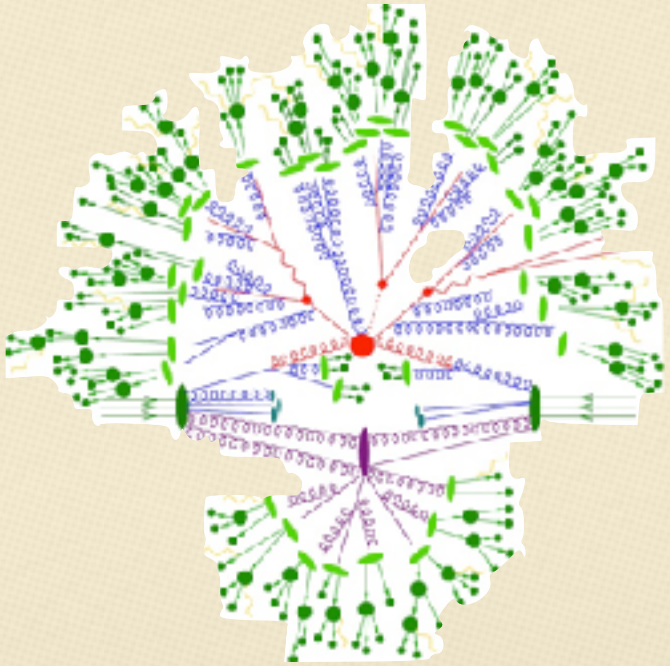
$$\frac{1}{4}\mathrm{Tr}[G^2] - \bar{\psi}(\not{D} - m)\psi$$ ☞



LHC and perturbative QCD

2022 Erice School of Subnuclear Physics
Gravity and matter in the subnuclear world

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Why these lectures?

- In particle physics, QCD is everywhere. The LHC collides protons, which are made up of quarks and gluons (partons). So every collision there involves partons in the initial state, and produces hadrons in the final state
    - To see anything new, beyond the Standard Model, we must be able to "remove the QCD foreground"
- But QCD is very interesting by itself. How can such complicated final states in colliders arise out of beautifully simple Lagrangian

$$\frac{1}{4}\mathrm{Tr}[G^2] - \bar{\psi}(\not{D} - m)\psi$$

- QCD is the <u>only unbroken, non-abelian gauge theory we have</u>, and we better study it as best as we can.
- I focus on ideas and methods, less on the latest results

Theme of these lectures: precision!

- We are in an era where new discoveries in particle physics must be found “behind the decimal point”
- Our mission: stress-test the Standard Model, especially QCD
  - With perturbation theory

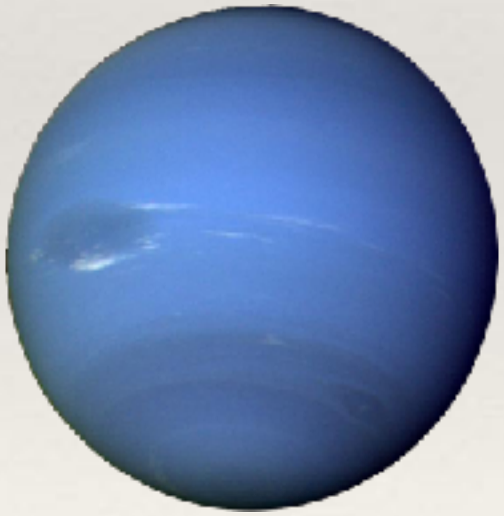
$$O = c_0 + g^2\, c_1 + g^4\, c_2 + \ldots$$

- Successful in the past:

Discovering new worlds through precision

- Careful measurements of Uranus' [Herschel, 1781] orbit showed deviations from the Standard Model of planetary orbits: Kepler's laws

- Adams and Le Verrier [1843]: discrepancies could be explained by presence of new planet. They also prediction its position



- Neptune discovery in 1846 [Galle]

Precision, accuracy, error and uncertainty

- A bit of terminology: for predictions for observable O

$$O^{[m]} = \sum_{n}^{m} c_n \alpha^n + \delta O^{[m]}$$

  - Precision: compute to order "m", large enough for $\delta O^{[m]}$ to be small enough
  - But beware: it can a be small variation on an incorrect result. It is then precise, but not accurate
  - Errors: a measure of accuracy
    - ✓ experimental: statistical and systematical
  - Uncertainty: indicates range in which true value could lie
- Confront prediction with measurement, all the more meaningful with small $\delta O^{[m]}$, and update hypotheses
- This is what we *should* be doing: a highly sophisticated instance of The Scientific Method

QCD Genesis

- The strong force at the beginning of the 1960's was not well understood.
- Lots of mesons found and baryons as well
  - π, ρ, K, η, K*, ω, φ, …, p, n, Λ, Σ, Ξ ("cascade"), Ω, Δ, ..
- Organized by Gell-Mann and Zweig with SU(3) of flavour ("eightfold way"), using "quarks"
  - ✓ p(uud), Λ (uds), Δ^{++} (uuu), Δ^{+}(uud), Δ^{0}(udd), Δ^{-}(ddd), etc
- To maintain of Pauli principle, and to explain absence of degenerate states:

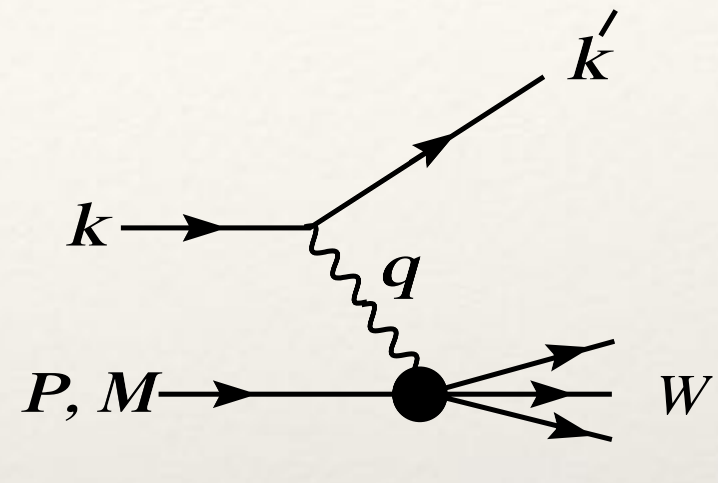
I. Quarks have 3 colours. II Bound states are singlets of this symmetry

- This led to a Lagrangian like QED, but now for quarks and gluons

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) - \sum_{f=1}^{n_f} \overline{\psi_f}(\not{D} + m_f)\psi_f$$

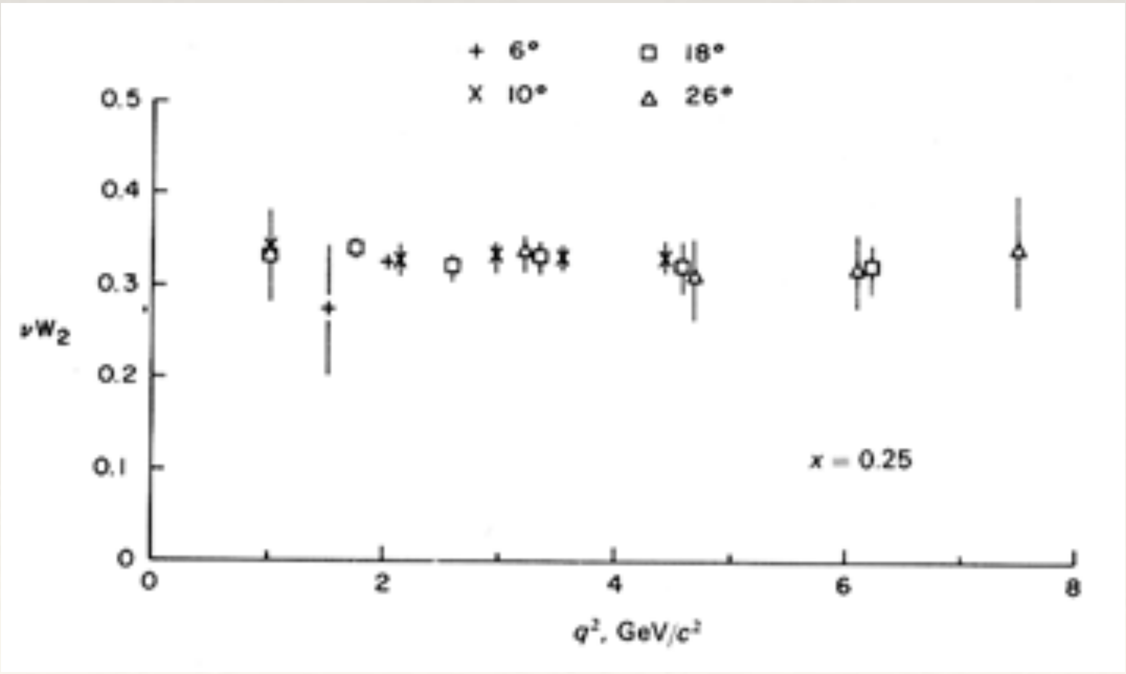
Seeing quarks

- In the late sixties, early seventies, deep-inelastic scattering experiments (SLAC-MIT) were done.
- Relation of cross section to "inelastic form factors" of proton F_1, F_2, F_3:



$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y}\right)^{\gamma} = \frac{8\pi\alpha^2 ME}{(Q^2)^2}\left\{\frac{1+(1-y)^2}{2}2xF_1^{\gamma}(x,Q^2) + (1-y)[F_2^{\gamma}(x,Q^2) - 2xF_1^{\gamma}(x,Q^2)] - \frac{M}{2E}xyF_2^{\gamma}(x,Q^2)\right\}$$

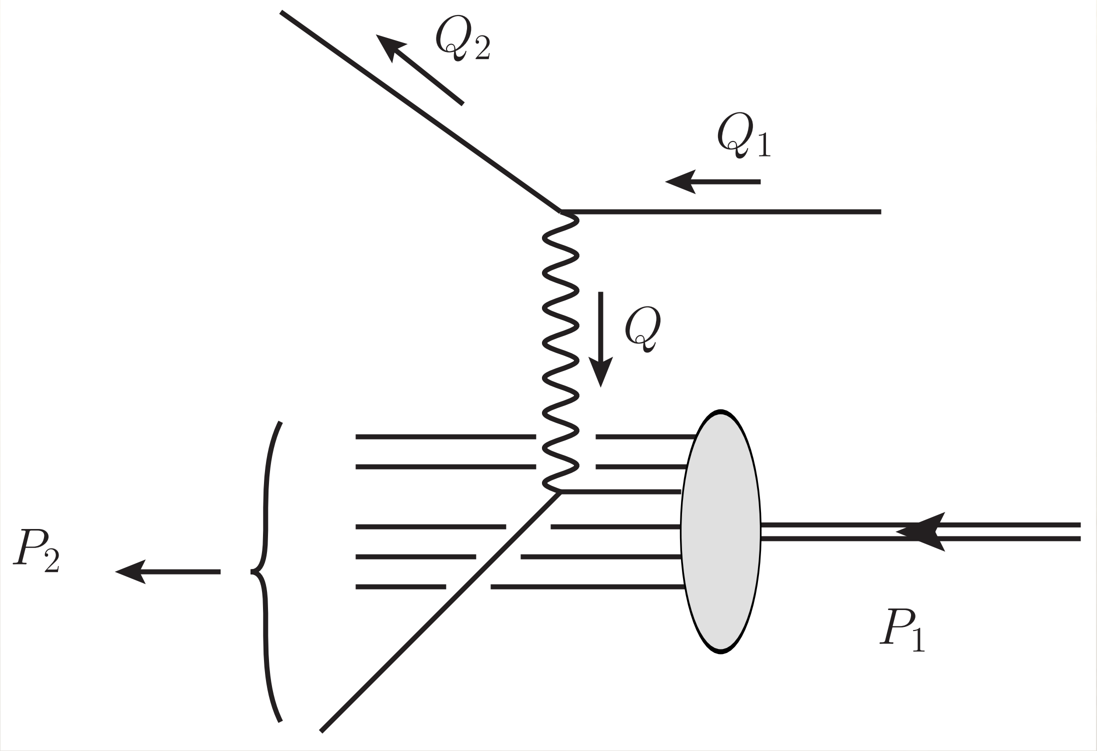
- Outcome: F_2 can depend on x and Q^2, but seemed to only depend on x
  - "Scaling"



Parton model

Feynman, Paschos

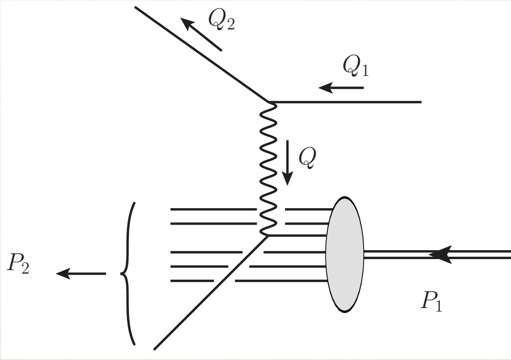
- Solution: the Parton model, wonderfully elegant idea, still at the basis of our predictions for the LHC. An electron hits a proton at high energy
- From the electron point of view, two relativistic effects occur
  - The proton is length contracted, looks like a disk
  - The internal proton dynamics is slowed down, due to time dilation
  - Assume interactions beween constituent “partons” are absent (rather wild assumption at the time)



Parton model

Feynman, Paschos

- How does the parton model explain scaling?
  - First, the collision takes place between the electron and a **constituent** (parton) of the proton
  - The parton has a fraction of the proton energy and momentum $p^\mu = \xi P_1^\mu$
    - ✓ Assume it is a spin-1/2 fermion
  - Some kinematics related to electon-parton



$$Q = Q_1 - Q_2 \qquad 0 = p_2^2 = (\xi P_1 + Q)^2 = 2\xi P_1 \cdot Q + Q^2 \qquad \xi = \frac{Q^2}{-2P_1 \cdot Q} \equiv x$$

  - Bjorken-x has therefore the meaning of parton momentum fraction. Electon-parton scattering can now be computed, and gives

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}\right)^\gamma = \frac{8\pi\alpha^2 ME}{(Q^2)^2} q_i^2 \frac{(1-y)^2+1}{2} \delta(x - \xi_i)$$

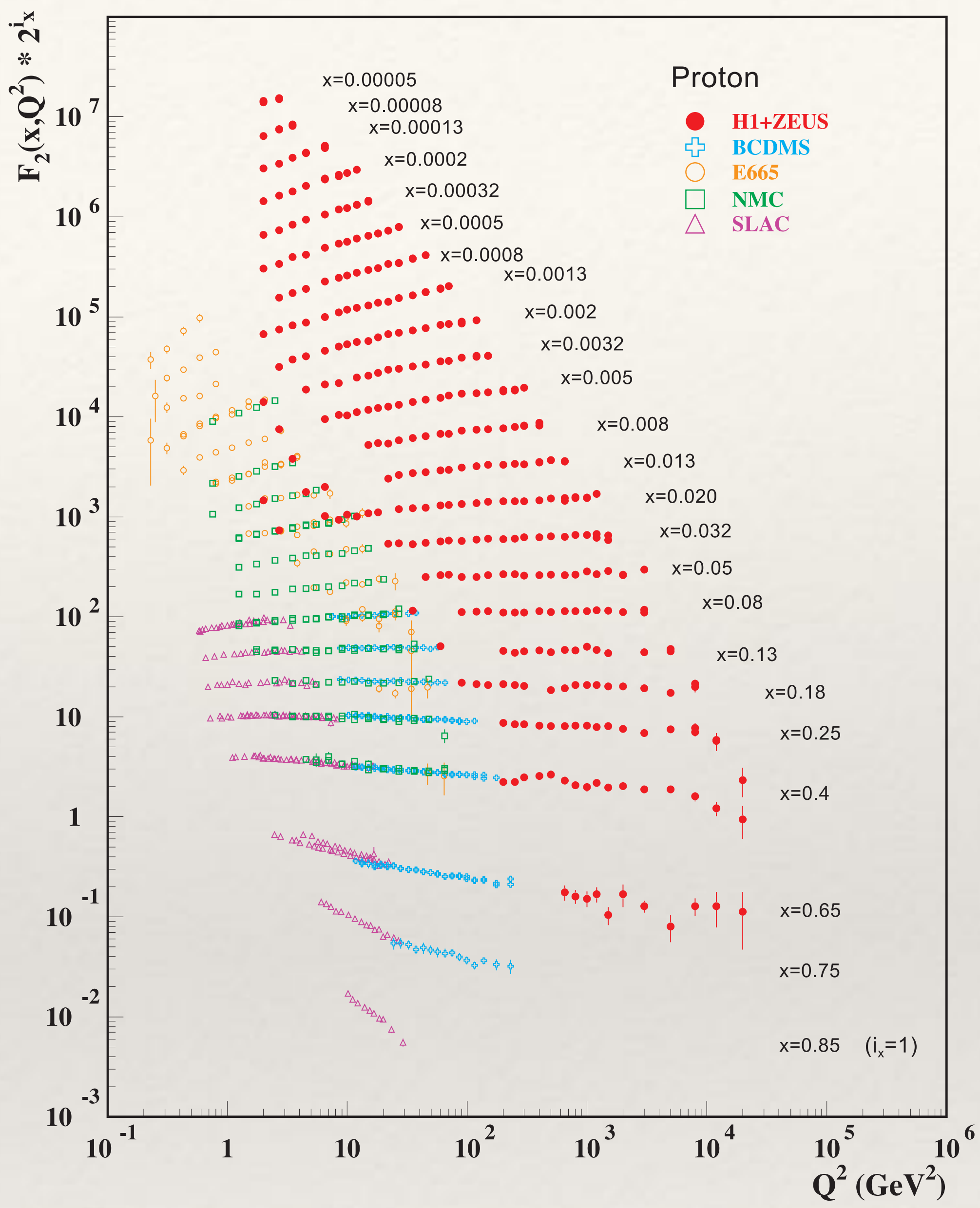
- Introduce now the parton distribution function (PDF) $\phi_{i/p}(\xi)$, and integrate over all allowed momentum fractions ξ

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}\right)^\gamma = \frac{8\pi\alpha^2 ME}{(Q^2)^2} \frac{(1-y)^2+1}{2} \sum_i q_i^2\, x\phi_{i/p}(x)$$

  - Scaling!

Scaling violation

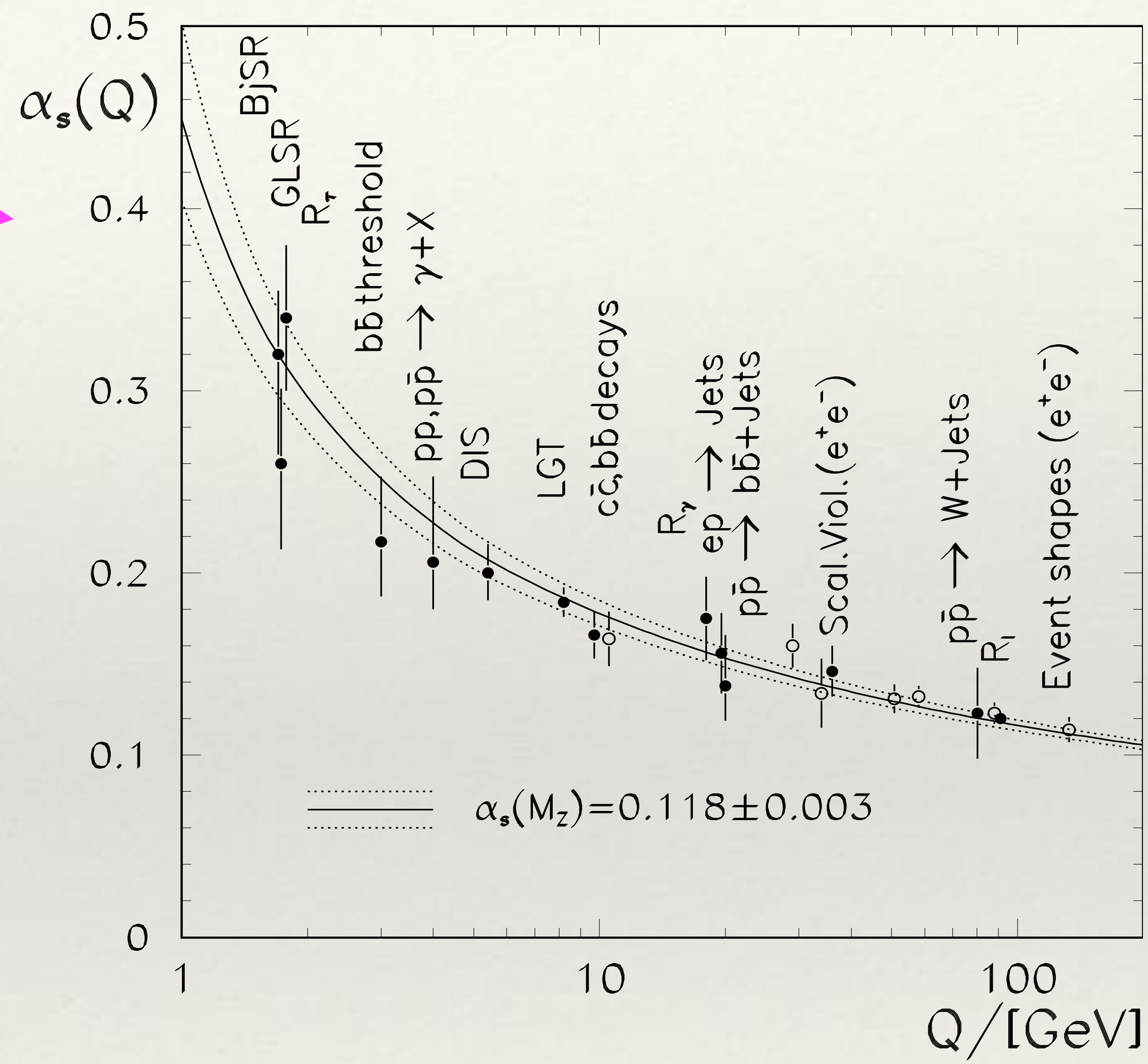
- But in better measurements: no scaling anymore
    - But "violation" very mild.
- => Parton model is the right way to think about collisions with hadrons
- "Easy" generalisation to hadron-hadron collision
    - Predictive power! Use PDF's measured in DIS for hadronic collisions.
- But there is fundamental paradox:



How can quarks be both strongly bound into hadrons, and act as free "partons" in high-energy scattering?

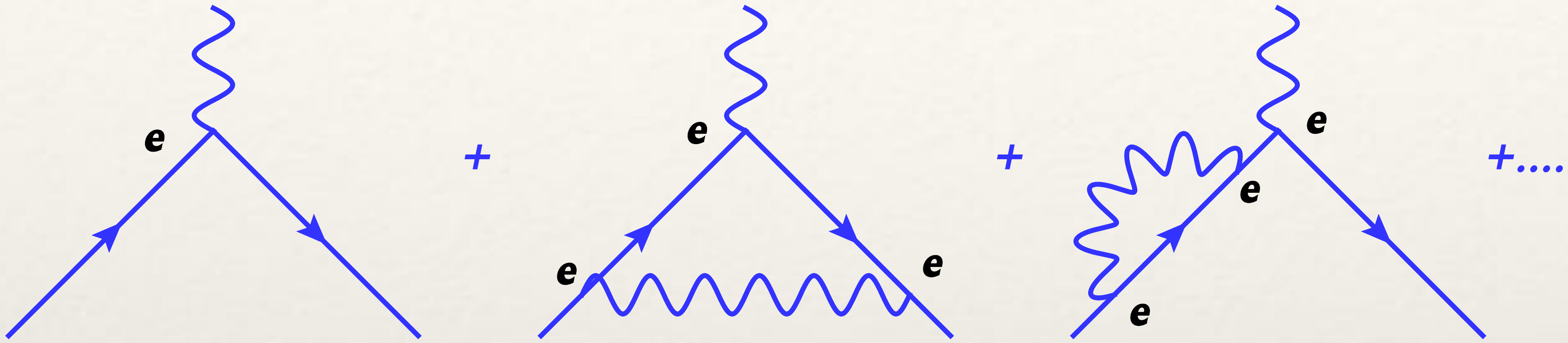
Towards a solution of the paradox

- To solve this paradox, the coupling would have to behave like this
    - At low Q coupling is strong
    - For increasing Q, the coupling decreases
- But: how does a coupling become Q dependent in the first place. In the Lagrangian it is just a number: "g"?
- To understand we need to consider the effect of renormalization
- Only non-abelian gauge theory behaves this way!
    - Nobelprize 2004: Gross, Wilczek, Politzer



Loops and regularization

- Quantum effects lead to a scale-dependent coupling, through renormalization.
- Computing any Green function at higher orders in a coupling leads to loops.



- Some loop integrals are divergent, and need to be regularized before being able to “handle” them

$$\int d^4l \, \frac{1}{[l^2 - m^2][(l+p)^2 - m^2]} \sim \int \frac{d^4l}{(l^2)^2} \sim \mathrm{i} \int \frac{d\Omega \, l^3 dl}{l^4} \sim 2\pi^2 \mathrm{i} \int^\infty \frac{dl}{l}$$

- One can put a cut-off on the l integral, but everyone uses dimensional regularization: 4⟶ 4-2ε

$$\int^\infty \frac{dl}{l} \to \int^\infty \frac{dl}{l^{1+2\varepsilon}} = \frac{-1}{2\varepsilon}$$

- Very elegant. So loop integral results are divergent. How to get rid of this? Renormalize!

Renormalization

- We focus on the key point. Rewrite the coupling as

$$e = Z_e\left(\frac{1}{\varepsilon}, e_R(\mu)\right)\, e_R(\mu)$$

$$Z_e = 1 + e_R^2(\mu)\left(z^{1,1}\frac{1}{\varepsilon} + z^{1,0}\right) + \mathcal{O}(e_R^4)$$

- So beside the loop integrals, there is now a <u>second source of 1/ε</u>: the renormalization of the coupling e in the tree-level graph, through Z
  - Choose now the number $z^{1,1}$ such that the 1/ε from the loop is cancelled.
  - Is this not ridiculous? I could cancel any 1/ε divergence in that way…!
  - BUT: the magic of renormalizable theories is that fixing $z^{1,1}$ in this way, will fix this type of 1/ε divergence in any other one-loop diagram in this theory.
  - One can renormalize a finite number of quantities: couplings, fields and masses. Fix the Z-factors in a few calculations, which then will cancel the 1/ε everywhere else

- Observe that on the right hand side a scale μ appears, in both Z-factor and renormalized coupling e_R. The product does not depend on it. This is the renormalization scale. Sketchwise:

$$\left(1 + e_R^2 \ln(\frac{\Lambda}{\mu}) + \mathcal{O}(e_R^4)\right) \times \left(1 + e_R^2 \ln(\frac{\mu}{Q}) + \mathcal{O}(e_R^4)\right) = 1 + e_R^2 \ln(\frac{\Lambda}{Q}) + \mathcal{O}(e_R^4)$$

QCD Beta-function

- In analogy to e_R, now for $\alpha_s = g^2/4\pi$

$$\alpha_s = Z_\alpha\left(\frac{1}{\varepsilon}, \alpha_{s,R}(\mu)\right)\alpha_{s,R}(\mu)$$

$$Z_\alpha = 1 + \frac{\alpha_{s,R}(\mu)}{4\pi}\left(\frac{11C_A - 2n_f}{3}\frac{1}{\varepsilon} + c_\alpha\right) + \mathcal{O}(\alpha_{s,R}^2)$$

- We derive from this

$$\mu\frac{d}{d\mu}\ln\alpha_{s,R}(\mu) = -\mu\frac{d}{d\mu}\ln Z_\alpha\left(\frac{1}{\varepsilon}, \alpha_{s,R}(\mu)\right) = -\frac{\beta(\alpha_{s,R}(\mu)}{\alpha_{s,R}(\mu)}$$

- The QCD beta function is known to 5th order by now. Keep only the first term

$$\mu\frac{d}{d\mu}\alpha_s(\mu) = -\frac{\alpha_s^2(\mu)}{2\pi}\left(\frac{11C_A - 2n_f}{3}\right) = -\frac{\beta_0}{2\pi}\alpha_s^2$$

  - An increase in μ leads to **decrease** in α (due to minus sign)
  - Solution

$$\alpha_s(\mu) = \frac{4\pi/\beta_0}{\ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)}$$

-

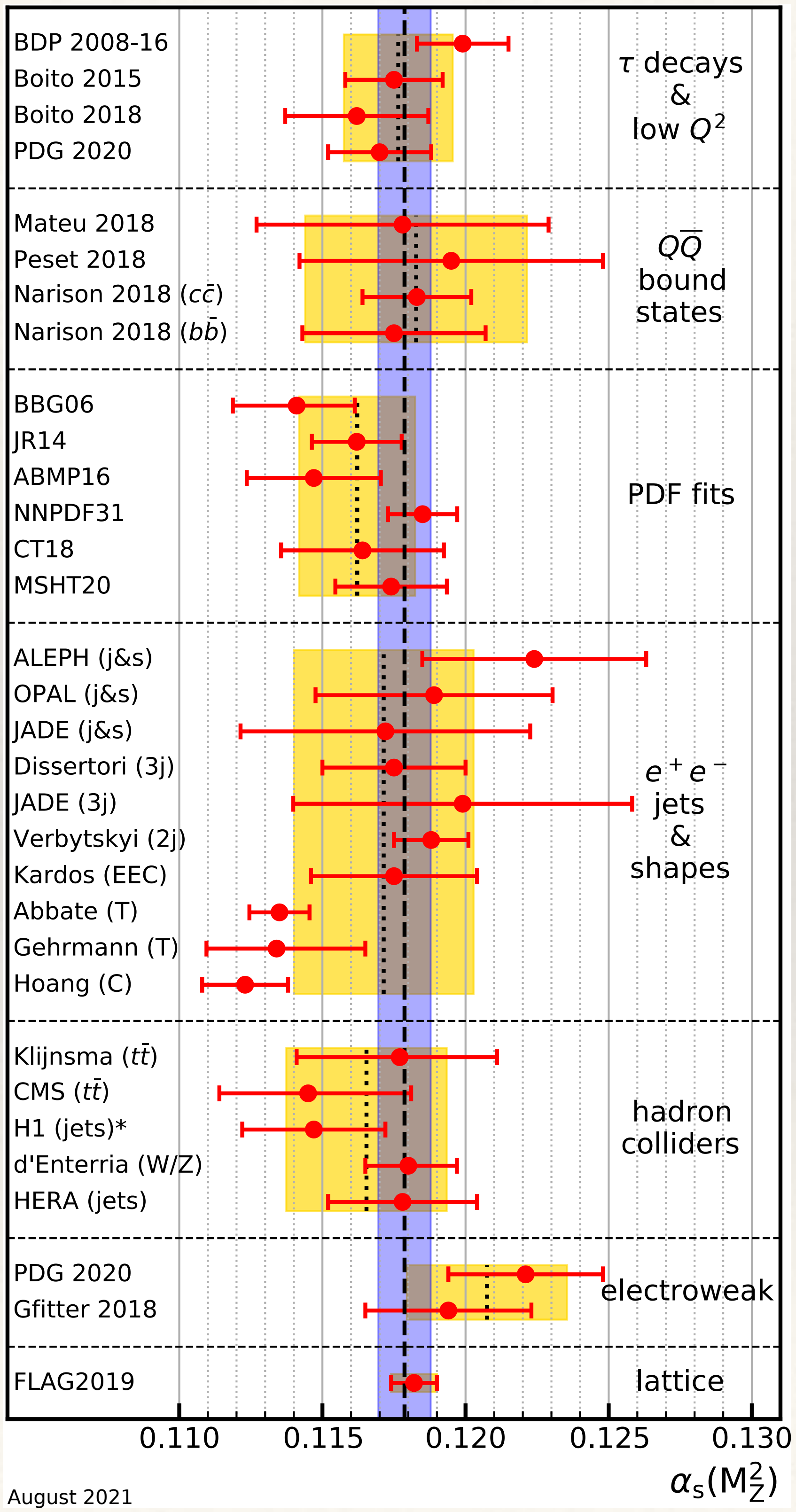
This solves the paradox!

Modern determinations of α_s

- α_s can be determined from comparing a perturbative expression for an observable with its measurement
- Particle Data Group collects these, performs subfield averages, and then world average

  WA: $\alpha_s(M_Z)$= 0.1179 +- 0.0009

- Criteria
  - Result must be published
  - At least NNLO calculation
  - Reliable uncertainty estimates



QCD parameter measurement

- α_S is a parameter in the QCD Lagrangian, just like quark masses. Should we know them precisely?
- For α_S almost always necessary for precision of observables
  - E.g. for e+e—> hadrons the lowest order is EM, first order term is proportional to α_S, so making $\delta\alpha_S$ very small does not matter too much
  - For pp -> ttjj the lowest order is of order α_S^4, so the uncertainty easily reaching 5-10% from α_S alone.
- For quark masses the same holds true
  - For high-energy jet production the bottom mass occurs mostly in logarithms, so mild dependence on δm_b
    - ✓ For the universe's sake, we should know the top quark mass very, very precisely..

QCD gauge/local symmetry

- Before diving into perturbative QCD, a few formal aspects of this non-abelian gauge theory.
- We would like to build a theory that is invariant under local SU(3) transformations. SU(3) is a non-abelian group (elements don't commute).
- We take a fermion field that has 3 components, that transforms as

$$\psi(x)' = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \end{pmatrix}' = U(x)\psi(x) = \begin{pmatrix} & & \\ & U(x) & \\ & & \end{pmatrix} \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \end{pmatrix}$$

  - The three components are also called R,B,G sometimes.
  - This is a "covariant transformation". We also have $\overline{\psi}' = \overline{\psi} U^\dagger(x) = \overline{\psi}' U^{-1}(x)$

- Problem: derivative of fermion field does not transform covariantly.

$$\partial_\mu \psi(x)' = U(x)\partial_\mu \psi(x) + (\partial_\mu U(x))\psi$$

- Solution: introduce better, covariant derivative D_μ, that does transform nicely, so that…

QCD gauge/local symmetry

- ..we have

$$\overline{\psi}'\gamma^\mu D'_\mu \psi' = \overline{\psi}\gamma^\mu \underbrace{U^{-1}(x)U(x)}_{=1} D_\mu \psi$$

- To construct the covariant derivative, introduce the gauge field (any many as there are SU(3) generators)

$$D_\mu \psi(x) = \left(\begin{pmatrix} \partial_\mu & & \\ & \partial_\mu & \\ & & \partial_\mu \end{pmatrix} - g A^a_\mu \underbrace{(T_a)}_{3\times 3 \text{ matrices}} \right) \psi(x)$$

$$\equiv (\partial_\mu - g A_\mu)\psi(x)$$

  - Notice there are now quark-gluon interactions!

$$\overline{\psi}\gamma^\mu D_\mu \psi = \overline{\psi}\gamma^\mu \partial_\mu \psi - g\overline{\psi}_i \gamma^\mu [\mathbf{T_a}]_{ij} \psi_j \, A^a_\mu$$

- The field strength is also derived from covariant derivative

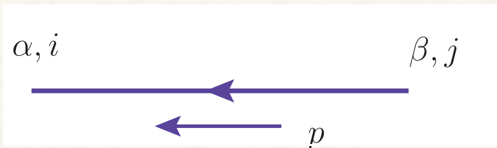
$$[D_\mu, D_\mu]\,\psi = -gG_{\mu\nu}\psi = -gG^a_{\mu\nu}\mathbf{T_a}\psi$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{bc}{}^a A^b_\mu A^c_\nu$$

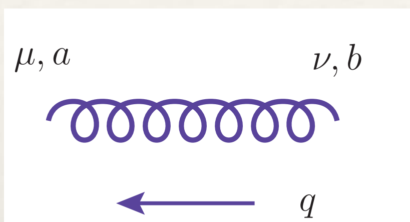
- Full invariant Lagrangian

$$\mathcal{L}_{QCD} = \frac{1}{4}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \sum_f \bar{\psi}_f \not{D} \psi_f - m_f \bar{\psi}_f \psi_f$$

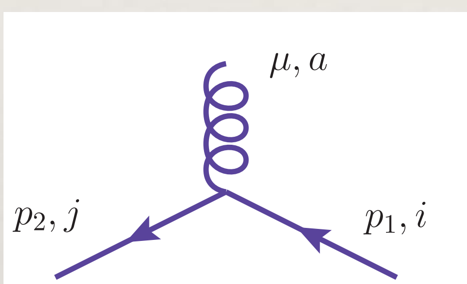
QCD Feynman Rules



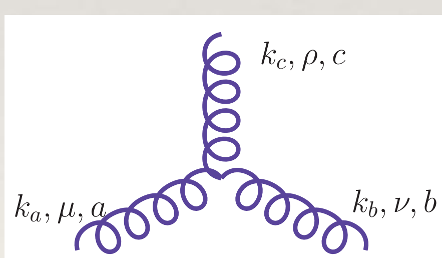
$$\frac{1}{\mathrm{i}(2\pi)^4}\frac{\delta_{ij}(-\mathrm{i}\not{p}+m)_{\alpha\beta}}{p^2+m^2}$$



$$\frac{1}{\mathrm{i}(2\pi)^4}\frac{\delta_{ab}}{q^2}\left(\eta_{\mu\nu}-(1-\lambda^{-2})\frac{q_\mu q_\nu}{q^2}\right)$$



$$\mathrm{i}(2\pi)^4\delta^4(p_1-p_2+q)(-g)[\mathbf{T_a}]_{ij}(\gamma_\mu)_{\alpha\beta}$$



$$\mathrm{i}(2\pi)^4(-ig)f^{abc}\left[\eta_{\mu\nu}(k^a-k^b)_\rho+\eta_{\nu\rho}(k^b-k^c)_\mu+\eta_{\rho\mu}(k^c-k^a)_\nu\right]$$

Feynman rules are direct link to the full quantum field theory

QCD and UV divergences

- When computing loop integrals, and UV divergences result from them, not all of them can be cancelled by renormalization of just the QCD coupling

$$\alpha_s = Z_\alpha\left(\frac{1}{\varepsilon}, \alpha_{s,R}(\mu)\right)\alpha_{s,R}(\mu)$$

$$Z_\alpha = 1 + \frac{\alpha_{s,R}(\mu)}{4\pi}\left(\frac{11C_A - 2n_f}{3}\frac{1}{\varepsilon} + c_\alpha\right) + \mathcal{O}(\alpha_{s,R}^2)$$

- In fact, in general, all the fields, couplings and parameters get their Z-factors.

$$W_\mu{}^a \to \sqrt{Z_W}\, W_\mu{}^a\,, \quad \psi \to \sqrt{Z_\psi}\,\psi\,, \quad c^a \to \sqrt{Z_c}\, c^a\,, \quad b^a \to \sqrt{Z_b}\, b^a\,,$$
$$g \to Z_g\, g\,, \quad m \to Z_m\, m\,, \quad \lambda \to Z_\lambda\, \lambda$$

- As a consequence many more sources of 1/ε, enough to cancel all?

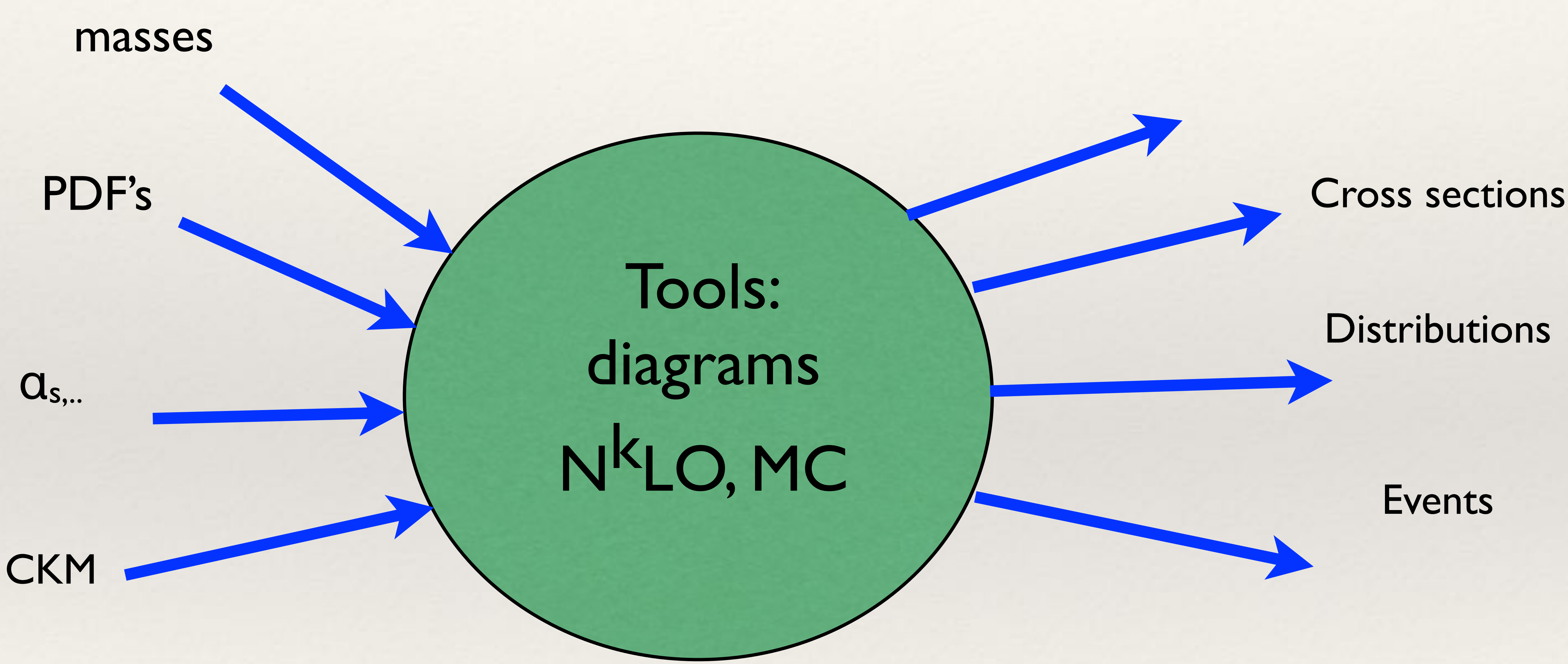
Renormalizability of QCD

- In fact, with these Z-factors, *every* UV divergence in *any* one-loop QCD amplitude is cancelled.
- But if it goes wrong at higher orders, all is for naught..
- This was a key worry in the early 70's. Renormalizability of QED was known, and of numerous scalar, Yukawa and other field theories. Non-abelian gauge seemed too hard.
- This was the problem that Gerard 't Hooft tackled as a PhD student, together with his advisor Martinus Veltman [after a summer school!]
- The solution was presented by 't Hooft at a EPS meeting in Amsterdam in 1971, leaving most participants stunned. He and Veltman proved that no new Z-factors are needed to any order. One just needs to determine them to higher order -> Nobel prize 1999
- They used lots of diagrammatic clever techniques. Afterwards a more efficient proof used "BRST symmetry"
    - Only then was QCD, and in fact the Standard Model, taken more seriously, now that it was a legitimate theory.
- Perturbative QCD however struggles mostly with infrared divergences…

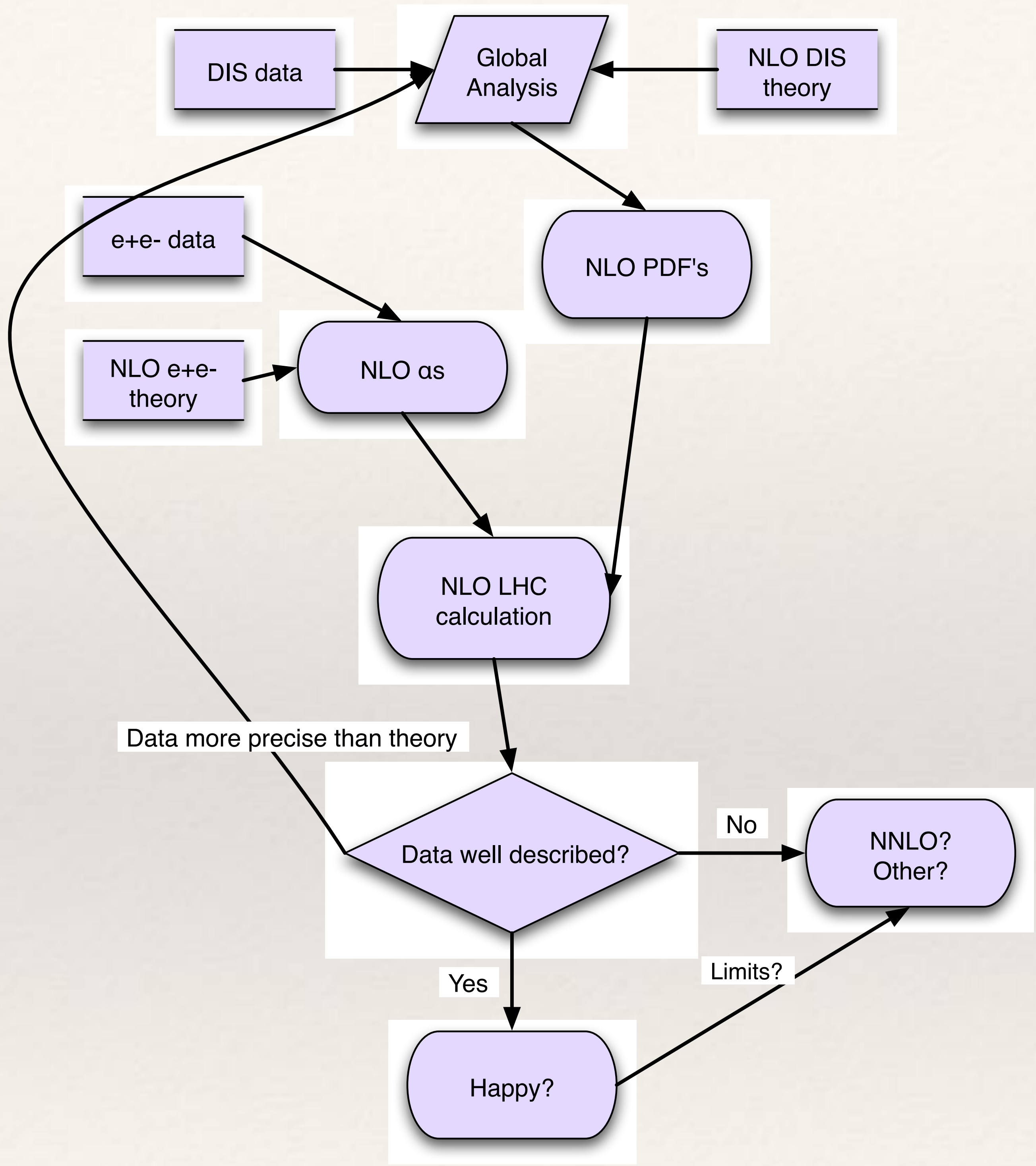
QCD for LHC in practice, simplified

Input quantities

Output

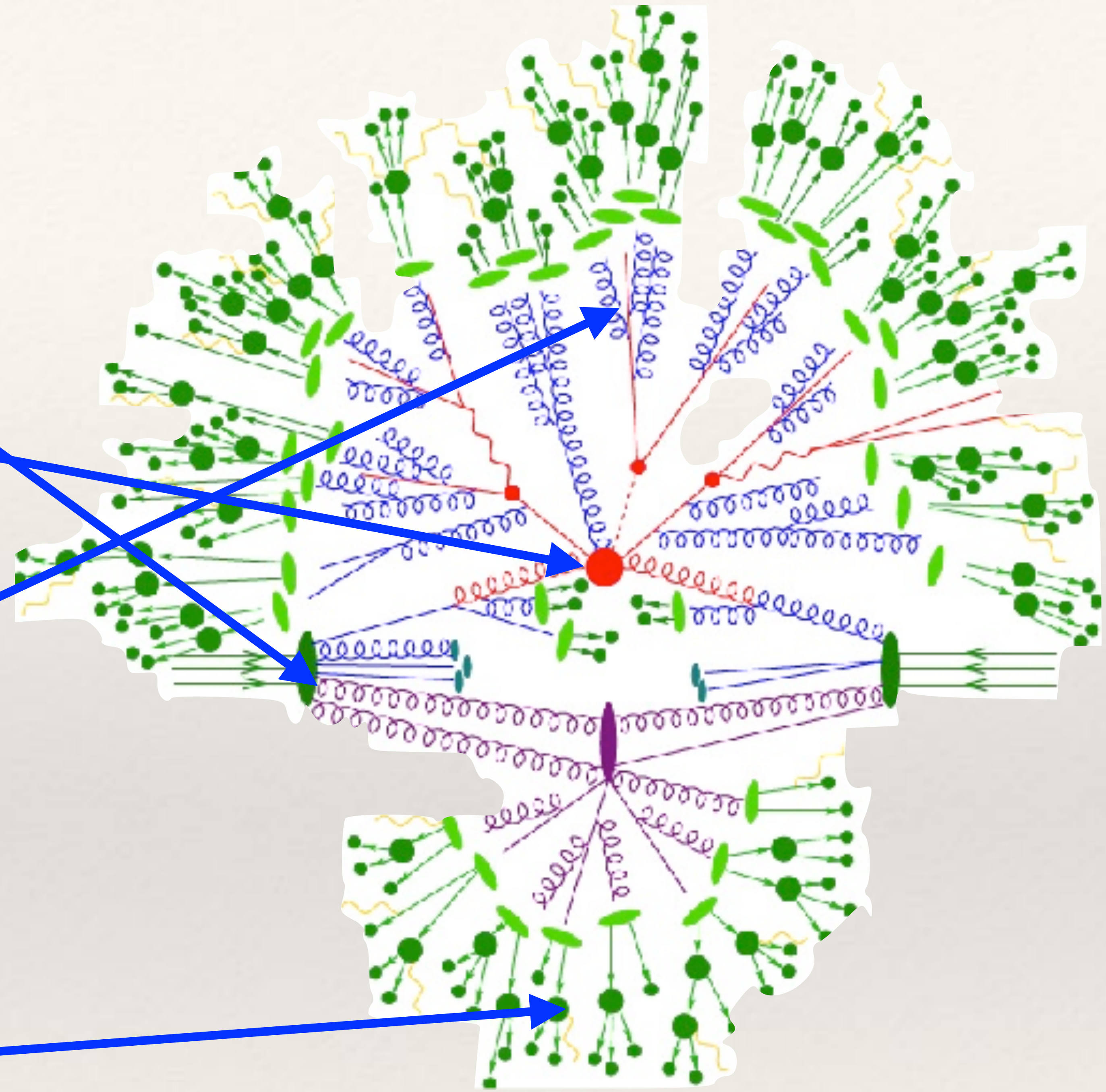


How to use QCD in practice, less simplified



QCD cross section in a picture

- PDFs
- Partonic processes
- Parton showers
- Hadronization



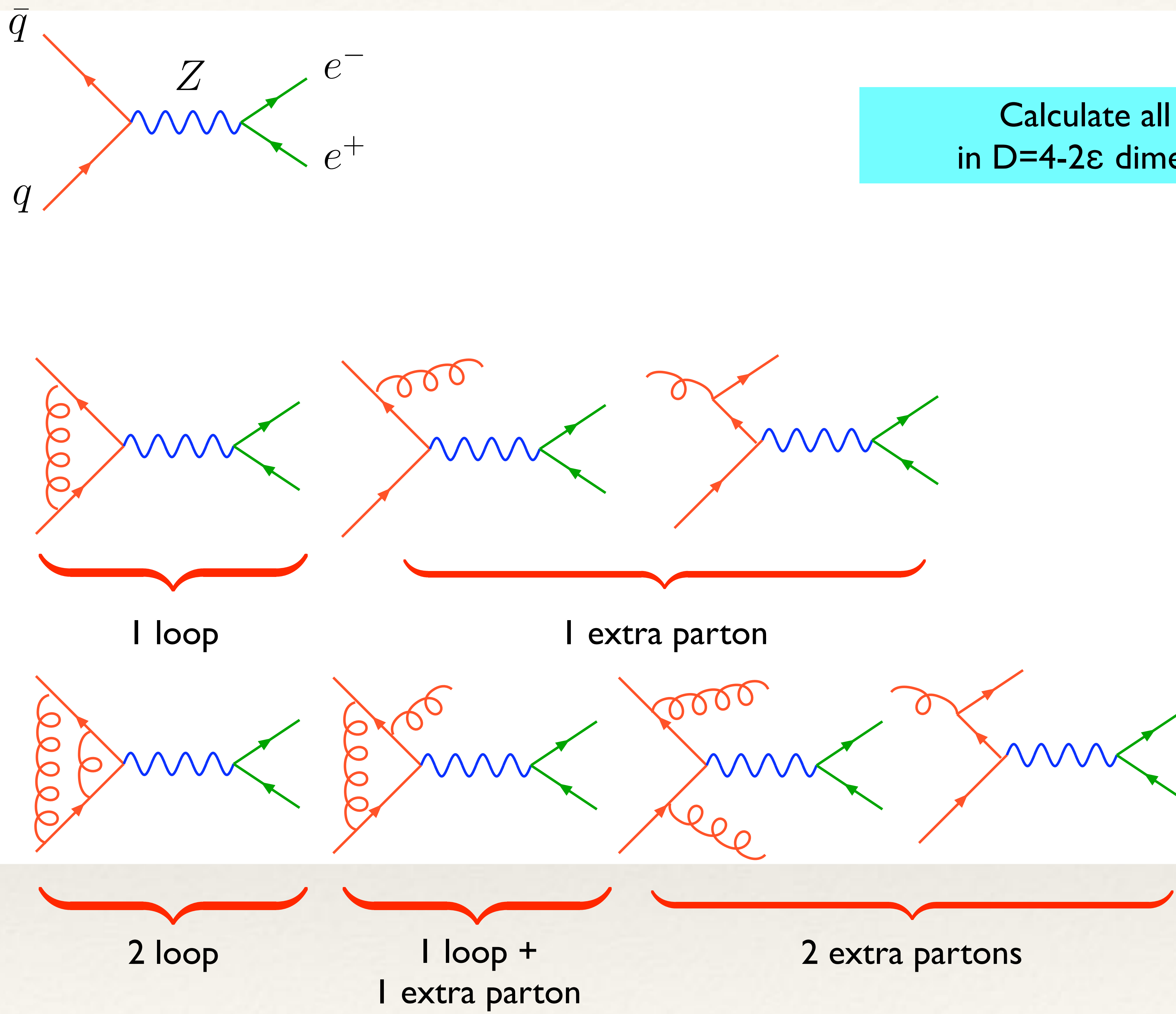
LO and higher orders

Calculate all this
in D=4-2ε dimensions

LO $\hat{\sigma}^{(0)}$

NLO $\hat{\sigma}^{(1)}$

NNLO $\hat{\sigma}^{(2)}$



Professional formula for QCD cross sections

Multi-differential hadronic N^{k}LO cross section

N^{k}LO PDF's

$$\frac{d\sigma^{pp\to X}}{d^3p_1\dots d^3p_n} = \sum_{a,b}\int dx_1 dx_2 \phi_{a/p}(x_1,\mu_F)\phi_{b/p}(x_2,\mu_F)$$

$$\times\hat{\sigma}_{ab}(p_a+p_b\to p_X,\alpha_s(\mu_R),\mu_R,\mu_F)+\mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

Multi-differential partonlevel N^{k}LO cross section

Power corrections. Hard!

Renormalization and Factorization scale

This formula is chain of ingredients, each must be determined very precisely!

Parton distribution functions: why and how

- Crucial at hadron colliders, must be known very accurately. But they cannot be computed from first principles.
- Answer: use their universality:
  - We need to determine 11 PDF (5 quarks + antiquarks + gluon), *and their uncertainties*
  - Choose with care a set of measurements/observables [e.g. DIS structure functions, or hadron collider cross sections] Each is described as a PDF ⊗ partonic cross sections. We then have the set of equations

$$(O_n \pm \Delta O_n)^{\mathrm{exp}} = \sum_{j=1}^{n_f} \phi_{j/p} \otimes [\hat{\sigma}_{n,j} \pm \delta\sigma_{n,j}]^{\mathrm{th}}$$

  - From the comparison one fits the $\varphi_{j/P}(x,\mu)$.
    - ✓ Various groups, employing different approaches
      - MSTW, CTEQ, NNPDF, GJR, HERAPDF, ABKM…
  - If the partonic calculation is LO, NLO, NNLO etc, then the PDF thus fitted are also labelled LO, NLO etc.
    - ✓ NLO PDF's must be used with NLO calculations, etc

PDF's as operator matrix elements

- Although they cannot yet be fully computed from first principles, one can give a precise definition of PDF's, in terms of operators. Essentially, these are counting operators (cf a†a in QM)

$$\phi_{q/p}(\xi) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dy^- e^{-i\xi p^+ y^-} \langle p|\bar{q}(0,y^-,0_T)\gamma^+ q(0,0,0_T)|p\rangle$$

proton state

Quark field

$$p^{\pm} = \frac{p^0 \pm p^3}{\sqrt{2}}$$

$$p \cdot q = -p^+ q^- - p^- q^+ + p_1 q_1 + p_2 q_2$$

  - in a certain gauge. The non-perturbative part sits in the hadronic state in which this counting operator is inserted.
  - Benefit: once you have an operator, one can compute its renormalization, and derive an RG equation for it (just like for the coupling constant). This is in fact the DGLAP equation
    - ✓ There are other ways of deriving it.
  - To do so, just replace the proton states with quark states (and keep the operator). At lowest order this is just

$$\delta(1-\xi)$$

  - At next order it has the form

$$\frac{\alpha_s}{2\pi} C_F \frac{1}{\varepsilon}\left(\frac{1+\xi^2}{1-\xi}\right)_+ + \ldots$$

quark-to-quark splitting function!

    - Plus distribution:

$$\int_0^1 dz \left[\frac{a(z)}{1-z}\right]_+ g(z) = \int_0^1 (g(z)-g(1)) \left[\frac{a(z)}{1-z}\right]$$

Parton distribution functions

- The logic is thus very similar to running coupling, we now have “running functions”:

$$\mu \frac{d}{d\mu} \phi_{i/H}(x,\mu) = \int_x^1 \frac{dz}{z} P_{ij}(z, \alpha_s(\mu))\, \phi_{j/H}\left(\frac{x}{z}, \mu\right) \qquad \left[\equiv P_{ij} \otimes \phi_{j/H}\right](x,\mu)$$

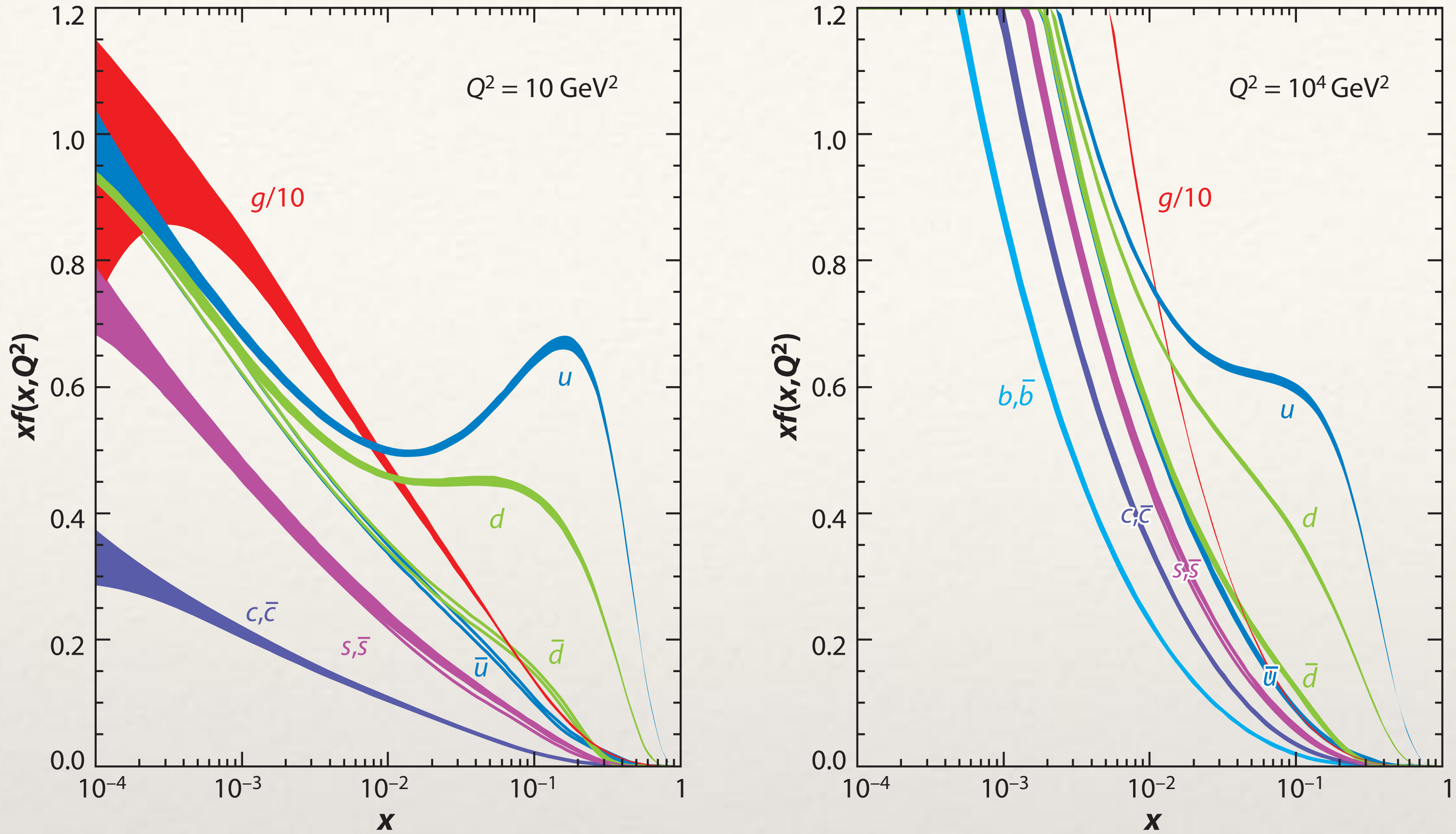
  - DGLAP equations
  - P_{ij} are the splitting functions. They are now known to NNLO (3rd order) [Moch, Vermaseren, Vogt]
  - Use determine the PDF’s at some scale Q, then compute them at all other scales by solving the DGLAP equations.

- Note: to determine the PDF’s precisely from the equation

$$(O_n \pm \Delta O_n)^{\mathrm{exp}} = \sum_{j=1}^{n_f} \phi_{j/p} \otimes \left[\hat{\sigma}_{n,j} \pm \delta\sigma_{n,j}\right]^{\mathrm{th}}$$

- one must choose the data on the lhs well.

Form of PDF’s



MSTW08 at two values of Q^2

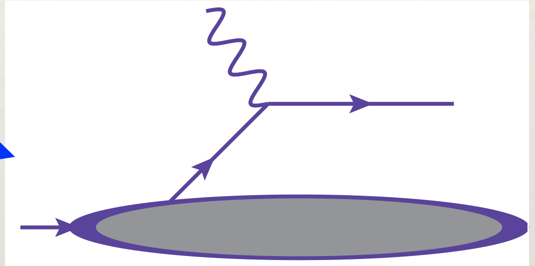
- Notice how evolving the sets to high scale narrows the uncertainty.
  - and how all PDF’s grow towards small x: driven by the gluon density in the evolution
- Only u and d still show some bumps: a memory of them being partly valence quarks

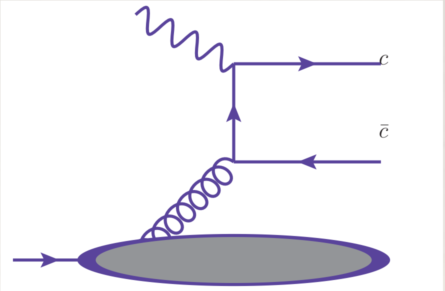
PDF input data

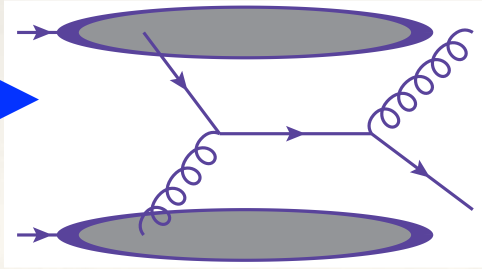
See e.g. Forte, Watt '13

- What data to choose as inputs to fit to?
  - Those that single out particular parton distributions
    - ✓ DIS structure functions most sensitive to valence (u-ū etc) quarks. Prompt photon production sensitive to gluon density etc.
  - Those that provide extra information in certain x ranges (e.g. jet production gives large-x gluon information)

Process	**Subprocess**	**Partons**	***x* range**
$\ell^{\pm}\{p,n\} \to \ell^{\pm} X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^{\pm} n/p \to \ell^{\pm} X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u},\, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})\, N \to \mu^-(\mu^+)\, X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu\, N \to \mu^-\mu^+\, X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu}\, N \to \mu^+\mu^-\, X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^{\pm}\, p \to e^{\pm}\, X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^{+}\, p \to \bar{\nu}\, X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^{\pm} p \to e^{\pm} c\bar{c}\, X$	$\gamma^* c \to c,\, \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^{\pm} p \to \mathrm{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \mathrm{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^{\pm} \to \ell^{\pm}\nu)\, X$	$ud \to W,\, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-)\, X$	$uu,\, dd \to Z$	d	$x \gtrsim 0.05$







Theory of PDF set formation

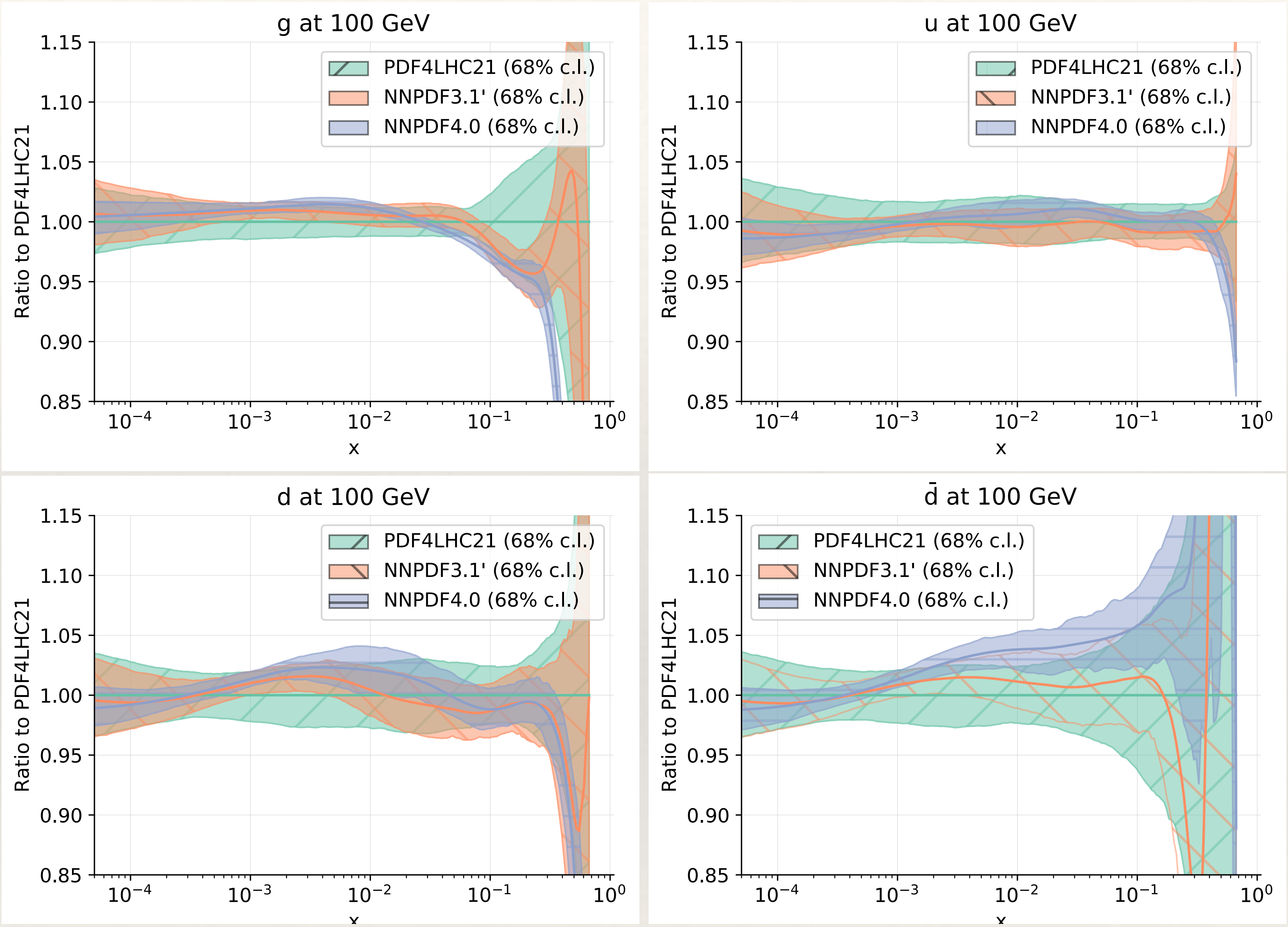
- Some theoretical constraints: sum rules
  - Charge sum rule: $\int_0^1 dx(\phi_{i/p}(x,Q^2) - \phi_{\bar{i}/p}(x,Q^2)) = \{2,1,0,0,0\}, \quad i = \{u,d,s,c,b\}$
  - Momentum sum rule: $\sum_{i\in\{g,u,d,s,\dots\}} \int_0^1 dx\, x\, \phi_{i/p}(x,Q^2) = 1$
- In principle, must solve 7x7 matrix evolution equation. But one can cleverly arrange this to have five independent equations, and one 2x2 equation.
  - Subtle issue: how to think about charm and bottom PDF's? In principle they can be computed from the gluon and light flavor PDF's. Also here different approaches, but won't go into details.
- Fitting: not easy. Use χ2 as goodness-of-fit [D_i = data, T_i = theory, V=exp. covariance matrix]
-
$$\chi^2 = \sum_{i=1}^{N_{\rm data}} \sum_{j=1}^{N_{\rm data}} (D_i - T_i)(V^{-1})_{ij}(D_j - T_j)$$
- We need a probability measure on the space of functions (in principle ∞-dimensional). To make things tractable, groups choose some parametrization for initial PDF. Many choose a physically motivated form with a limited set of parameters $\phi_i(x,Q_0^2) = x^{\alpha_i}(1-x)^{\beta_i} g_i(x)$
- Can also choose a (very redundant) set of unbiased functions, with hundreds of parameters. But then minimization difficult.

PDF uncertainties

- Two approaches to establish probability measure: 1) Hessian 2) Monte Carlo
  - Hessian: 1-σ confidence interval by moving parameters that make up χ^2 to χ^2_{min}+T. Note that "tolerance" T=1 is theoretically correct, but problematic in practice
    - ✓ Advantage: compact representation of uncertainties.
    - ✓ Product: S_0 central set, and then N_{par} 1-σ error S_i sets.
  - Monte Carlo: create a large number of replica sets
    - ✓ E.g. by constructing data replica's with the right average and covariance
    - ✓ Fit then PDF sets S^k to data replicas.
    - ✓ Now best fit is MC mean over sets S^k., also 1-σ straightforward
  - Both methods agree overall reasonably well. So far uncertainties based only on experimental ones.
- A comparison of some modern NNLO sets:

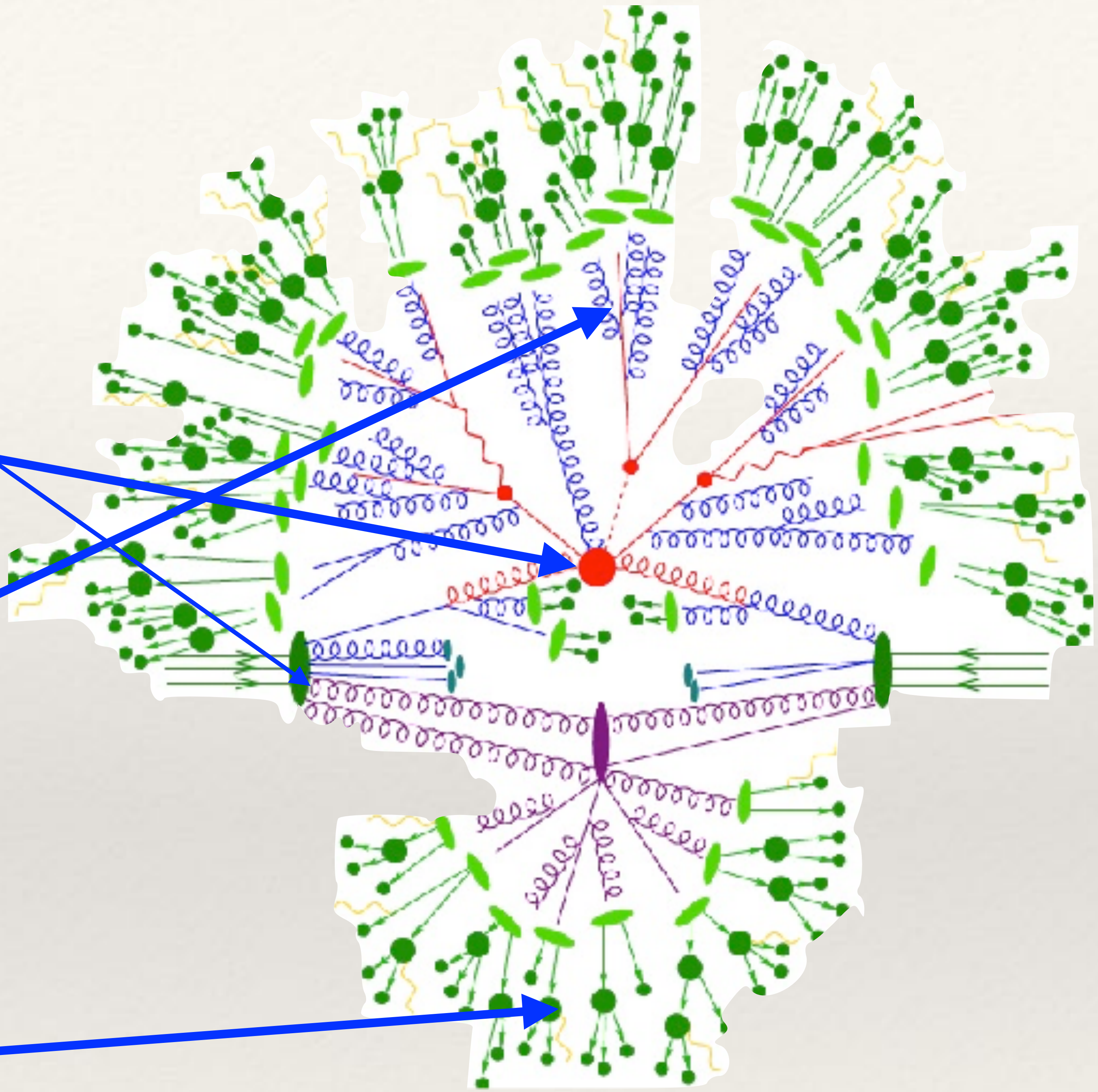
Recent comparison of NNLO sets

- PDF4LHC21
  - Recent combination of various sets
- NNPDF3.1'
  - NNPDG set used in PDF4LHC21
- NNPDF4.0
  - Latest NNPDF set, not included in combination.
  - More data included, methodology improvements, positivity constraint



Where are we?

- PDFs
- Partonic processes
- Parton showers
- Hadronization



Parton showers: elementary Monte Carlo

- Consider a process in which branchings take place (radioactive decays, or parton showers).
  - f(t): chance of branching for time t. Then probability for branching at time t

$$P(t) = -\frac{d\Delta(t)}{dt} = f(t)\Delta(t)$$

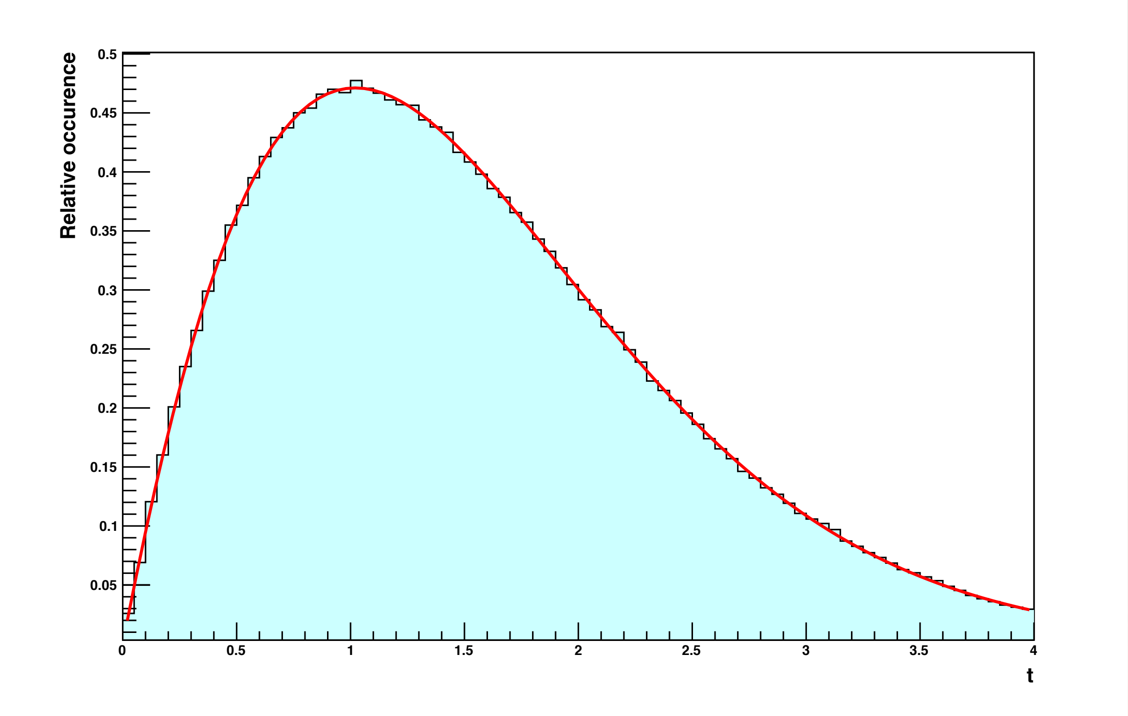
  - Δ(t): probability that no branching has occurred until t. ["Sudakov form factor"]

$$P(t) = f(t)\exp\left\{-\int_0^t dt' f(t')\right\}$$

  - Prototype for parton shower!
- How to choose t values based on random numbers such that the likelihood of selecting a t value is given by this expression?
- Veto-algorithm!

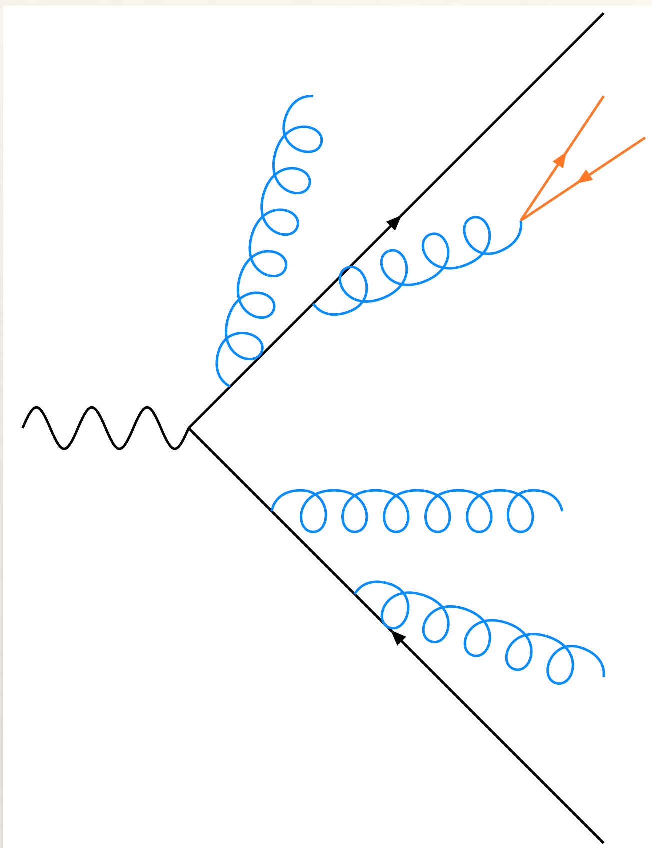
Standard Veto Algorithm

- Example
  - $f(t) = t$, $F^{-1}(x) = (2x)^{(1/2)}$, $g(t) = t+1$ larger than $f(t)$, G the primitive of g
  - Algorithm
    1. start with $i=0$, $t_0=0$
    2. i++, then select t_i according to $t = G^{-1}(G(t_{i-1}) - \ln R)$, $t_i > t_{i-1}$.
    3. compare a new R with $f(t_i)/g(t_i)$. If $f(t_i)/g(t_i) < R$, return to 2
    4. otherwise accept t_i.
  - Result: nice agreement between analytical and veto-algorithm result.

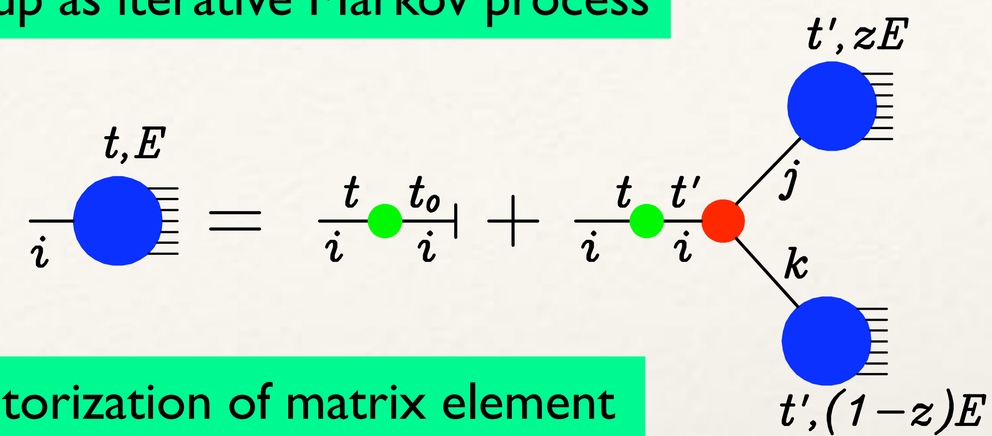


Parton branchings

An approximate description of full matrix elements when radiation is mostly collinear and soft



Set up as iterative Markov process



based on factorization of matrix element and phase

Sudakov form factor: probability of no emission between two emissions (related to virtual graphs)

$$\Delta_i(t_1, t_2) = \exp\left[-\sum_{(jk)} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_S(t)}{2\pi} \int dz\, P_{i,jk}(z) \int \frac{d\varphi}{2\pi} \right]$$

In contrast to NLO, NNLO calculations, with parton showers the number of partons per event is not fixed. But it is still a unitary process

$$\sum_n P_n(t) = 1$$

Accuracy of parton showers and matching

- Very flexible, simulation, all 4-vectors of all final state particles available.
    - But, at a price: only Leading Logarithmic (LL) accuracy
    - Recent progress in improving the intrinsic accuracy of the PS to next-to-leading logarithmic (NLL),

PanScales group

- Matching to fixed order calculations a bit tricky at NLO etc
    - NLO has one extra parton
    - The shower can also provide extra partons. Don't double count!
- But combined NLO + PS is best of both worlds. E.g. top quark pair production