Probing dense matter with neutron star mergers

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Washington University in St. Louis



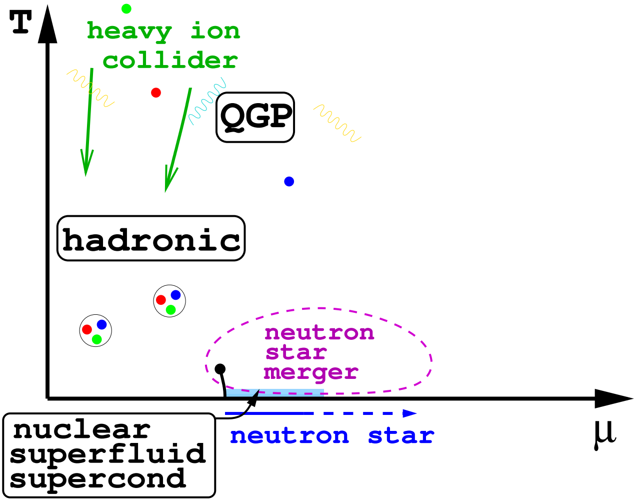
Outline

1. Neutron star mergers as a probe of dense matter
2. Disequilibrium: thermal conductivity; shear and bulk viscosity
3. Is thermal conductivity important in mergers?
   Dissipation time for temperature inhomogeneities
4. Is bulk viscosity important in mergers?
   - Bulk viscosity is a resonance
   - Damping time for density oscillations
5. Damping of density oscillations:
   - Urca processes, direct and modified
   - Fermi Surface approximation
   - Detailed balance—how it can fail

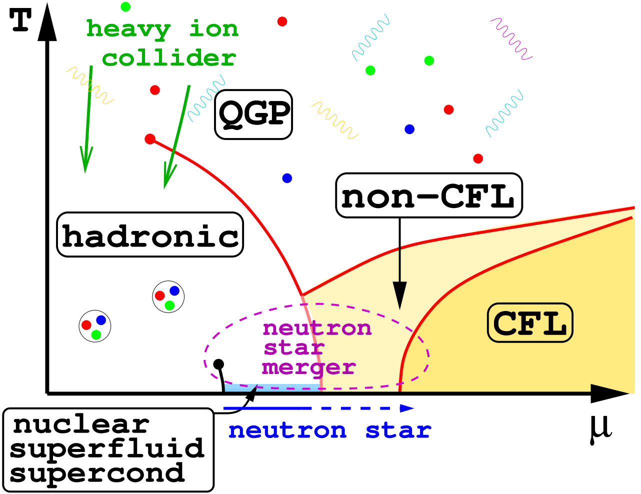
(1) Neutron star mergers

QCD Phase diagram

We want to know the properties of matter under extreme conditions



Conjectured QCD Phase diagram

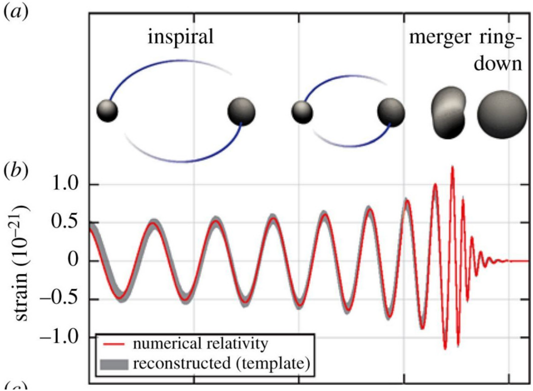


heavy ion collisions: deconfinement crossover and chiral critical point

neutron stars: quark matter core?

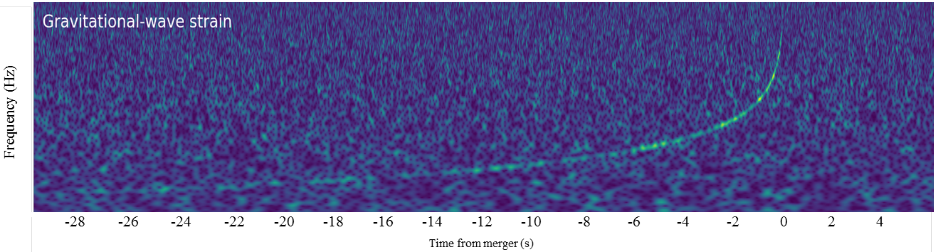
neutron star mergers: dynamics of warm and dense matter

Grav waves from mergers: prediction



Grav waves from mergers: observation

LIGO Data from the event GW170817

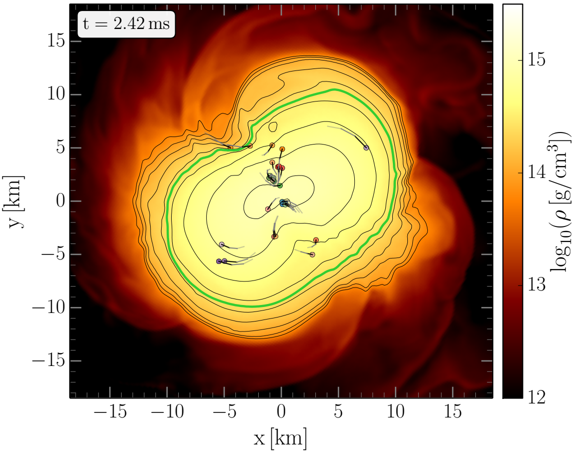


Omega Scan of LIGO's First Neutron Star Merger Detection

With LIGO we only see the inspiral, not the merger itself.

Neutron star mergers

Mergers probe the properties of nuclear/quark matter at high density (up to $\sim 4n_{\rm sat}$) and temperature (up to $\sim 60\,$MeV)

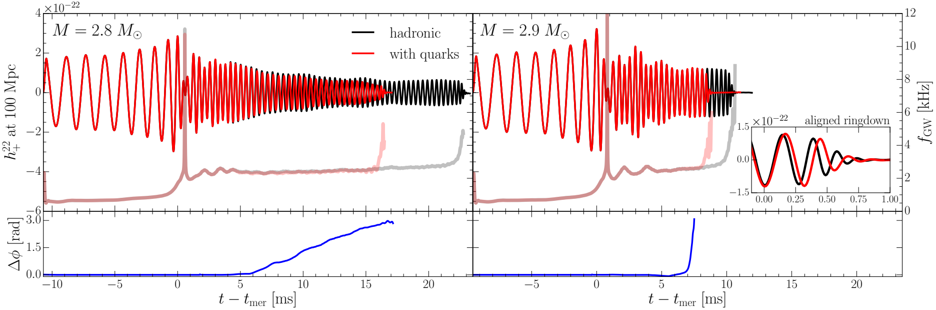


If we want to use mergers to learn about nuclear matter, we need to include all the relevant physics in our simulations.

Rezzolla group, Frankfurt [Video]

Using grav waves to probe dense matter

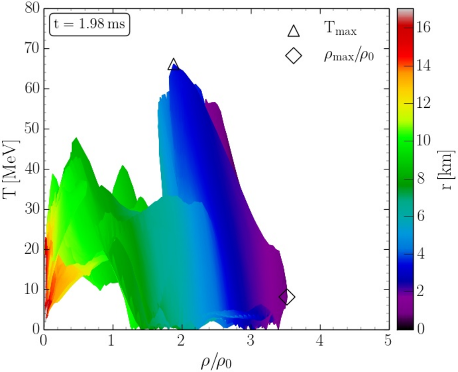
Current simulations try to connect the gravitational wave signal with features of the **Equation of State**, such as a first-order phase transition:

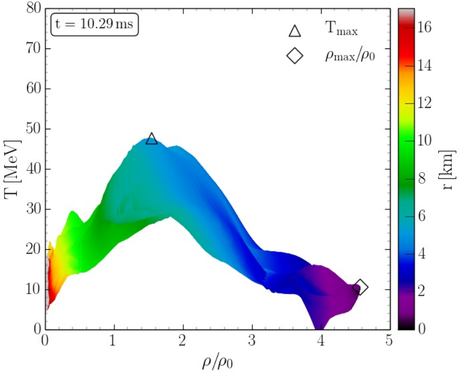


Most et. al., `arXiv:1807.03684`

solid lines: gravitational wave strain
translucent lines: instantaneous frequency

Nuclear material in a neutron star merger





M. Hanauske, Rezzolla group, Frankfurt

Equilibrium: **Equation of State** $\varepsilon(n_B, s)$ or $P(\mu, T)$; but...

Significant spatial/temporal variation in:
- temperature
- fluid flow velocity
- density

so we need to allow for
- thermal conductivity
- shear viscosity
- bulk viscosity

(2) Disequilibrium

Equilibration phenomena in mergers

The important mechanisms are the ones whose equilibration time is $\lesssim 20\,\mathrm{ms}$

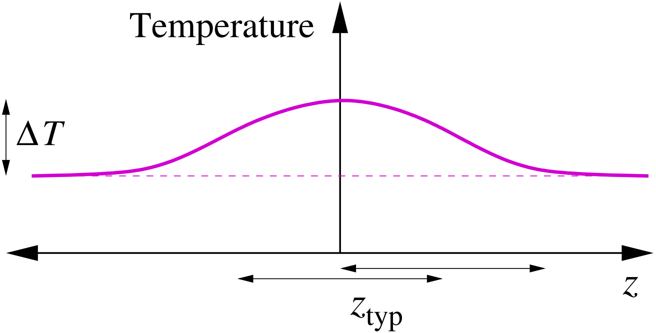
Executive Summary:

- Thermal equilibration: If neutrinos are trapped, and there are short-distance temperature gradients then thermal transport might be fast enough to play a role.
- Shear viscosity: similar conclusion.
- Bulk viscosity: could damp density oscillations on the same timescale as the merger.

(3) Thermal equilibration

Does thermal conductivity smooth out temperature gradients on the 20 ms timescale of the merger?

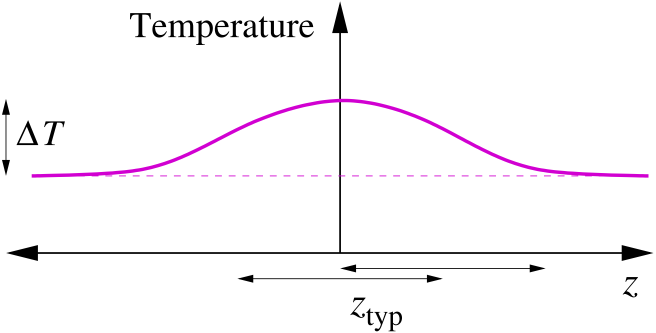
Thermal equilibration



Volume
$V \sim z_{\text{typ}}^3$

Surface area
$A \sim 6z_{\text{typ}}^2$

Thermal equilibration



Volume
$V \sim z_{\rm typ}^3$

Surface area
$A \sim 6 z_{\rm typ}^2$

Extra heat in region: $E_{\rm therm} = c_V V \Delta T \approx c_V z_{\rm typ}^3 \Delta T$

Rate of heat outflow: $W_{\rm therm} = \kappa \dfrac{dT}{dz} A \approx \kappa \dfrac{\Delta T}{z_{\rm typ}} 6 z_{\rm typ}^2$

$$\boxed{\text{Time to equilibrate:} \quad \tau_\kappa = \frac{E_{\rm therm}}{W_{\rm therm}} \approx \frac{c_V z_{\rm typ}^2}{6\kappa}}$$

Thermal equilibration time

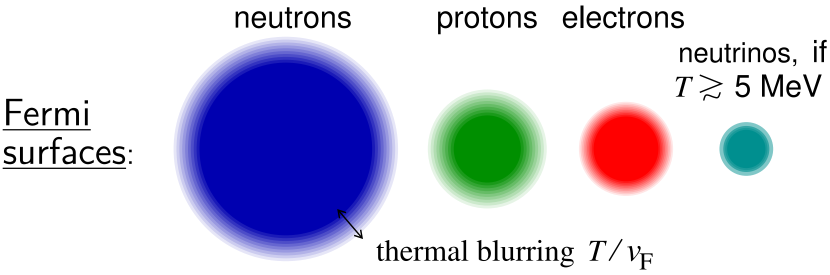
$$\boxed{\text{Time to equilibrate:} \quad \tau_\kappa = \frac{E_{\mathrm{therm}}}{W_{\mathrm{therm}}} \approx \frac{c_V z_{\mathrm{typ}}^2}{6\kappa}}$$

Thermal diffusion is important if $\tau_\kappa \lesssim 20\,\mathrm{ms}$

To calculate the thermal equilibration time τ_κ, we need

- specific heat capacity c_V
- thermal conductivity κ

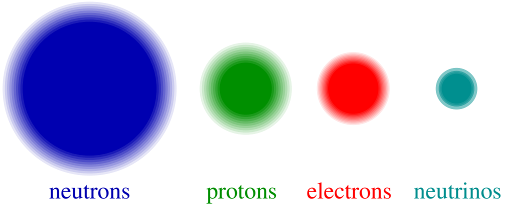
Nuclear material constituents



neutrons:	$\sim 90\%$ of baryons	$p_{Fn} \sim 350\,\text{MeV}$
protons:	$\sim 10\%$ of baryons	$p_{Fp} \sim 150\,\text{MeV}$
electrons:	same density as protons	$p_{Fe} = p_{Fp}$
neutrinos:	only present if mfp $\ll 10\,\text{km}$	i.e. when $T \gtrsim 5\,\text{MeV}$

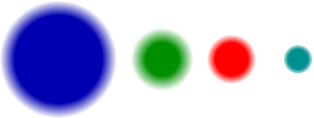
Specific heat capacity

What determines the specific heat capacity?



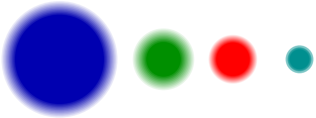
Specific heat capacity

Dominated by **neutrons**



$$
\begin{aligned}
c_V &\sim \text{number of states available} \\
&\quad\ \text{to carry energy} \lesssim T \\
&\sim \text{vol of mom space with states available to carry energy} \lesssim T \\
&\sim {p_{Fn}}^2 \delta p
\end{aligned}
$$

Specific heat capacity



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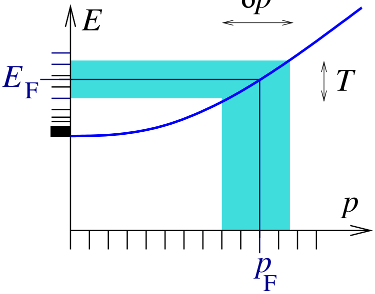

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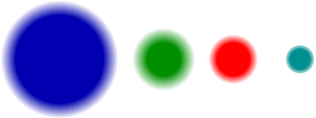

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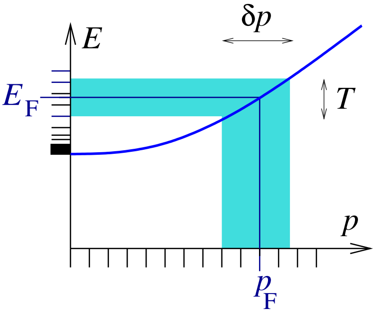


$$\delta p = \frac{T}{v_{Fn}} = T \times \frac{m_n^*}{p_{Fn}}$$

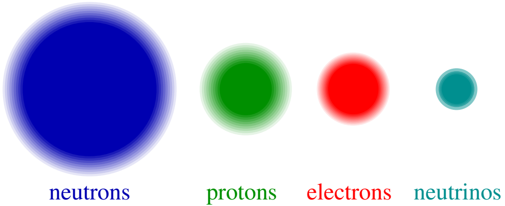
$$c_V \sim {p_{Fn}}^2 \delta p \sim {p_{Fn}}^2 \frac{m_n^*}{p_{Fn}} T \sim m_n^* p_{Fn} T$$

(Note: neutron density $n_n \sim {p_{Fn}}^3$)

$$\boxed{c_V \approx 1.0\, m_n^*\, {n_n}^{1/3}\, T}$$

Thermal conductivity

What determines the thermal conductivity?



Thermal conductivity

Thermal conductivity $\quad \kappa \propto n\, v\, \lambda$

Dominated by the species with the right combination of

- high density
- weak interactions $\Rightarrow$ long mean free path (mfp) λ

neutrons:

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neutrons: high density, but strongly interacting (short mfp) ✘

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neutrons: high density, but strongly interacting (short mfp)

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electrons: low density, only E-M interactions (long mfp) *

neutrinos:

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neutrons: high density, but strongly interacting (short mfp) ✗

protons: low density, strongly interacting (short mfp) ✗ ✗

electrons: low density, only E-M interactions (long mfp) ✓*

neutrinos:
- $T \lesssim 5\,\text{MeV}$: $\lambda >$ size of merged stars, so they all escape, density $= 0$ ✗
- $T \gtrsim 5\,\text{MeV}$: $\lambda <$ size of merged stars, but still very long mfp! ✓✓

* E-M interactions can be long-range, reduces mfp below that of neutrons

Shternin & Ofengeim `arXiv:2202.05794`

Electrons vs Neutrinos

$$\tau_\kappa \approx \frac{c_V z_{\rm typ}^2}{6\kappa}$$

electron-dominated ($T \lesssim 5\,\mathrm{MeV}$)

$$\kappa^{(e)} \approx 1.5\,\frac{n_e^{2/3}}{\alpha}$$

neutrino-dominated ($T \gtrsim 5\,\mathrm{MeV}$)

$$\kappa^{(\nu)} \approx 0.33\frac{n_\nu^{2/3}}{G_F^2 m_n^{*2} n_e^{1/3}\, T}$$

Equilibration time for hot spot of size $z_{\rm typ}$:

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Equilibration time for hot spot of size $z_{\rm typ}$:

$$\tau_\kappa^{(e)} = \boxed{5\times 10^8\,\mathrm{s}}\left(\frac{z_{\rm typ}}{1\,\mathrm{km}}\right)^2\left(\frac{T}{1\,\mathrm{MeV}}\right)\times\left(\frac{m_n^*}{0.8\,m_n}\right)\left(\frac{n_0}{n_n}\right)^{1/3}\left(\frac{0.1}{x_p}\right)^{2/3}$$

$$\tau_\kappa^{(\nu)} \approx \boxed{0.7\,\mathrm{s}}\left(\frac{z_{\rm typ}}{1\,\mathrm{km}}\right)^2\left(\frac{T}{10\,\mathrm{MeV}}\right)^2\times\left(\frac{\mu_e}{2\mu_\nu}\right)^2\left(\frac{0.1}{x_p}\right)^{1/3}\left(\frac{m_n^*}{0.8\,m_n}\right)^3$$

Electron thermal transport is *slow*!
electron mfp is too short

Neutrino thermal transport…
maybe if gradients on 0.1 km scale?

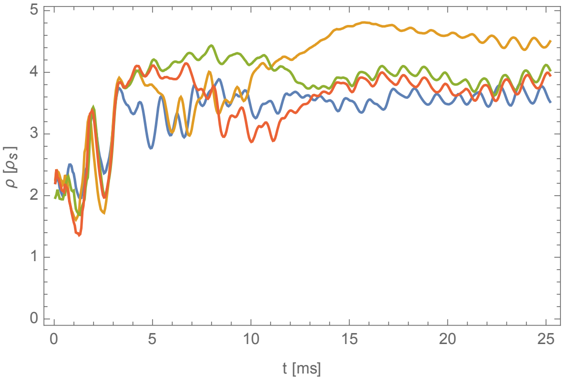
(4) Damping of density oscillations

Are density oscillations damped on the 20 ms timescale of the merger?

Density oscillations in mergers

Density vs time for tracers in merger

Bulk viscosity neglected



Tracers (co-moving fluid elements) show dramatic density oscillations, especially in the first 5 ms.

Amplitude: up to 50%
Period: 1–2 ms

How long does it take for bulk viscosity to dissipate a sizeable fraction of the energy of a density oscillation?

What is the damping time τ_ζ?

Density oscillation damping time τ_ζ

Density oscillation of amplitude Δn at angular freq ω:

$$n(t) = \bar{n} + \Delta n \cos(\omega t)$$

Density oscillation damping time τ_ζ

Density oscillation of amplitude Δn at angular freq ω:

$$n(t) = \bar{n} + \Delta n \cos(\omega t)$$

Energy of density oscillation:
(K = nuclear incompressibility)

$$E_{\rm comp} = \frac{K}{18}\bar{n}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

Compression dissipation rate:
(ζ = bulk viscosity)

$$W_{\rm comp} = \zeta\frac{\omega^2}{2}\left(\frac{\Delta n}{\bar{n}}\right)^2$$

$$\boxed{\text{Damping Time:} \quad \tau_\zeta = \frac{E_{\rm comp}}{W_{\rm comp}} = \frac{K\bar{n}}{9\omega^2\zeta}}$$

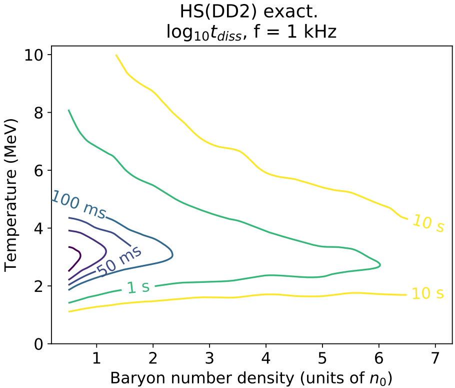
Bulk visc is only important if $\tau_\zeta \lesssim 20\,\mathrm{ms}$

Damping time calculation (ν-transparent)

Damping time:

$$\boxed{\tau_\zeta = \frac{K\bar{n}}{9\omega^2\zeta}}$$

Damping can be fast enough to affect merger dynamics!



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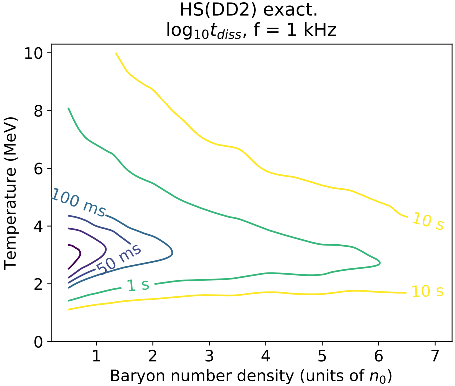

- Damping gets *slower at higher density*.
  Baryon density $\bar{n}$ and incompressibility K are both increasing.
  Oscillations carry more energy $\Rightarrow$ slower to damp

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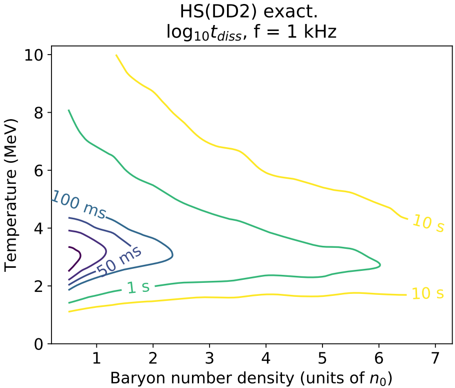

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- Damping gets *slower at higher density*.
  Baryon density $\bar{n}$ and incompressibility K are both increasing.
  Oscillations carry more energy $\Rightarrow$ slower to damp
- *Non-monotonic T-dependence:* damping is fastest at $T \sim 3\,\text{MeV}$.
  Damping is slow at very low or very high temperature.
  Non-monotonic dependence of bulk viscosity on temperature

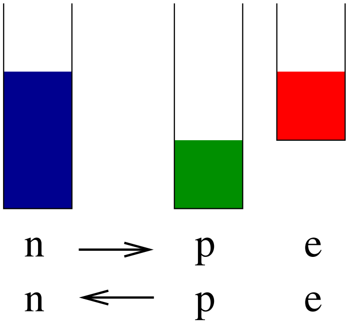
Bulk viscosity and beta equilibration

- Why is there bulk viscosity in nuclear matter?
- Why does it peak at $T \sim 3\,\text{MeV}$?

Bulk viscosity and beta equilibration

- Why is there bulk viscosity in nuclear matter?
- Why does it peak at $T \sim 3\,\mathrm{MeV}$?

When you compress nuclear matter,
the proton fraction wants to change.



Only the weak interaction can change proton fraction;
It operates on a macroscopic time scale, comparable to the merger ($\sim$ ms)

Bulk viscosity: phase lag in system response

Some property of the material (proton fraction) takes time to equilibrate.

Baryon density n and hence fluid element volume V gets out of phase with applied pressure P:

$$\text{Dissipation} = -\int P\, dV = -\int P\, \frac{dV}{dt} dt$$

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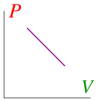
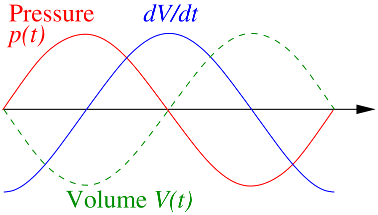
Dissipation $= 0$





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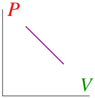
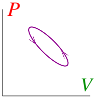
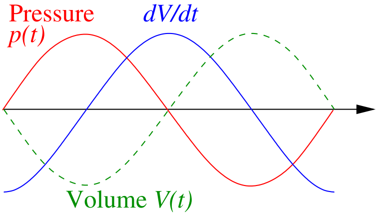
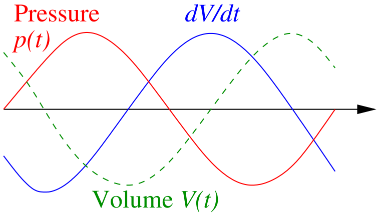


Some phase lag.
Dissipation > 0





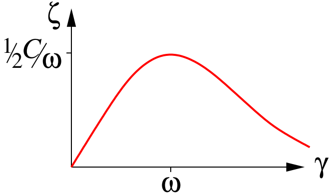


Bulk viscosity: a resonant phenomenon

Bulk viscosity is **maximum** when

$$\underset{\gamma}{\text{(internal equilibration rate)}} \quad = \quad \underset{\omega}{\text{(freq of density oscillation)}}$$

$$\zeta = C\frac{\gamma}{\gamma^2+\omega^2}$$



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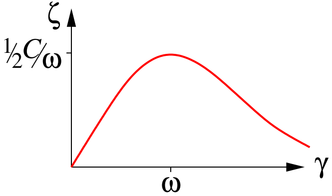

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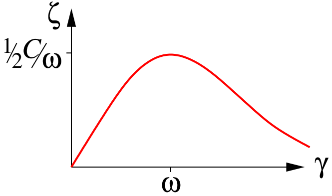

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- Slow equilibration: $\gamma \to 0 \;\Rightarrow\; \zeta \to 0$.
  System does not try to equilibrate: proton number and neutron number are both conserved. Proton fraction fixed.

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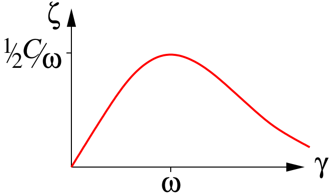

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- Slow equilibration: $\gamma \to 0 \;\Rightarrow\; \zeta \to 0$.
  System does not try to equilibrate: proton number and neutron number are both conserved. Proton fraction fixed.
- Maximum phase lag when $\omega = \gamma$.

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We now see why bulk visc is a <u>non-monotonic</u> fn of temperature.

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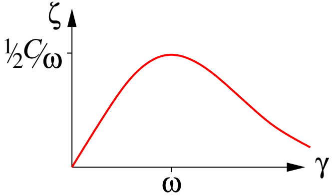

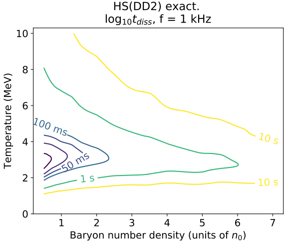

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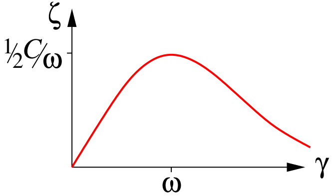
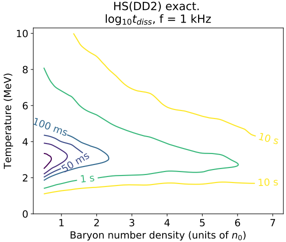
We now see why bulk visc is a <u>non-monotonic</u> fn of temperature.

$$\zeta = C \frac{\gamma(T)}{\gamma(T)^2 + \omega^2}$$





Beta equilibration rate $\gamma(T)$ rises monotonically with temperature (phase space at Fermi surface)

Maximum bulk viscosity in a neutron star merger will be when

$\boxed{\text{equilibration rate matches typical compression frequency } f \approx 1\,\text{kHz}}$

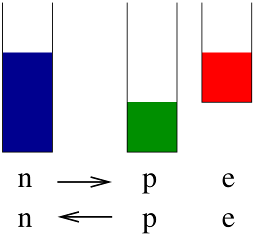
I.e. when $\gamma \sim 2\pi \times 1\,\text{kHz}$

How do we calculate the beta equilibration rate γ?

Bulk viscosity and beta equilibration

When you compress nuclear matter, the proton fraction wants to change.

Only weak interactions can change proton fraction, via "Urca processes"



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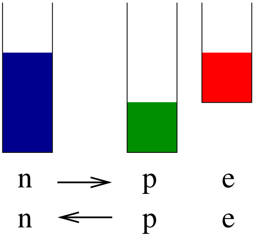


Urca process	neutrino-transparent ($T \lesssim 5\,\mathrm{MeV}$)*	neutrino-trapped ($T \gtrsim 5\,\mathrm{MeV}$)*
neutron decay	$n \to p + e^- + \bar{\nu}_e$	$\nu_e + n \to p + e^-$
electron capture	$p + e^- \to n + \nu_e$	$p + e^- \to n + \nu_e$
	forward $\neq$ backward	$A + B \leftrightarrow C + D$
equilibrium condition:	$\mu_n = \mu_p + \mu_e$?	$\mu_n + \mu_\nu = \mu_p + \mu_e$

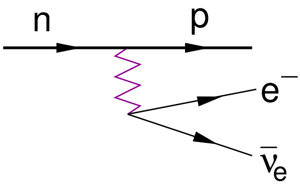
* Neutrino transparency is a finite volume effect, which occurs when the neutrino mean free path is greater than the size of the system. Our system is a neutron star, $R \sim 10\,\mathrm{km}$

Beta equilibration: direct Urca

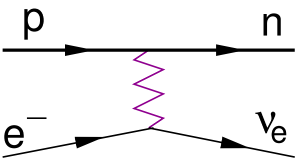
neutrino-transparent regime, $T \lesssim 5\,\mathrm{MeV}$

To calculate the beta equilibration rates, the obvious Feynman diagrams are the "direct Urca" ones

neutron decay



electron capture



Direct Urca rate

$$\Gamma_{n\to pe^-\bar{\nu}_e} = \int \frac{d^3p_n}{(2\pi)^3}\frac{d^3p_p}{(2\pi)^3}\frac{d^3p_e}{(2\pi)^3}\frac{d^3p_\nu}{(2\pi)^3}\frac{\sum_{\text{spins}}|\mathcal{M}_{\text{dU}}|^2}{2^4 E_n^* E_p^* E_e E_\nu}$$
$$\times (2\pi)^4\,\delta^4(p_n - p_p - p_e - p_\nu) f_n\,(1-f_p)\,(1-f_e)$$

$$\Gamma_{p\,e^-\to n\,\nu_e} = \text{same, with } f_i \to 1-f_i,\ E_\nu \to -E_\nu$$

where $f_i \equiv \dfrac{1}{1+e^{\frac{E_i-\mu_i}{T}}}$ (Fermi-Dirac distributions).

Matrix element is a function of the momenta. In non-rel approx:

$$\sum_{\text{spins}} |\mathcal{M}_{\text{dU}}|^2 = 32G^2 E_n^* E_p^* E_e E_\nu \left(1 + 3g_A^2 + \left(1-g_A^2\right)\frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_e E_\nu}\right)$$

where $G^2 = G_F^2\cos^2\theta_c$ and $g_A = 1.26$.

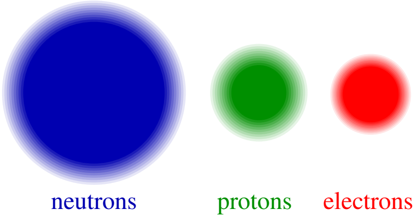
Looks complicated. Can we simplify it?

Beta equilibration phase space

Neutrino-transparent regime, $T \lesssim 5\,\mathrm{MeV}$, there is no neutrino sea.

$$n \longrightarrow p \quad e \quad \bar{\nu}$$

$$\nu \quad n \longleftarrow p \quad e$$



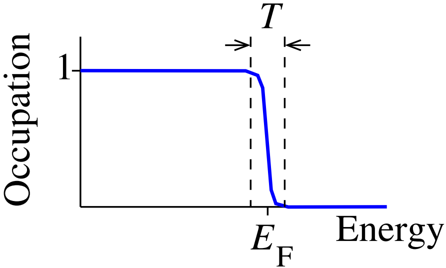
- Lots of low-energy neutrons, but their decay to $p + e$ is Fermi-blocked
- Not many high-energy neutrons

At low temperature, beta equilibration is dominated by modes near the Fermi surfaces

Fermi Surface approximation

If the temperature is low enough, we can analyse beta equilibration processes in a simple way using the *Fermi Surface* (FS) approximation.

$$T \ll (E_F - m)$$
$$\text{or} \quad (\mu - m)$$



In the FS approximation, all the particles participating in beta equilibration processes are close to their Fermi surfaces.
We can then evaluate the momentum integrals...

Direct Urca rate in FS approx

Using the Fermi Surface approximation,

$$\Gamma_{\mathrm{dU,nd}} - \Gamma_{\mathrm{dU,ec}} = \frac{17 G^2 (1 + 3g_A^2)}{240\pi} E^*_{Fn} E^*_{Fp} p_{Fe} T^4 \; \Theta_{\mathrm{dU}} \; (\mu_n - \mu_p - \mu_e)$$

$$\Theta_{\mathrm{dU}} \equiv \begin{cases} 0 & \text{if } p_{Fn} > p_{Fp} + p_{Fe} \\ 1 & \text{if } p_{Fn} < p_{Fp} + p_{Fe}, \end{cases}$$

(Electrical neutrality requires $p_{Fp} = p_{Fe}$)

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- Urca processes drive $(\mu_n - \mu_p - \mu_e)$ to zero so the equilibrium condition in ν-transparent matter is $\mu_n = \mu_p + \mu_e$?
  (only when the Fermi Surface approx is valid!)

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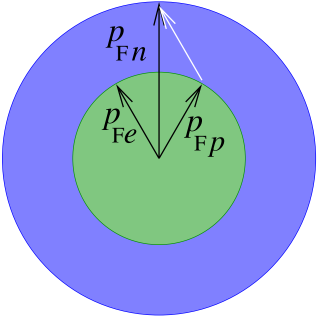
  (only when the Fermi Surface approx is valid!)
- Theta function: the rate is *zero* if the proton fraction is too small! Why?
  Why don't we see this sharp jump in the damping time plot?

When can direct Urca happen?

$$n \to p\ e^- \ \bar{\nu}_e, \quad p\ e^- \to n\ \nu_e$$

High density
High proton fraction
Direct Urca open



$\vec{p}_n = \vec{p}_p + \vec{p}_e$ is possible
because $p_{Fn} < p_{Fp} + p_{Fe}$

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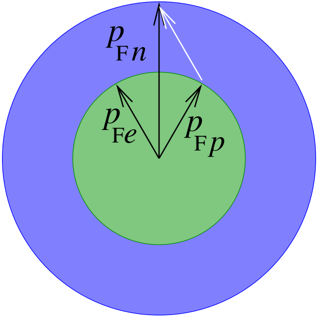
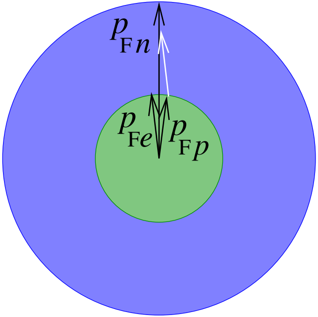


$\vec{p}_n = \vec{p}_p + \vec{p}_e$ is possible
because $p_{Fn} < p_{Fp} + p_{Fe}$

Low density
Low proton fraction
Direct Urca closed

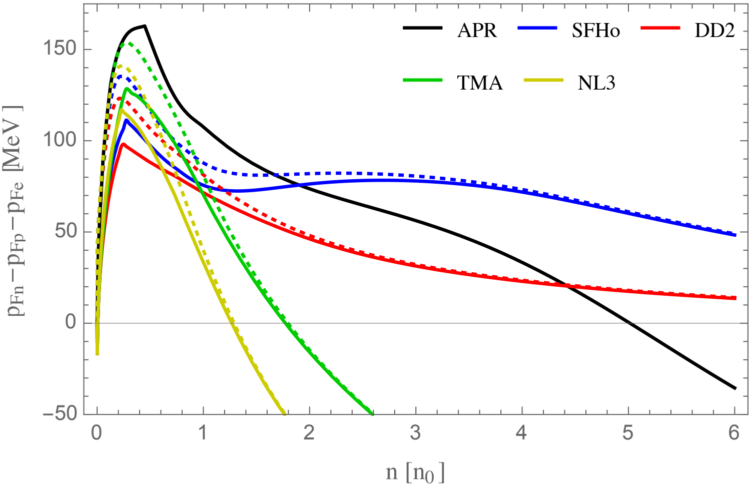


$\vec{p}_n = \vec{p}_p + \vec{p}_e$ is impossible
because $p_{Fn} > p_{Fp} + p_{Fe}$

Direct Urca threshold

Some examples of the direct Urca kinematic constraint

$\Delta p \equiv p_{Fn} - p_{Fp} - p_{Fe}$ When $\Delta p < 0$ direct Urca can happen.

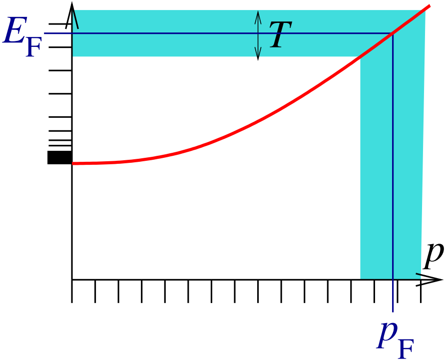


At $T \ll 1\,\text{MeV}$ we will get wildly different rates depending on the EoS.

When is the FS approx valid?

neutrons

$$p_F \sim 350\,\text{MeV}$$
$$E_F - m \sim 60\,\text{MeV}$$



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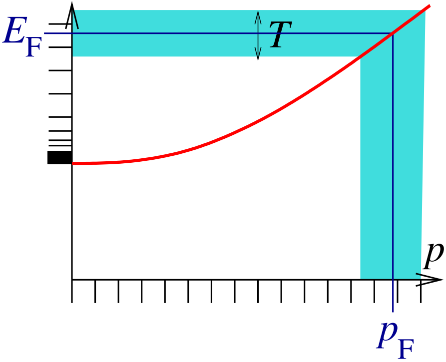
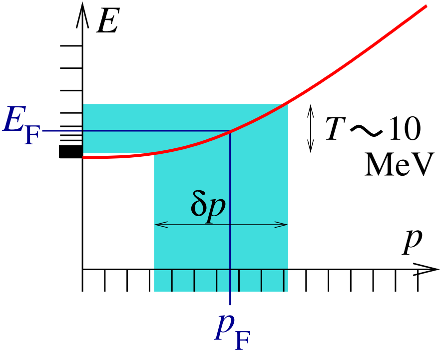


protons

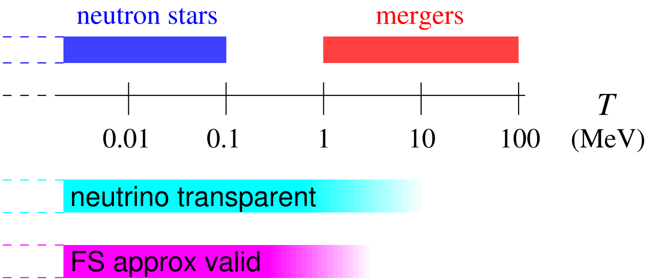
$$p_F \sim 150\,\text{MeV}$$
$$E_F - m \sim 10\,\text{MeV}$$



Fermi Surface approx clearly becomes invalid as T rises to 10 MeV.

But we will see that it becomes misleading above $T \sim 1\,\text{MeV}$.

Temperature regimes for neutron stars



We want to understand bulk viscosity in mergers.

Bulk viscosity arises from beta equilibration on the 1 ms timescale.

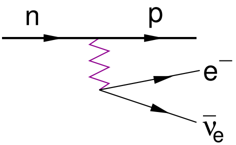
- First, understand beta equilibrium in the "cold" regime where FS approx is valid
- Then, for mergers, do the "warm" regime where the star is still neutrino transparent but FS approx is unreliable

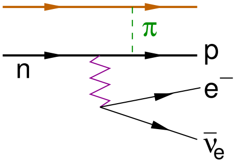
Other Urca processes

What happens when the density is below the direct Urca threshold?
A subleading process becomes relevant: "modified Urca".

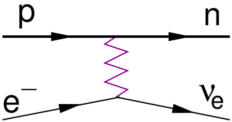
Direct Urca | Modified Urca

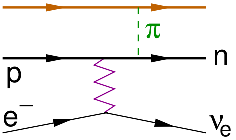
n decay





e^- capture

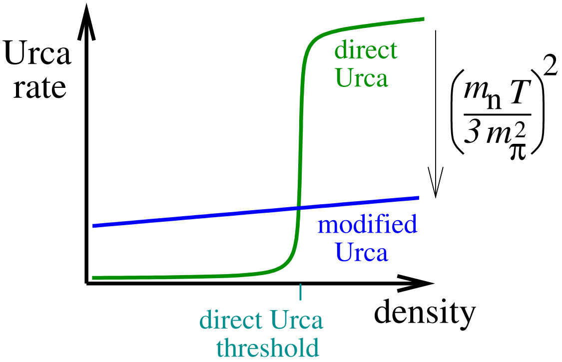




direct Urca only occurs above
direct Urca threshold density

Urca in the cold regime

So in the cold regime, $T \ll 1\,\text{MeV}$, the picture is

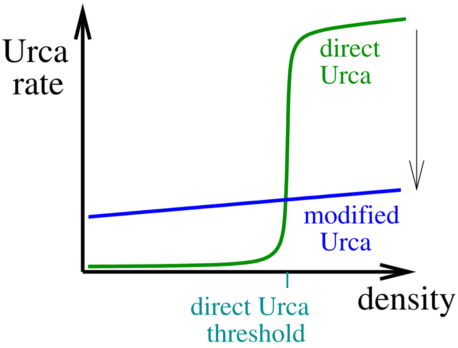


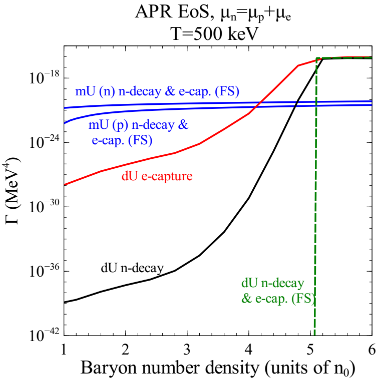
Is this picture still valid at merger temperatures: $T = 1$ to $100\,\text{MeV}$?

NO. Thermal blurring of the proton Fermi surface opens up direct Urca at $T \gtrsim 1\,\text{MeV}$.

Rethinking β-equilibrium, I

In the "warm" regime, $1\,\text{MeV} \lesssim T \lesssim 5\,\text{MeV}$, the direct Urca threshold is thermally blurred, and this affects electron capture more than neutron decay





When $\mu_n = \mu_p + \mu_e$, the forward (n decay) and backward (e^- capture) rates are not equal!

Rethinking β equilibrium, II

Typical equilibration scenario:

$$A + B \leftrightarrow C + D$$
$$\mu_A + \mu_B = \mu_C + \mu_D$$

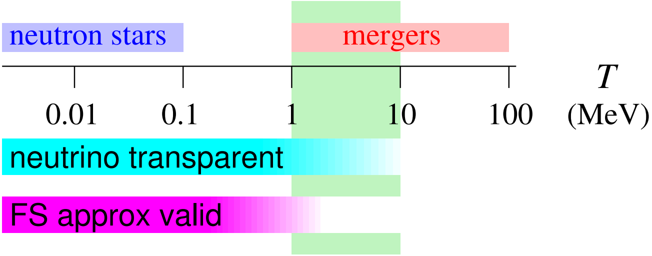
"Detailed balance": energy cost is the same for the forward and backward reactions.

Urca equilibration in neutrino-transparent regime:

$$n \rightarrow p + e^- + \bar{\nu}_e$$
$$\nu_e + n \leftarrow p + e^-$$

Forward and backward reactions are *not the same*.

Detailed balance does not apply.



Correct criterion for β equilibrium

The real criterion for β equilibrium in neutrino-transparent matter is

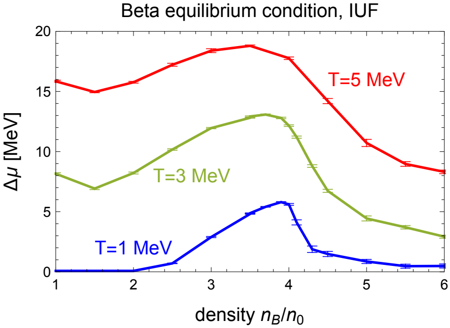
$$\Gamma(n \to p\ e^-\ \bar{\nu}_e) = \Gamma(p\ e^- \to n\ \nu_e)$$

If the forward and backward reactions are not the same, this will occur at a non-zero value of

$$\mu_\delta = \mu_n - \mu_p - \mu_e$$

- At $T \ll 1\,\text{MeV}$ the Fermi Surface approx is valid, neutrino energy can be ignored, so the reaction is approximately $n \leftrightarrow e^- + p$ so β equilibrium is when μ_δ is negligible, i.e. $\mu_n = \mu_p + \mu_e$.
- At $1\,\text{MeV} \lesssim T \lesssim 5\,\text{MeV}$, $\mu_n = \mu_p + \mu_e + \mu_\delta$
- At $T \gtrsim 5\,\text{MeV}$, neutrinos are trapped, the reaction is $\nu_e + n \leftrightarrow p + e^-$, and detailed balance holds again, beta equilibrium is when $\mu_n + \mu_\nu = \mu_p + \mu_e$.

Beta equilibrium in warm matter

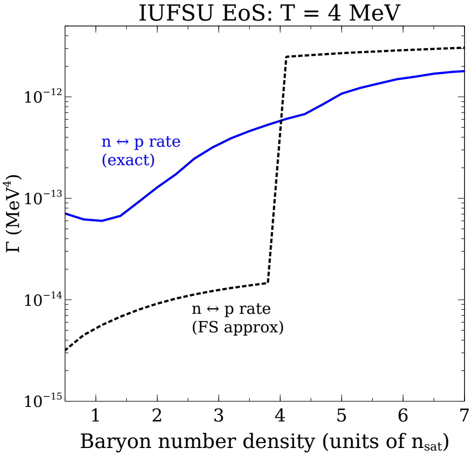


As T rises above 1 MeV, FS approx breaks down and the value of μ_δ needed to achieve β equilibrium gets larger.

What does the breakdown of FS approx mean for β equilibration rates?

Beyond the Fermi Surface approx

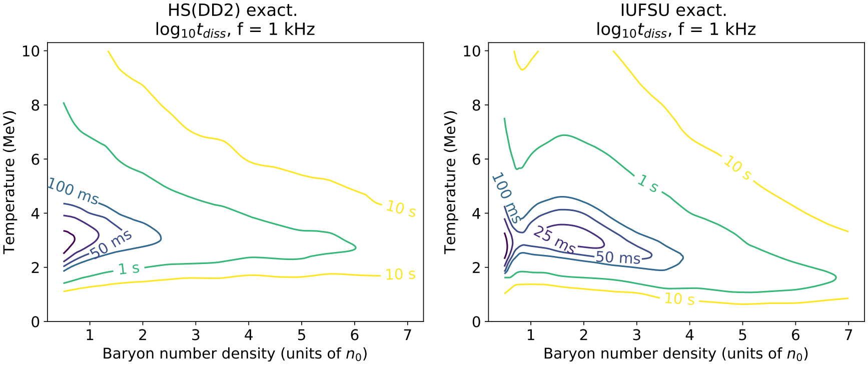
It is possible to do the full 5D phase space integral numerically.



At $T \gtrsim 1\,\mathrm{MeV}$ the proton Fermi surface is sufficiently thermally blurred to smooth out the switch-on of direct Urca.

This is why the direct Urca threshold is not clearly visible in the contour plots of the dissipation time.

Damping time



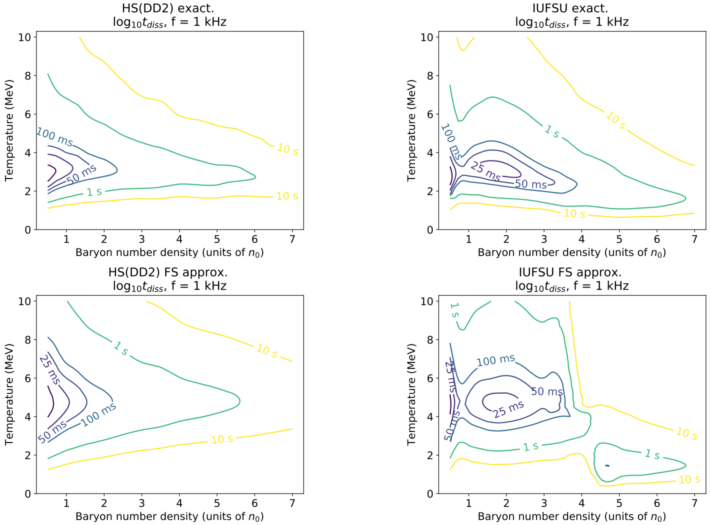
The damping time for density oscillations is shortest around $T \sim 3\,\text{MeV}$, independent of the EoS.

It is short enough to be relevant for neutron star mergers, especially at low density.

Testing the Fermi Surface Approx

Exact:

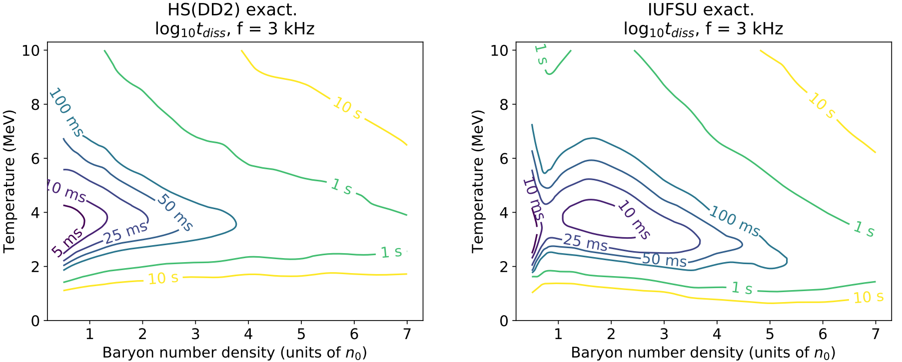
FS approx:



FS approx exaggerates the sharpness of the onset of direct Urca (IUFSU, at $n = 4n_{\mathrm{sat}}$)

Higher frequency oscillations

If 3 kHz oscillations occur then they would be damped even faster.



Note that max damping occurs at a slightly higher temperature, to get the beta equilibration rate to match the higher oscillation frequency.

Why is resonance with $1\,\mathrm{kHz}$ at $T \sim$ MeV?

Let's estimate $\gamma(T)$ and see when it is $2\pi \times 1\,\mathrm{kHz}$.

$$
\begin{aligned}
\frac{dn_a}{dt} &= -\gamma\,(n_a - n_{a,\mathrm{equil}}) \\
\Gamma_{n\to p} - \Gamma_{p\to n} &\sim -\gamma\,\frac{\partial n_a}{\partial \mu_a}\,\mu_a
\end{aligned}
$$

In FS approx, at β-equilibrium,

$$\Gamma_{n\to p} = \Gamma_{p\to n} \sim G_F^2 \times (p_{Fn}^2 T) \times (p_{Fp} T) \times T^3$$

If we push it away from β equilibrium by adding μ_a, the leading correction is to replace one power of T with μ_a

$$\Gamma_{n\to p} - \Gamma_{p\to n} \sim G_F^2 (p_{Fn}^2 T) \times (p_{Fp} T) \times T^2 \mu_a$$

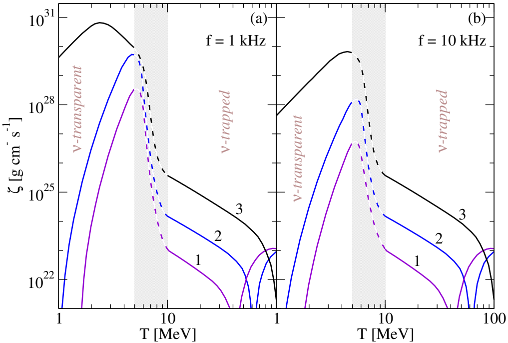
So

$$\gamma \sim \frac{\partial \mu_a}{\partial n_a} G_F^2\, p_{Fn}^2\, p_{Fp}\, T^4 \sim \frac{1}{(30\,\mathrm{MeV})^2} \frac{(350\,\mathrm{MeV})^2 (150\,\mathrm{MeV})}{(290\,\mathrm{GeV})^4}\, T^4$$

Solve for when $\gamma = 2\pi \times 1\,\mathrm{kHz} = 4 \times 10^{-18}\,\mathrm{MeV}$:

$$T \sim 1\,\mathrm{MeV}$$

The "hot" (neutrino-trapped) regime



Beta equilibration now includes neutrinos in the initial state too:

$\nu_e + n \leftrightarrow p + e^-$

Bulk viscosity is lower in hot matter ($T \gtrsim 5\,\text{MeV}$).

- β equilibration is too fast, above resonant temperature, because there so much phase space at the Fermi surfaces
- The relevant susceptibilities are smaller, so the peak bulk visc is smaller

Summary

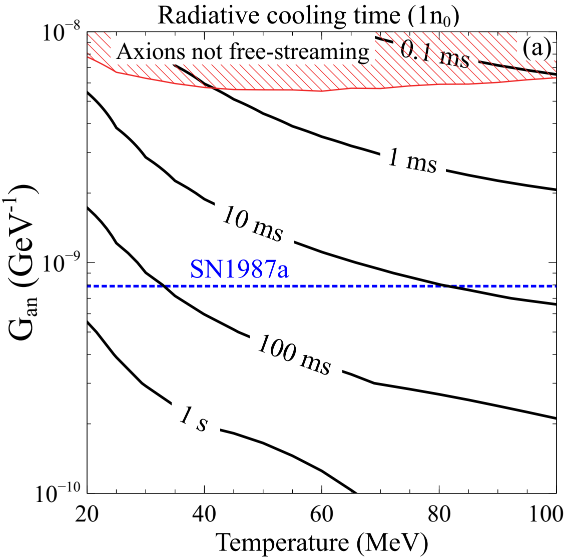
- Neutron star mergers probe the dynamical response of high-density matter, including dissipation properties.
- Thermal conductivity and shear viscosity may become significant in the neutrino-trapped regime ($T \gtrsim 5\,\mathrm{MeV}$) if there are fine-scale gradients ($z \lesssim 100\,\mathrm{m}$).
- In neutrino-transparent nuclear matter (at low density and $T \sim 3\,\mathrm{MeV}$) bulk viscosity will be significant in damping density oscillations.
- Under these conditions, the Fermi Surface approximation and detailed balance are not valid.
  Rate calculations must include the whole phase space.

Next steps

- Include beta equilibration in merger simulations.
- Do better calculations of beta equilibration rates in warm ($T \sim$ MeV) nuclear matter
- Calculate bulk viscous damping for other forms of matter: hyperonic, pion condensed, nuclear pasta, quark matter, etc
- Other manifestations? (Heating, neutrino emission,...)
- Beyond Standard Model physics?

Cooling by axion emission

Time for a hot region to cool to half its original temperature:



Harris, Fortin, Sinha, Alford
`arXiv:2003.09768`