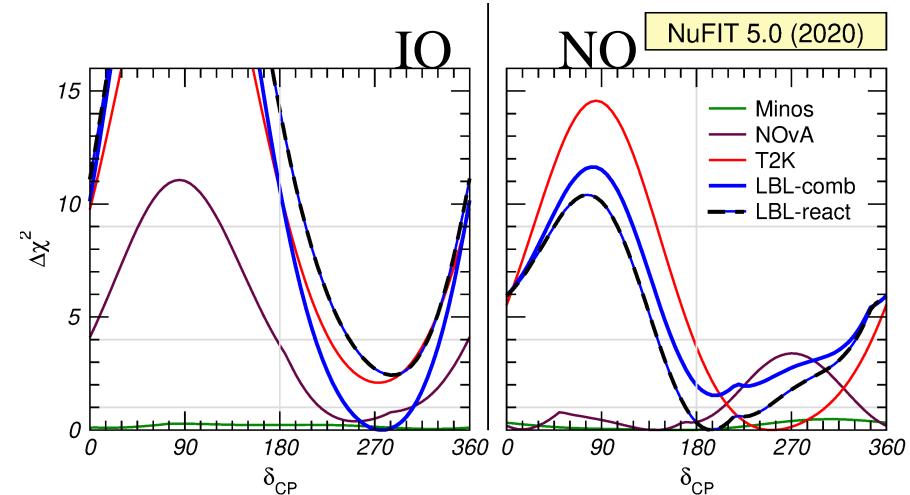
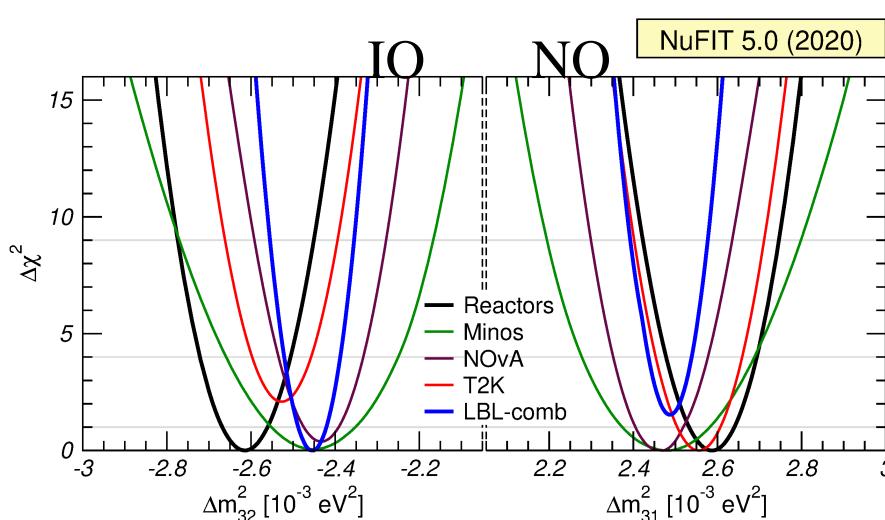
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- LBL/Reactor complementarity in Δm_{3l}^2 \Rightarrow NO best fit in LBL+Reactors
- in NO: b.f $\delta_{CP} = 195^\circ$ \Rightarrow CPC allowed at 0.6σ
- in IO: b.f $\delta_{CP} \sim 270^\circ$ \Rightarrow CPC disfavoured at 3σ

Near Future for CP and Ordering: Strategies

- $\nu/\bar{\nu}$ comparison with or without Earth matter effects in $\nu_\mu \rightarrow \nu_e$ & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ at LBL:
DUNE (wide band beam, L=1300 km), HK (narrow band beam, L=300 km)

$$P_{\mu e} \simeq s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31} \pm V} \right)^2 \sin^2 \left(\frac{\Delta_{31} \pm V L}{2} \right) + 8 J_{CP}^{\max} \frac{\Delta_{12}}{V} \frac{\Delta_{31}}{\Delta_{31} \pm V} \sin \left(\frac{V L}{2} \right) \sin \left(\frac{\Delta_{31} \pm V L}{2} \right) \cos \left(\frac{\Delta_{31} L}{2} \pm \delta_{CP} \right)$$

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- Reactor experiment at $L \sim 60$ km (vacuum) able to observe
the difference between oscillations with Δm_{21}^2 and Δm_{32}^2 : JUNO, RENO-50

$$P_{\nu_e, \nu_e} = 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) - \sin^2 2\theta_{13} \left[c_{12}^2 \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + s_{12}^2 \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E} \right) \right]$$

- Challenge: Energy resolution

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- Challenge: Energy resolution
- Earth matter effects in large statistics ATM ν_μ disapp : HK, INO, PINGU, ORCA ...
- Challenge: ATM flux contains both ν_μ and $\bar{\nu}_\mu$, ATM flux uncertainties

Beyond 3ν 's: Light Sterile Neutrinos

a Gonzalez-Garcia

- Several Observations which can be Interpreted as Oscillations with $\Delta m^2 \sim \text{eV}^2$

LSND & MiniBoone

LSND 2001:

Signal $\nu_\mu \rightarrow \nu_e$ (3.8σ)

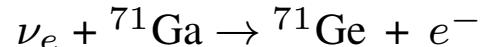
MiniBooNE 2020:

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ & $\nu_\mu \rightarrow \nu_e$
 $(639 \pm 132.8 \text{ events})$

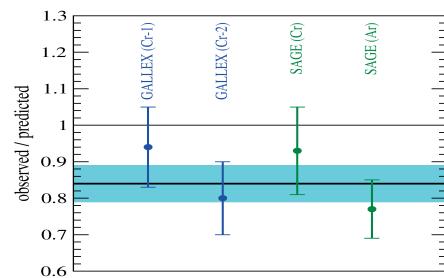
Gallium Anomaly

Acero, Giunti, Laveder, 0711.4222
Giunti, Laveder, 1006.3244

Radioactive Sources (^{51}Cr , ^{37}Ar)
in calibration of Ga Solar Exp;



Give a rate lower than expected



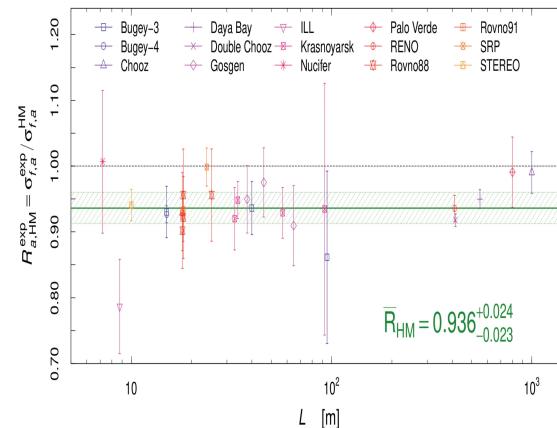
Explained as ν_e disappearance

Reactor Anomaly (2011)

Huber, 1106.0687
Mention et al., 1101.2755

New reactor flux calculation

\Rightarrow Deficit in data at $L \lesssim 100 \text{ m}$



Explained as $\bar{\nu}_e$ disappearance

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Jaime Gonzalez-Garcia

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$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ & $\nu_\mu \rightarrow (\bar{\nu}_e + \nu_e)$
 (639 ± 132.8)

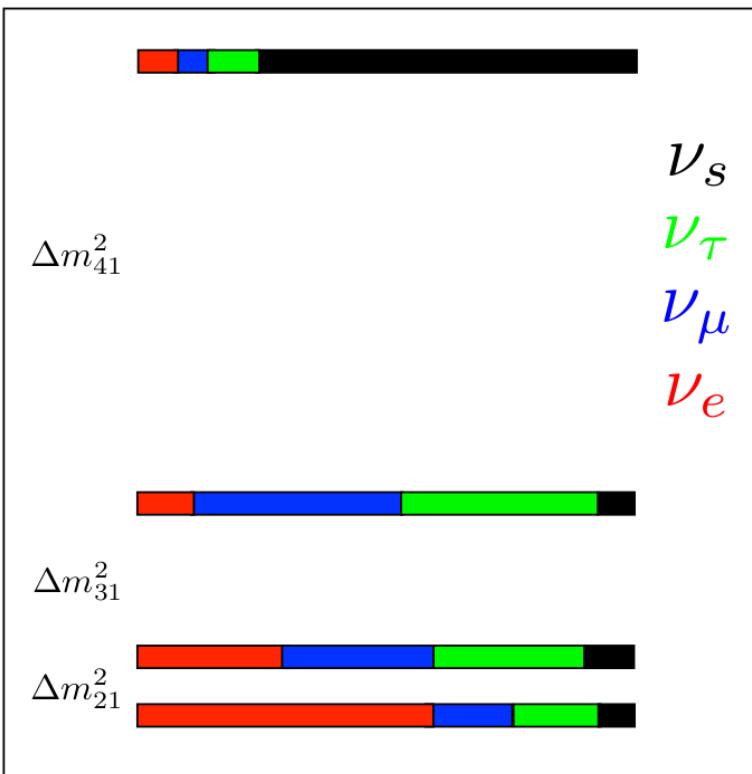
Gallium Anomaly

Acero, Giunti, Laveder, 0711.4222
Giunti, Laveder, 1006.3244

Reactor Anomaly (2011)

Huber, 1106.0687
Mention et al., 1101.2755

Oscillation Interpretation Requires new (sterile) ν 's



disappearance

Beyond 3ν 's: Light Sterile Neutrinos

a Gonzalez-Garcia

- Several Observations which can be Interpreted as Oscillations with $\Delta m^2 \sim \text{eV}^2$

LSND & MiniBoone

LSND 2001:

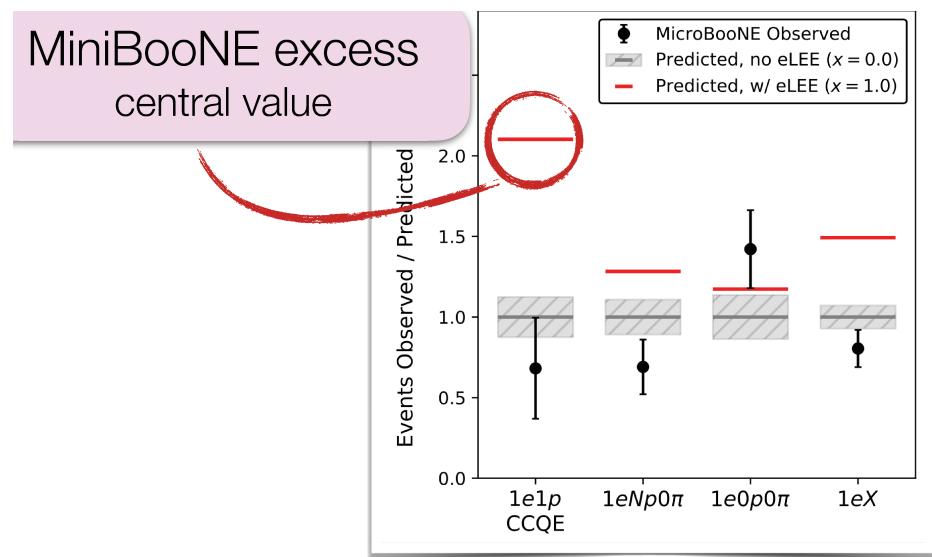
Signal $\nu_\mu \rightarrow \nu_e$ (3.8σ)

MiniBooNE 2020:

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ & $\nu_\mu \rightarrow \nu_e$

(639 ± 132.8) events

MicroBooNE 2021/2022:



No support for excess ν_e interpretation in MiniBooNE

(Fig from Kopp's ν 2022 talk)

MicroBooNE

Coll.

Beyond 3ν 's: Light Sterile Neutrinos

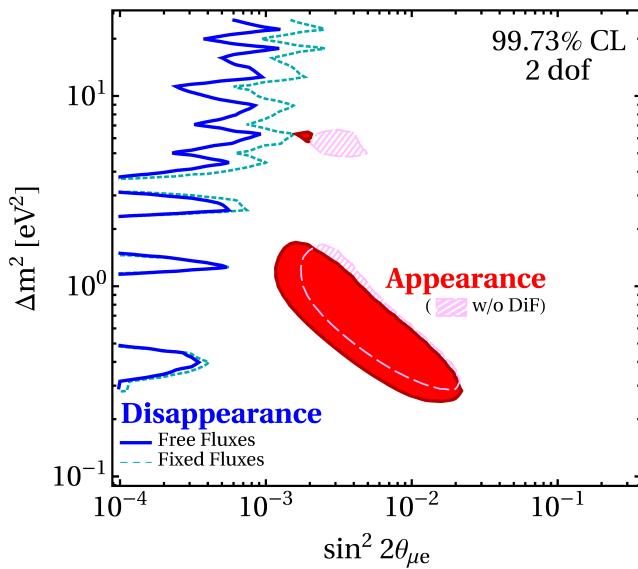
a Gonzalez-Garcia

LSND & MiniBoone

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e \text{ & } \nu_\mu \rightarrow \nu_e$$

$$\sin^2 2\theta_{\mu e} \sim \frac{1}{4} \sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}$$

Strong tension with
non-observation of ν_μ dissap



Dentler et al, 1803.10661

Purely sterile oscillation
robustly disfavoured
additional SM or NP effects?

Beyond 3ν 's: Light Sterile Neutrinos

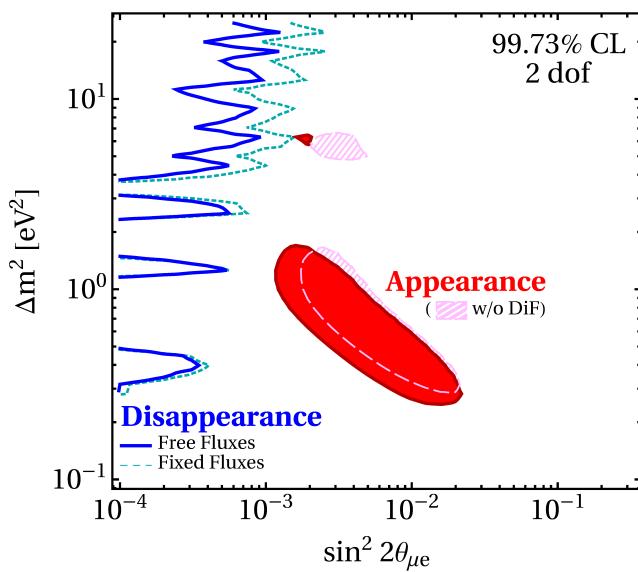
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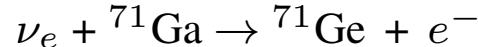


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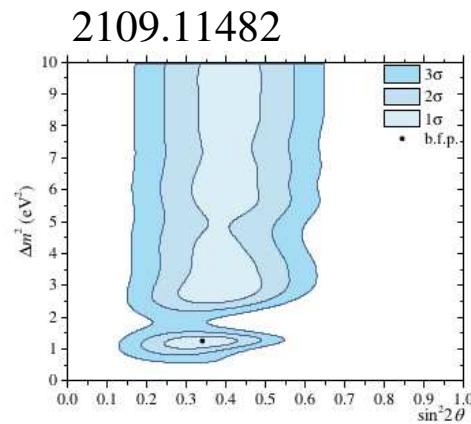
Acero et al, 0711.4222; Giunti, Laveder, 1006.3244



Rate lower than expected

Explained as ν_e disappearance

Confirming results from BEST



Re-

quires large mixings

Ruled out/tension by solar ν' s

Goldhagen et al 2109.14898

Berryman et al 2111.12530

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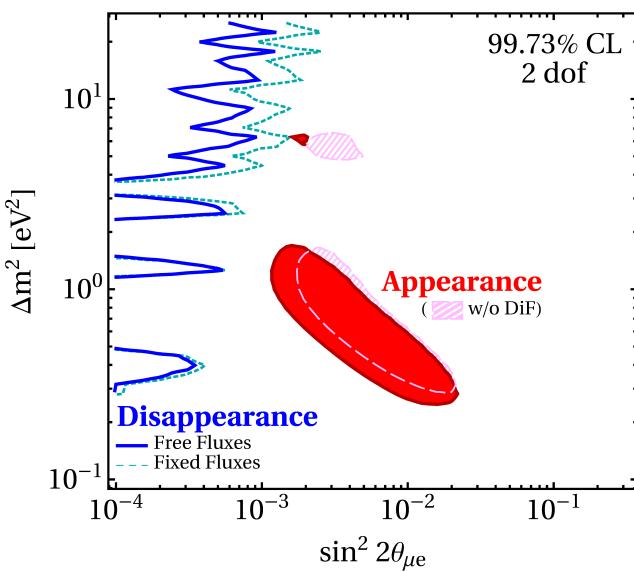
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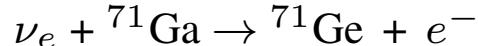


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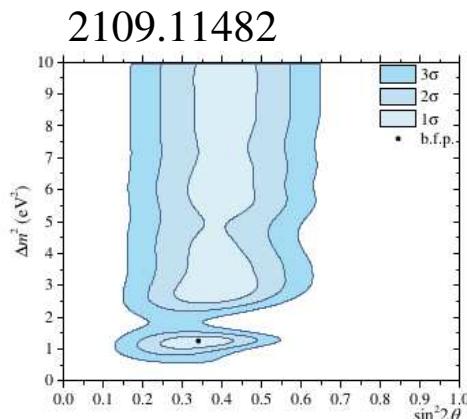
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Goldhagen et al 2109.14898

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Reactor Anomaly

Huber, 1106.068, Mention et al, 1101.2755

2011 reactor flux calculation \Rightarrow

Deficit in $R = \frac{\text{data}}{\text{predict}}$ at $L \lesssim 100$ m

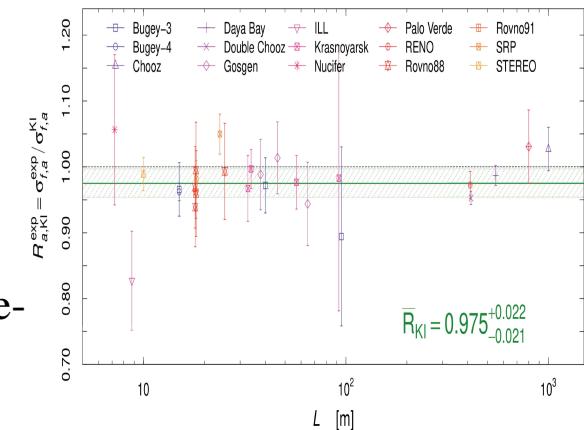
Explained as $\bar{\nu}_e$ disappearance

2022 with updated inputs (${}^{235}\text{U}$)

Berryman Huber, 2005.01756

Kipeikin et al, 2103.01486

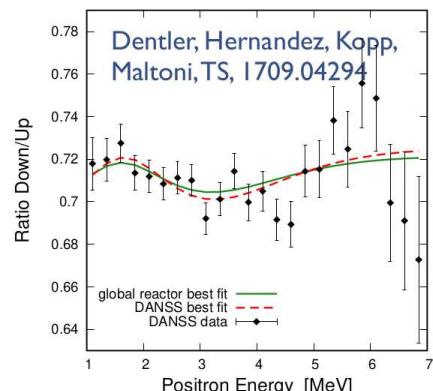
Giunti et al, 2110.06820



(Fig from Giunti et al, 2110.06820)

Anomaly $\sim 1\sigma$
with new fluxes

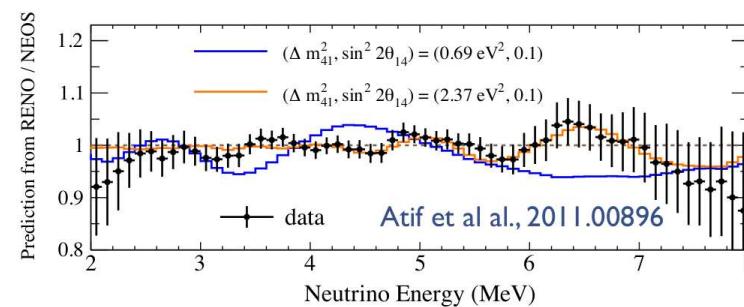
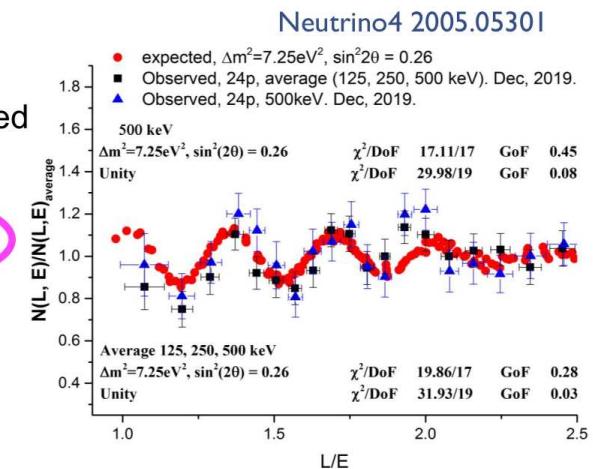
Recent relative spectral measurements



~~DANSS: relative spectra @ L = 10.7 and 12.7 m
prev. $\sim 2\sigma$ hint decr. $\sim 1.5\sigma$~~
DANSS talk @ ICHEP20 (update at EPS-HEP21)

segmented detectors:
STEREO [arXiv:1912.06582]
 $L = 9$ to 11 m $\Delta\chi^2(\text{no osc}) \approx 9$
PROSPECT [arXiv:2006.11210]
 $L = 6.7$ to 9.2 m

Neutrino4: segmented detector, $L = 6.25$ to 11.9 m, 216 bins in L/E , $\text{"}3\sigma\text{"}$ indication



NEOS: spectrum at $L = 24$ m, relative to RENO (or DayaBay) near detectors: $\Delta\chi^2(\text{no osc}) = 11.7$

Spectral ratios at different baselines \Rightarrow Independent of flux normalizations.

But low statistical significance (Wilks theorem fails) Berryman, et al 2111.12530

MC estimation of prob distribution \Rightarrow no significant indication of ν_s oscillations

Non Standard ν Interactions (NSI)

At dimension-6 new 4-fermion interactions involving ν 's.

Some can affect CC process in production and detection

$$(\bar{\nu}_\alpha \gamma_\mu P_L \ell_\beta)(\bar{f}' \gamma^\mu P f)$$

and can be strongly constrained with charged lepton processes

Some affect only NC ν interactions

$$(\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta)(\bar{f} \gamma^\mu P f)$$

and are more poorly constrained

NC-Non Standard ν Interactions in ν -OSC

z-Garcia

Including non-standard neutrino NC interactions with fermion f

$$\mathcal{L}_{\text{NSI}} = -2\sqrt{2}G_F \varepsilon_{\alpha\beta}^{fP} (\bar{\nu}_\alpha \gamma^\mu L \nu_\beta) (\bar{f} \gamma_\mu P f), \quad P = L, R$$

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$$H_{\text{mat}} = \sqrt{2}G_F N_e(r) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \sqrt{2}G_F N_e(r) \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix}$$

$$\varepsilon_{\alpha\beta}(r) \equiv \sum_{f=ued} \frac{N_f(r)}{N_e(r)} \varepsilon_{\alpha\beta}^{fV} \Rightarrow 3\nu \text{ evolution depends on 6 (vac) + 8 per } f \text{ (mat)}$$

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\Rightarrow Parameters degeneracies

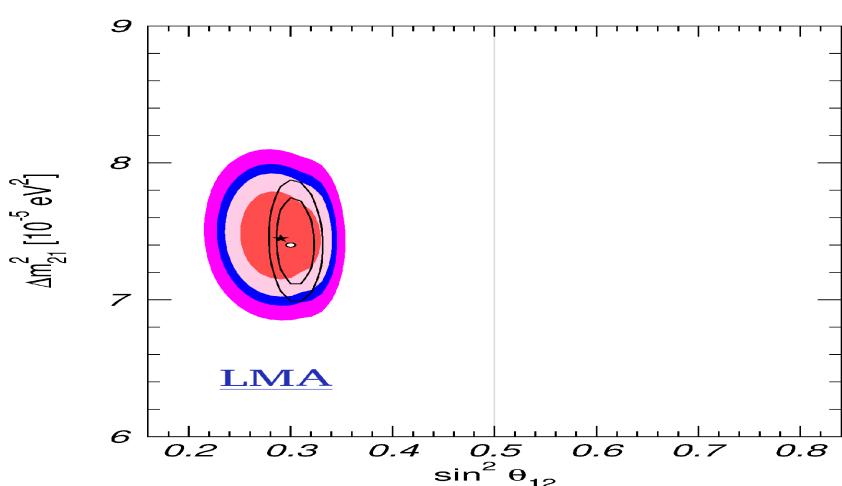
In particular $H \rightarrow -H^*$ \Rightarrow same Probabilities \Rightarrow invariance under simultaneously:

$$\begin{aligned} \theta_{12} &\leftrightarrow \frac{\pi}{2} - \theta_{12}, & (\varepsilon_{ee} - \varepsilon_{\mu\mu}) &\rightarrow -(\varepsilon_{ee} - \varepsilon_{\mu\mu}) - 2, \\ \Delta m_{31}^2 &\rightarrow -\Delta m_{32}^2, & (\varepsilon_{\tau\tau} - \varepsilon_{\mu\mu}) &\rightarrow -(\varepsilon_{\tau\tau} - \varepsilon_{\mu\mu}), \\ \delta &\rightarrow \pi - \delta, & \varepsilon_{\alpha\beta} &\rightarrow -\varepsilon_{\alpha\beta}^* \quad (\alpha \neq \beta), \end{aligned}$$

\Rightarrow Degeneracies in θ_{12} octant and mass ordering

NSI: Bounds/Degeneracies from/in Oscillation data

Esteban *et al.* JHEP'18[1805.04530] Coloma, Esteban, MCGG, Maltoni, JHEP'19[1911.09109] (updated 2020)

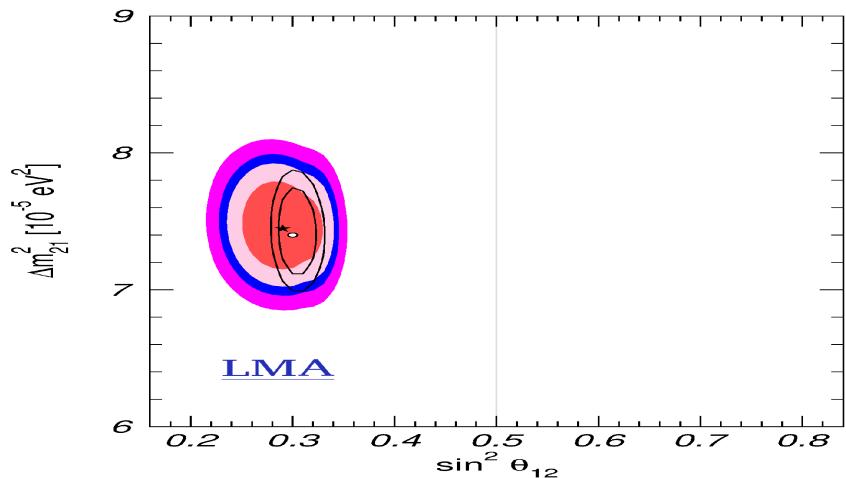


- Standard Solution $\equiv \text{LMA} \Rightarrow \text{Bounds } \mathcal{O}(1\% - 10\%)$

	LMA
$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$	$[-0.072, +0.321]$
$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$	$[-0.001, +0.018]$
$\varepsilon_{e\mu}^u$	$[-0.050, +0.020]$
$\varepsilon_{e\tau}^u$	$[-0.077, +0.098]$
$\varepsilon_{\mu\tau}^u$	$[-0.006, +0.007]$

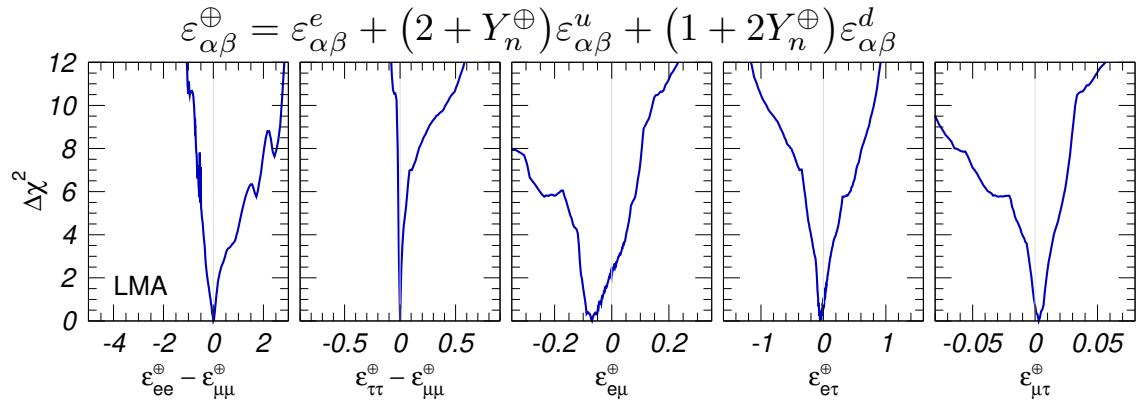
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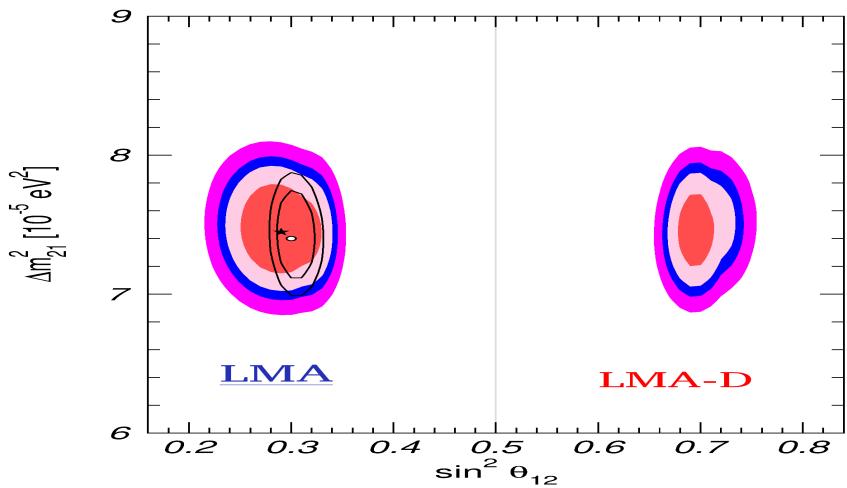
- Standard Solution \equiv LMA \Rightarrow Bounds $\mathcal{O}(1\% - 10\%)$
 \Rightarrow Maximum effect at LBL experiments:



\Rightarrow To be considered in effects/sensitivity studies
at DUNE, HK... (tables available)

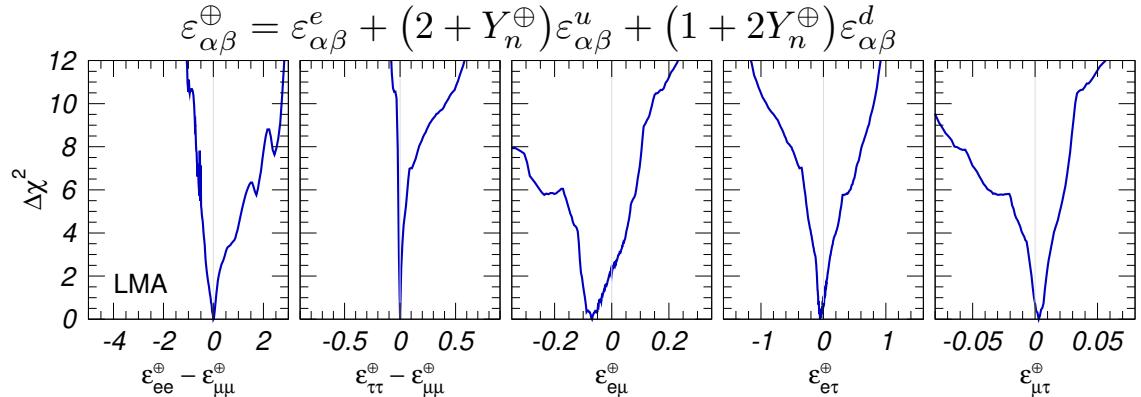
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	LMA	$\text{LMA} \oplus \text{LMA-D}$
$\varepsilon_{ee}^u - \varepsilon_{\mu\mu}^u$	$[-0.072, +0.321]$	$\oplus [-1.042, -0.743]$
$\varepsilon_{\tau\tau}^u - \varepsilon_{\mu\mu}^u$	$[-0.001, +0.018]$	$[-0.016, +0.018]$
$\varepsilon_{e\mu}^u$	$[-0.050, +0.020]$	$[-0.050, +0.059]$
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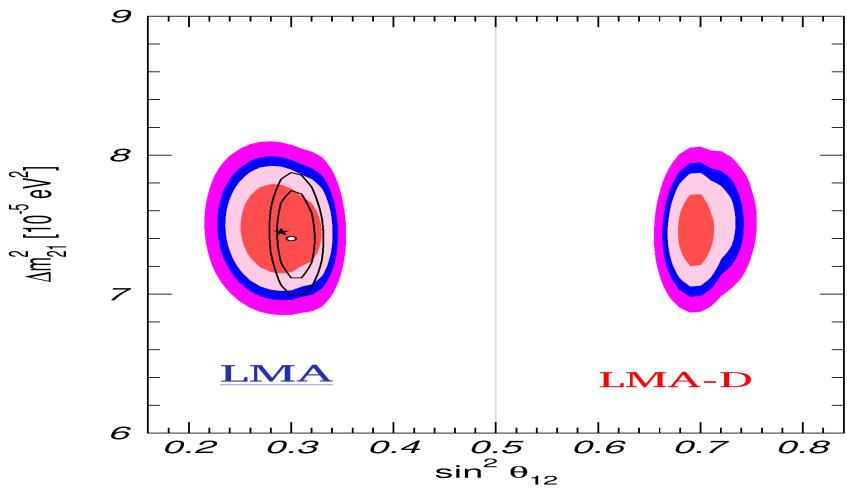
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- Degenerate solution $\equiv \text{LMA-D}$
Miranda, Tortola, Valle, hep-ph/0406280

$$\Rightarrow \theta_{12} \leftrightarrow \frac{\pi}{2} - \theta_{12} \quad \& \quad (\varepsilon_{ee} - \varepsilon_{\mu\mu}) \rightarrow -(\varepsilon_{ee} - \varepsilon_{\mu\mu}) - 2$$

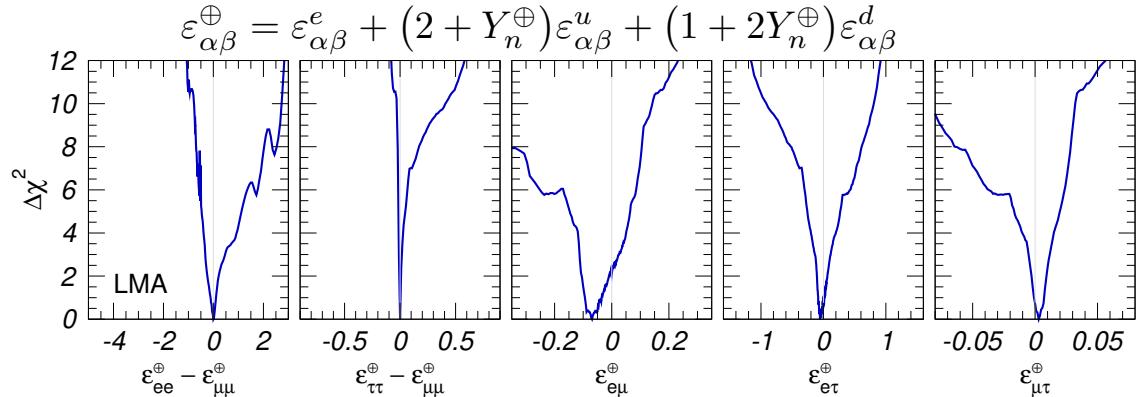
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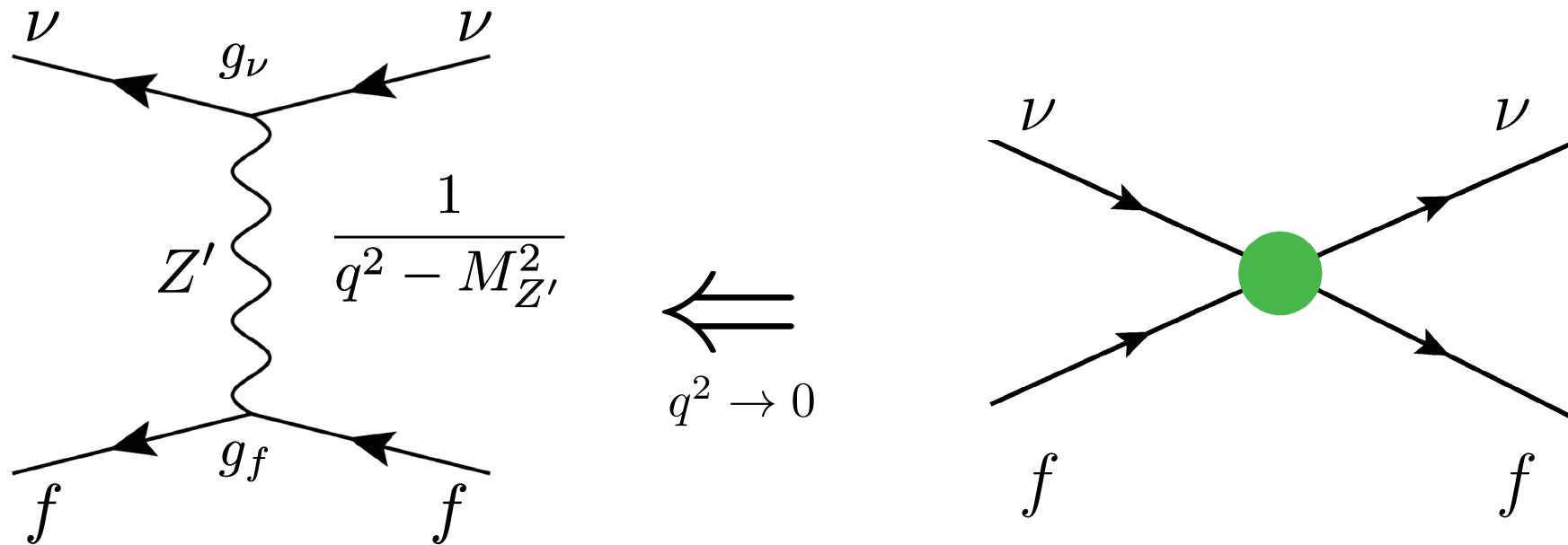
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- $\Rightarrow \theta_{12} \leftrightarrow \frac{\pi}{2} - \theta_{12}$ & $(\varepsilon_{ee} - \varepsilon_{\mu\mu}) \rightarrow -(\varepsilon_{ee} - \varepsilon_{\mu\mu}) - 2$
- \Rightarrow Requires NSI $\sim G_F$ (light mediators?)

Farzan 1505.06906, and Shoemaker 1512.09147

Oscillation bounds on Z'/Dark Photons

Coloma, MCGG, Maltoni, JHEP'21 [2009.14220]

Interpreting

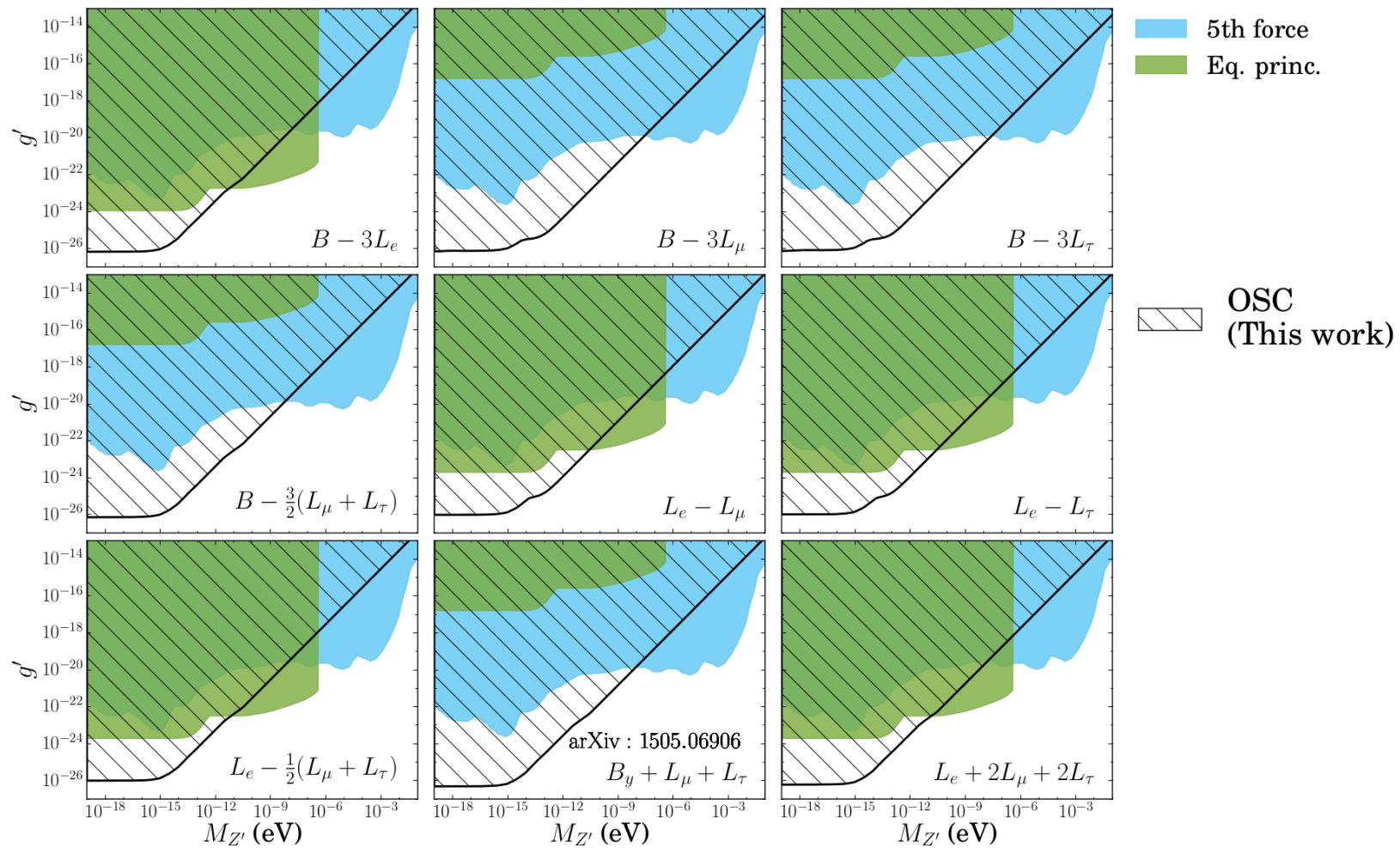


$$\frac{g'^2}{M_{Z'}^2} q'_f q'_\nu \quad \Leftarrow \quad \epsilon_{\alpha\beta}^f$$

$Z'/\text{Dark-photon}$: Bounds from ν Oscillations

Coloma, MCGG, Maltoni, JHEP'21 [2009.14220]

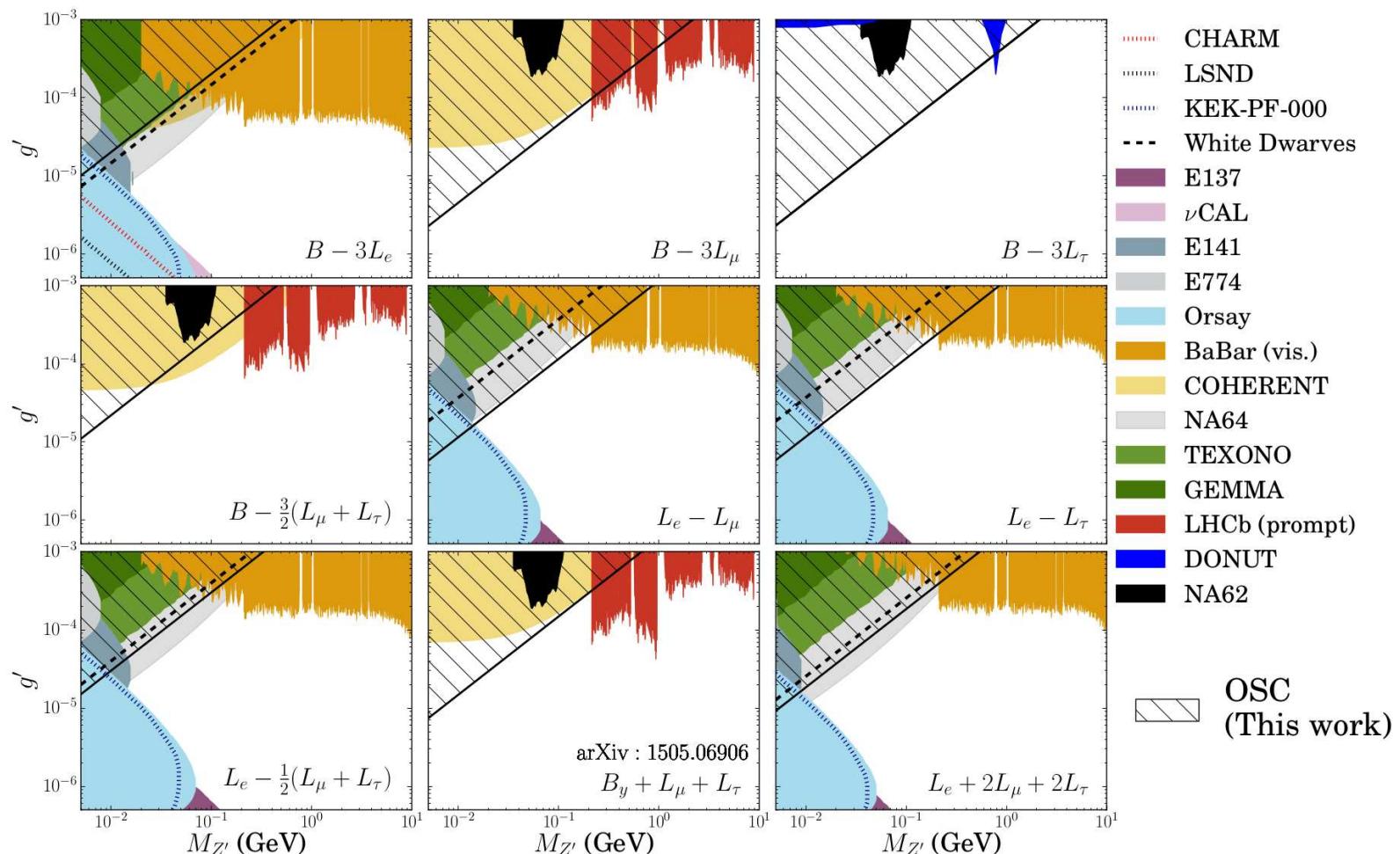
Very light ($M' \lesssim \mathcal{O}(\text{eV})$) mediator \Rightarrow Long Range Force to Contact Interaction in H_{mat}



Z' Models: ν Oscillations Bounds

Coloma, MCGG, Maltoni ArXiv:2009.14220

$M_{Z'} \gtrsim \mathcal{O}(\text{MeV}) \Rightarrow$ Contact Interaction in H_{mat}



Confirmed Low Energy Picture and MY List of Q&A

- At least two neutrinos are massive \Rightarrow There is NP
- Oscillations DO NOT determine the lightest mass
Only model independent probe of m_ν β decay: $\sum m_i^2 |U_{ei}|^2 \leq (0.8 \text{ eV})^2$ Katrin 2021
- Dirac or Majorana?: Anxiously waiting for ν -less $\beta\beta$ decay Lecture by S. Bettini
- Three mixing angles are non-zero (and relatively large) \Rightarrow very different from CKM
- Leptonic CP and Ordering: “Hints” but not confirmation
Definite answer may require new oscillation experiments
- Only three light states? Some old anomalies supporting light sterile neutrinos vanishing, and tensions with new experiments
- Other NP at play in oscillations Interesting effects of NP with light mediators

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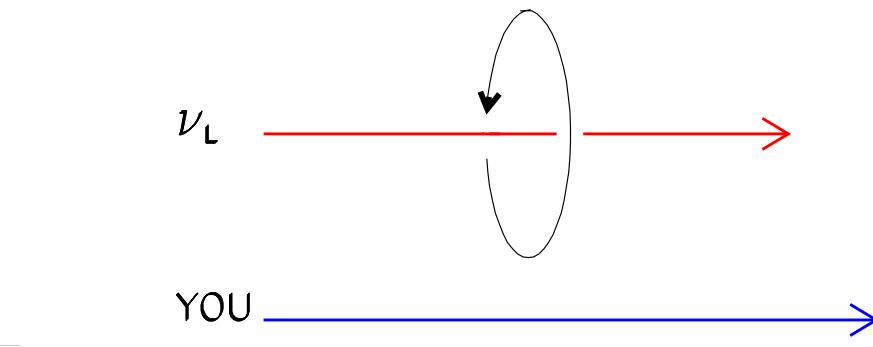
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- Neutrinos in Cosmology? Lectures by L. Verde
- Astrophysics/Astronomy with Neutrinos? Lecture by F.Halzen

...

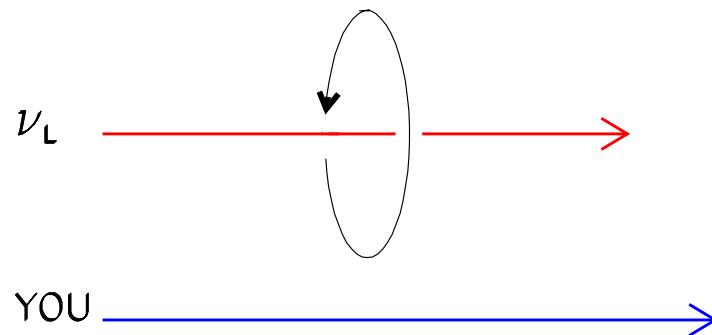
Neutrinos always “Left-Handed” \equiv Massless

- If ν had a mass they would not go to the speed of light:
 \Rightarrow the direction of its momentum depends on the reference frame
So in one reference frame

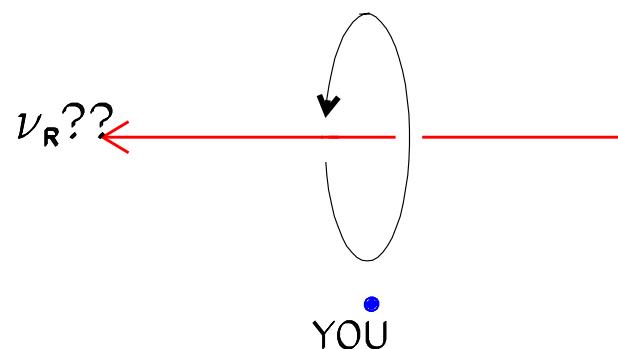


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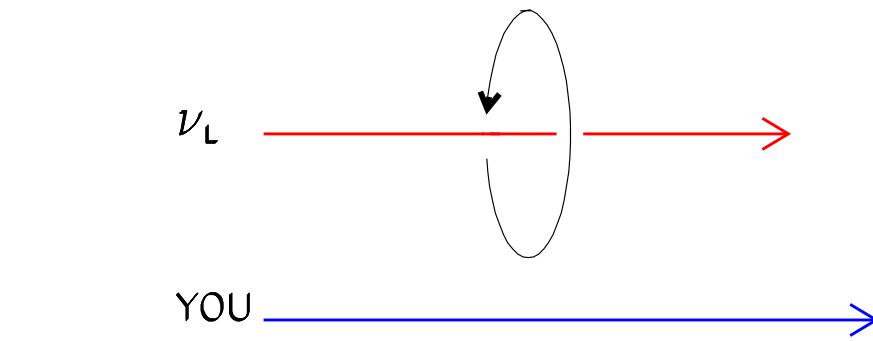
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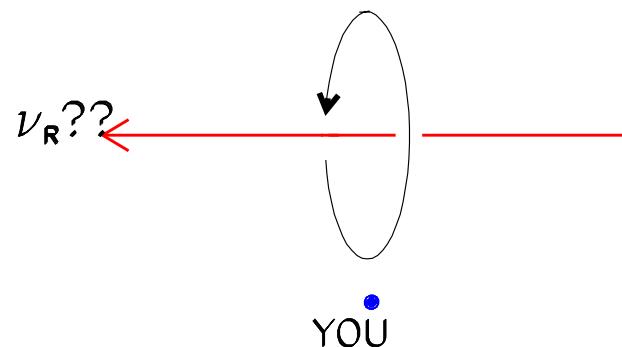
So in one reference frame



So if always left \Rightarrow

Strictly massless

And in another



The Other Flavours

ν coming out of a nuclear reactor is $\bar{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

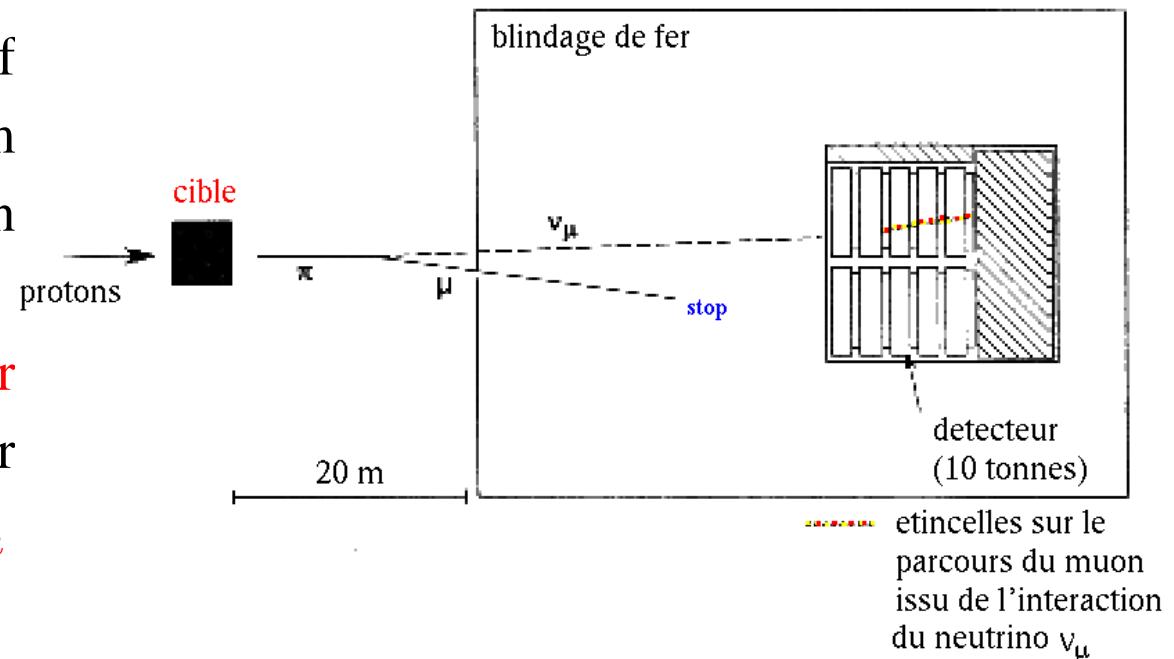
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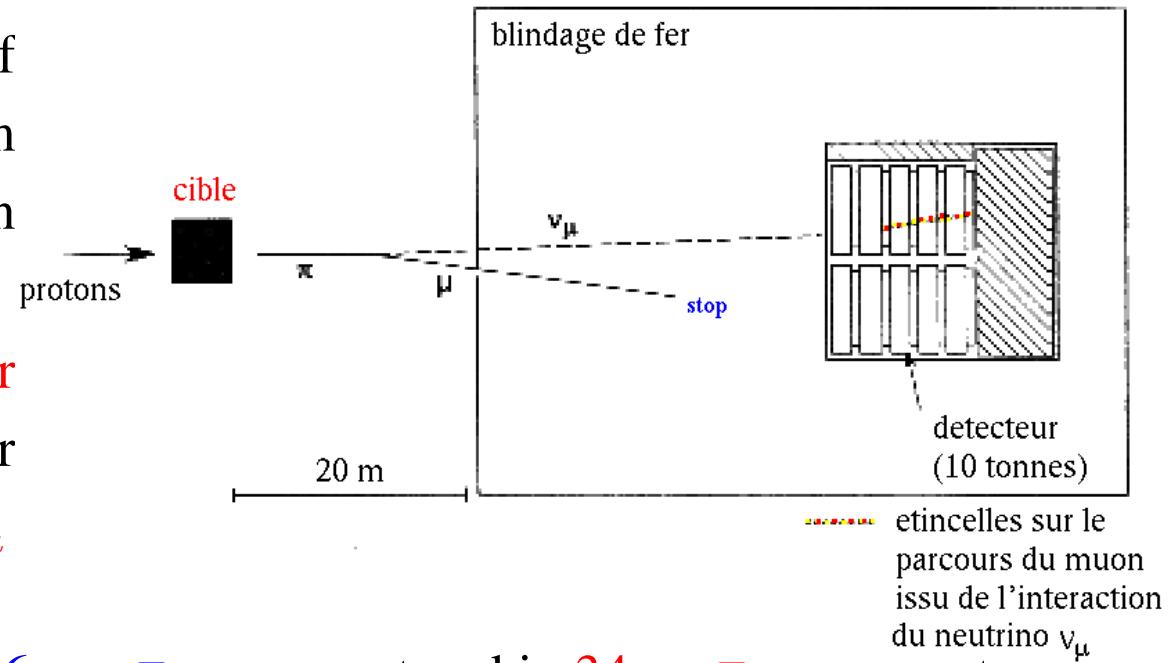


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If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and e^-

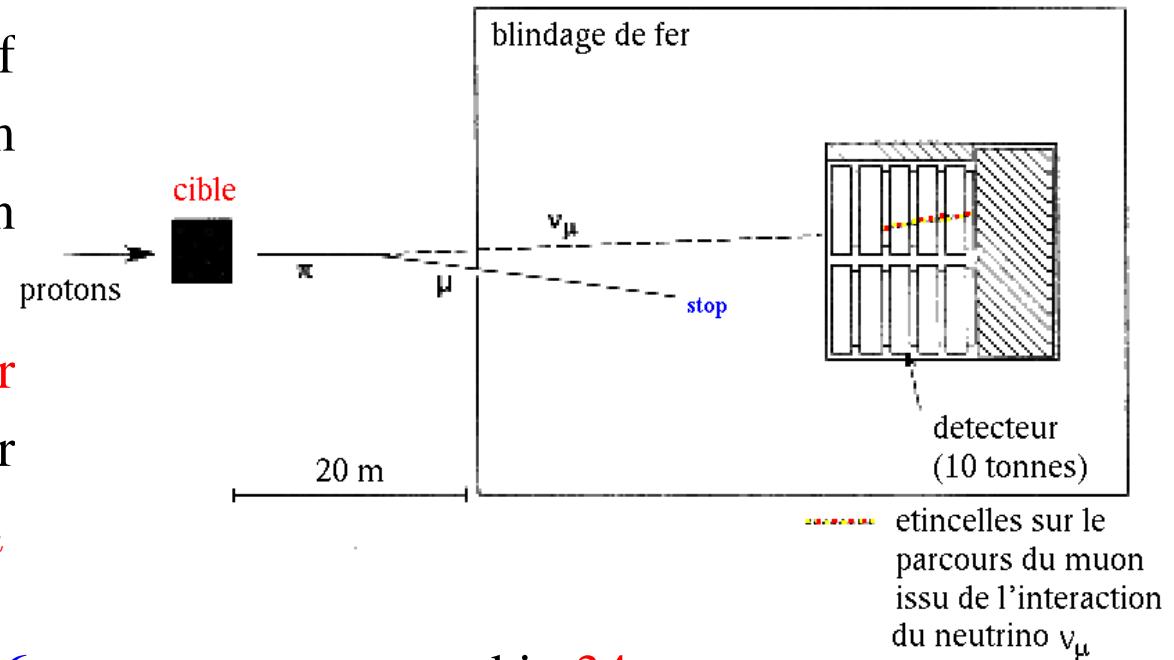
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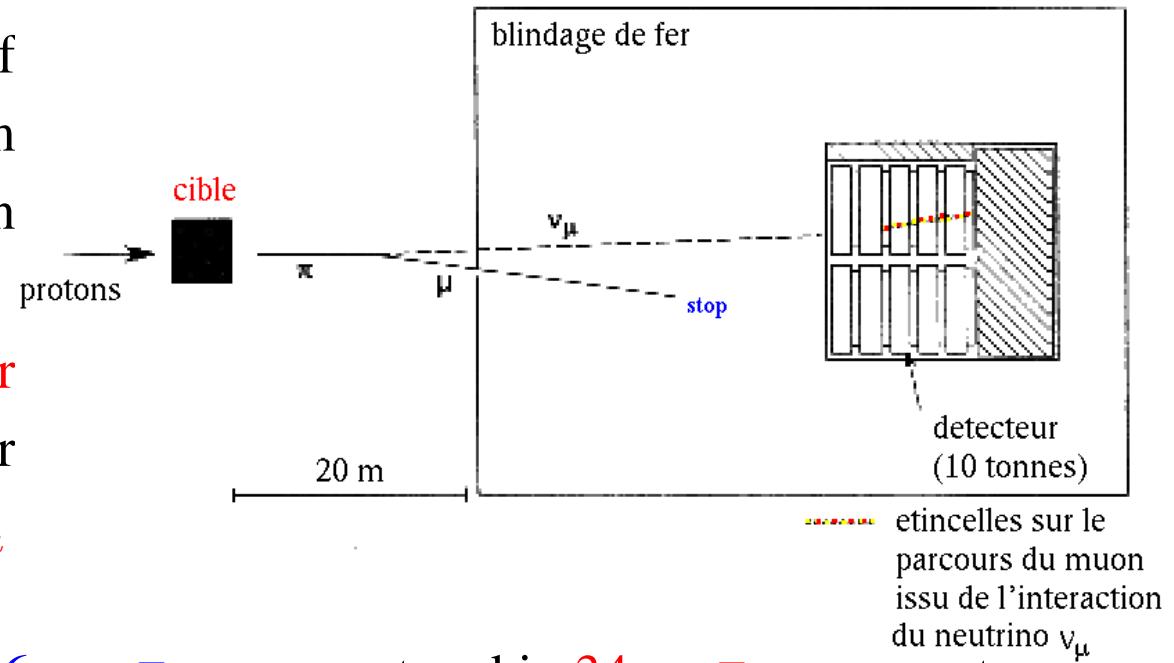
If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and $e^- \Rightarrow$ Conclusion: ν_μ is a different particle

The Other Flavours

ν coming out of a nuclear reactor is $\bar{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

In 1959 **M. Schwartz** thought of producing an intense ν beam from π 's decay (produced when a proton beam of GeV energy hits matter)



Schwartz, Lederman, Steinberger and **Gaillard** built a spark chamber (a 10 tons of neon gas) to detect ν_μ

They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

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In 1977 **Martin Perl** discovers the particle tau \equiv the third lepton family.

The ν_τ was observed by **DONUT** experiment at FNAL in 1998 (officially in Dec. 2000).

✓ Mass from Non-Renormalizable Operator

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\text{NP}}$

- The same particle content as the SM and same pattern of symmetry breaking
- But there can be non-renormalizable
(dim > 4) operators

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There is only one!

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Implications:

- It is natural that ν mass is the first evidence of NP
- Naturally $m_\nu \ll$ other fermions masses $\sim \lambda^f v$ if $\Lambda_{\text{NP}} \gg v$
- See-saw with heavy ν_R integrated out is a particular example of this

Neutrino Mass Scale: Other Channels

Muon neutrino mass

- From the two body decay at rest

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

- Energy momentum conservation:

$$m_\pi = \sqrt{p_\mu^2 + m_\mu^2} + \sqrt{p_\mu^2 + \cancel{m_\nu^2}}$$

$$\cancel{m_\nu^2} = m_\pi^2 + m_\mu^2 - 2 + m_\mu \sqrt{p^2 + m_\pi^2}$$

- Measurement of p_μ plus the precise knowledge of m_π and $m_\mu \Rightarrow \cancel{m_\nu}$
- The present experimental result bound:

$$m_{\nu_\mu}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\mu j}|^2} < 190 \text{ KeV}$$

Tau neutrino mass

- The τ is much heavier $m_\tau = 1.776 \text{ GeV}$
 \Rightarrow Large phase space \Rightarrow difficult precision for m_ν
- The best precision is obtained from hadronic final states

$$\tau \rightarrow n\pi + \nu_\tau \quad \text{with } n \geq 3$$

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⇒ If mixing angles U_{ej} are not negligible

Best kinematic limit on Neutrino Mass Scale comes from Tritium Beta Decay

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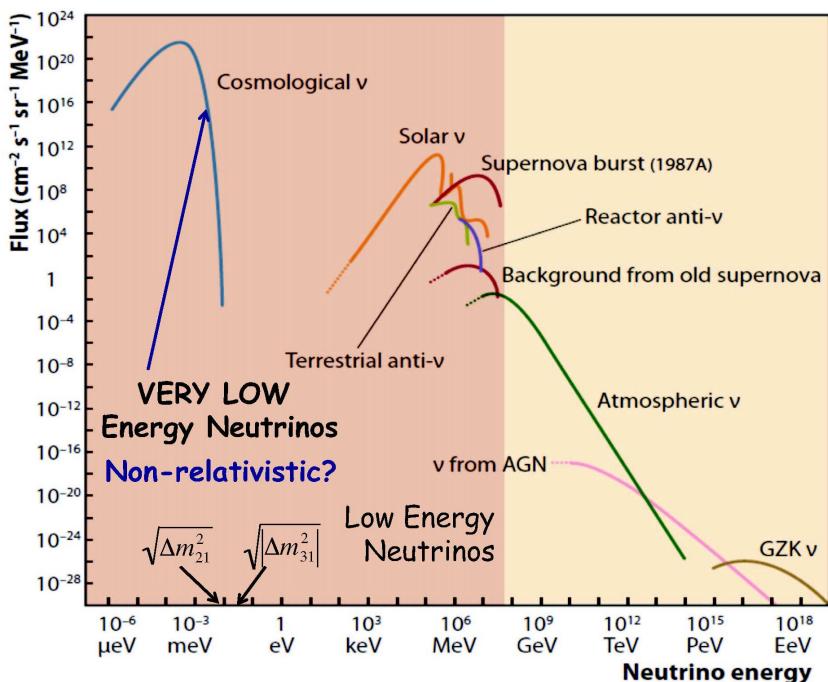
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To allow observation of neutrino oscillations:

- Nature has to be good: $\theta \neq 0$
- Need the right set up (\equiv right $\langle \frac{L}{E} \rangle$) for Δm^2

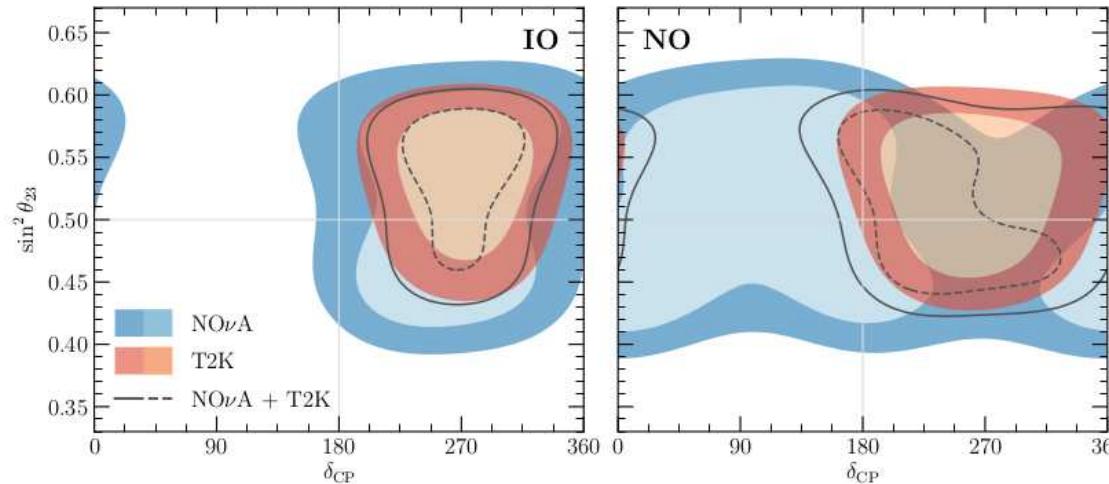


Source	E (GeV)	L (Km)	Δm^2 (eV ²)
Solar	10^{-3}	10^7	10^{-10}
Atmos	$0.1-10^2$	$10-10^3$	$10^{-1}-10^{-4}$
Reactor	10^{-3}	SBL: 0.1–1	$10^{-2}-10^{-3}$
		LBL: $10-10^2$	$10^{-4}-10^{-5}$
Accel	10	SBL: 0.1	$\gtrsim 0.01$
		LBL: 10^2-10^3	$10^{-2}-10^{-3}$

Compatibility T2K/NO ν A

Concha Gonzalez-Garcia

- 1 and 2 σ (2dof) allowed regions (for $s_{13}^2 = 0.0224$, marg over $|\Delta m_{3\ell}^2|$)

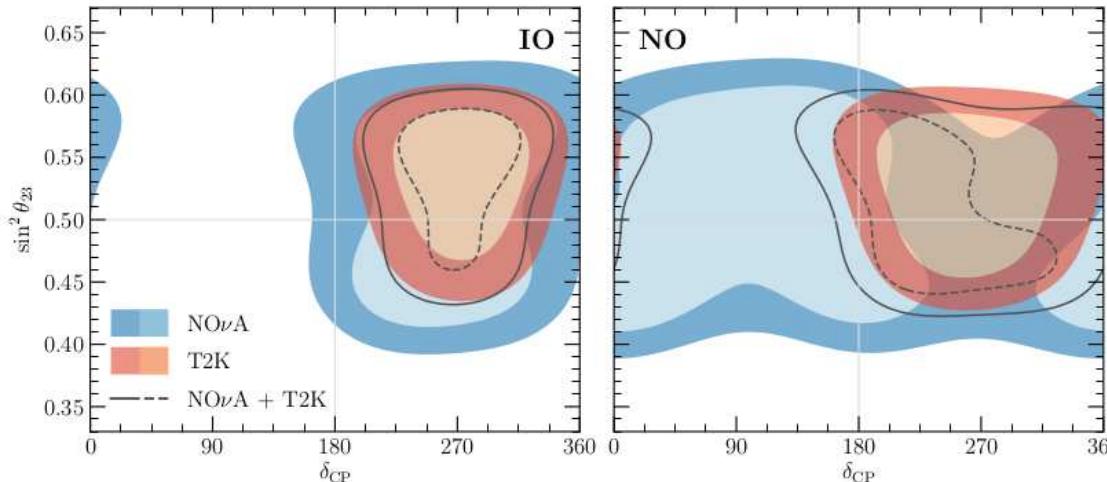


⇒ Better agreement in IO but NO 1σ regions “touch”

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- Parameter goodness-of-fit (PG) test:

	normal ordering			inverted ordering		
	χ^2_{PG}/n	p-value	# σ	χ^2_{PG}/n	p-value	# σ
T2K vs NOvA (θ_{13} free)	6.7/4	0.15	1.4 σ	3.6/4	0.46	0.7 σ
T2K vs NOvA (θ_{13} fix)	6.5/3	0.088	1.7 σ	2.8/3	0.42	0.8 σ

No significant
incompatibility

Z' Models: Viable models for LMA-D

Coloma, MCGG, Maltoni, JHEP'21 [2009.14220]

