Symmetry-improved Cornwall–Jackiw–Tomboulis Effective Action

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(but originally from around here...)

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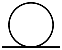
An exercise in thermal field theory

$\Pi =$ 

- $Z = \mathrm{Tr}(e^{-\beta H})$, $\beta = 1/T$
- $e^{-\beta H} \sim e^{-iHt}$ with $t = -i\beta$
  - euclidean QFT
  - discrete Matsubara frequencies $\omega_n = 2\pi n/\beta$
- effective IR cutoff $2\pi T$: $\Pi \simeq \Pi|_{T=0} + c\lambda T^2$ $(T \gg m)$

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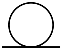
$\Pi =$ 

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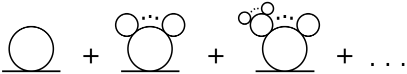

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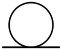
$\Pi =$ 

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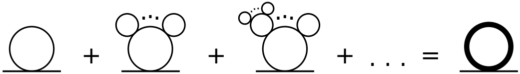
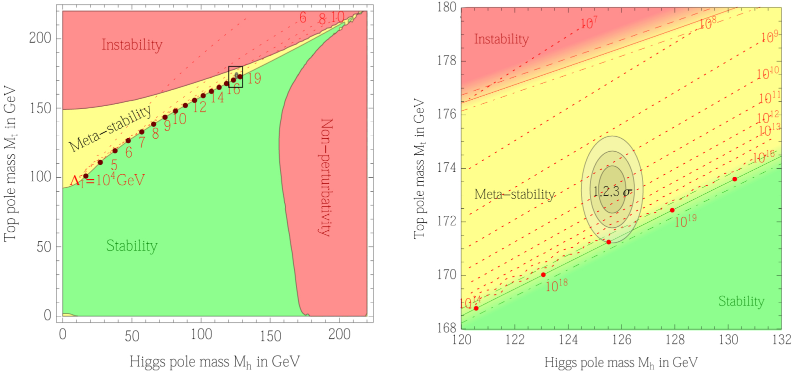


is there a **calculable** formulation of QFT beyond perturbation theory?

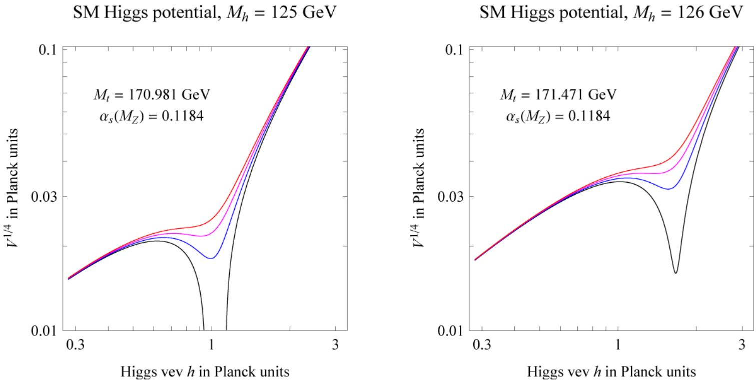
Metastability of the SM at NNLO



[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 2013]

Uncertainties of the SM potential at NNLO

Higgs Potential versus Variations in Top Mass by 0.1 MeV



[Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia, 2012]

2PI Cornwall–Jackiw–Tomboulis effective action

Generalize the standard 1PI Effective Action:

- local and bi-local sources $J(x)$, $K(x,y)$:
$$Z[J,K] \;=\; \int \mathcal{D}\phi \, e^{i(S[\phi] + J_x\phi_x + \frac{1}{2}K_{xy}\phi_x\phi_y)} \;=\; e^{iW[J,K]}$$
- $\frac{\delta W[J,K]}{\delta J_x} = \langle\widehat{\phi}_x\rangle \equiv \phi_x \qquad 2\frac{\delta W[J,K]}{\delta K_{xy}} = \langle\widehat{\phi}_x\widehat{\phi}_y\rangle \equiv \phi_x\phi_y + i\Delta_{xy}$
- double Legendre transform:
$$\Gamma[\phi,\Delta] = W[J,K] - J_x\phi_x - \tfrac{1}{2}K_{xy}(i\Delta_{xy} + \phi_x\phi_y)$$

Self-consistent Equations of Motion

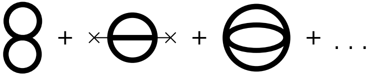
$$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi_x} = -J_x - K_{xy}\,\phi_y \qquad\qquad \frac{\delta\Gamma[\phi,\Delta]}{\delta\Delta_{xy}} = -\frac{i}{2}\,K_{xy}$$

physical solution: extremum of $\Gamma[\phi,\Delta]$

2PI effective action - explicit form

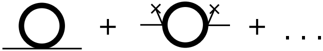
$$\Gamma[\phi,\Delta] = S[\phi] + \frac{i}{2}\,\mathrm{Tr}\ln\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,\Delta\Delta^{0^{-1}} - i\,\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta]$$

$$\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta] \quad =$$

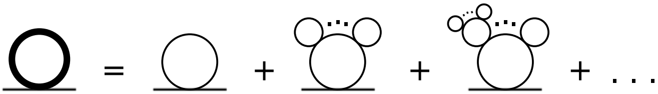


Equations of Motion

- $\dfrac{\delta\Gamma}{\delta\phi} = 0$
- $\dfrac{\delta\Gamma}{\delta\Delta} = 0 \;\rightarrow\;$ SDE: $\Delta^{-1} = \Delta^{0^{-1}} +$



2PI resummation



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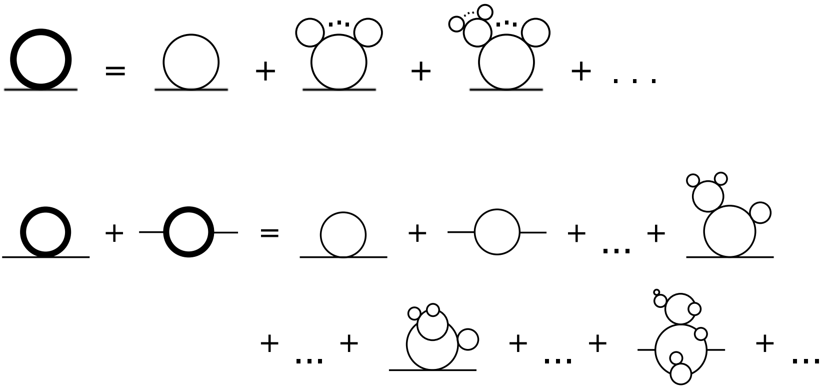
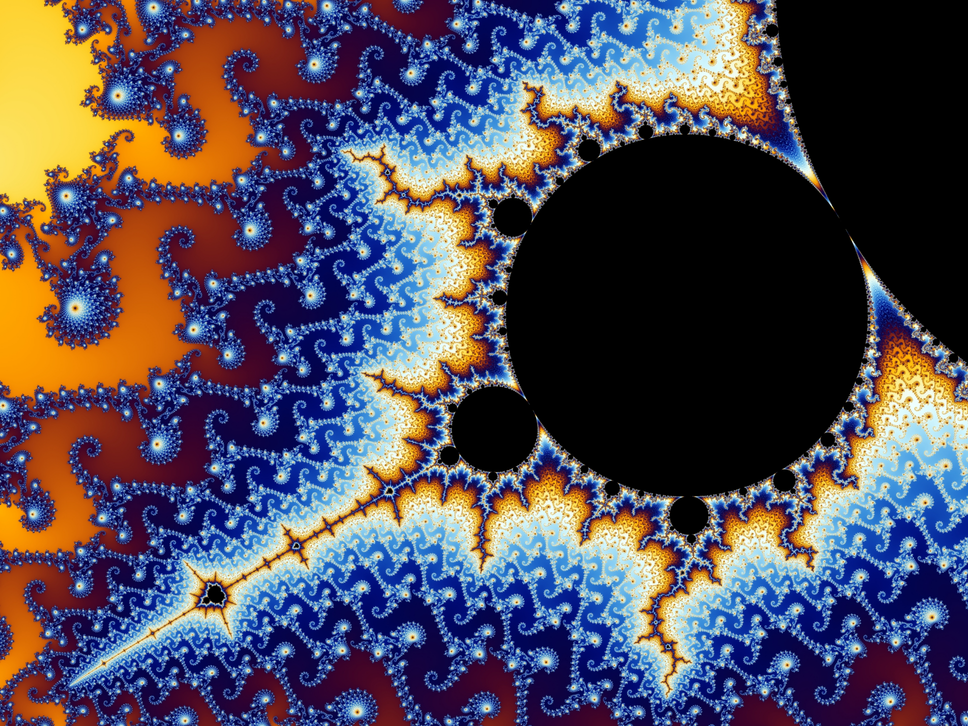




Symmetry-improved CJT

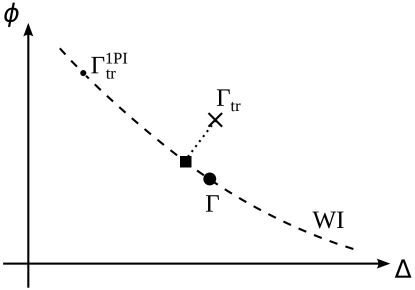
Truncations of CJT do not encode symmetries properly

[Baym, Grinstein, 1977; Amelino-Camelia, 1997]

Standard Ward Identities not satisfied $\Longrightarrow$ massive Goldstone bosons

Symmetry-improved CJT effective action

Pilaftsis and Teresi, Nucl. Phys. B 874 (2013) 2, 594. arXiv:1305.3221 [hep-ph]



Improved Effective Potential

$$-\frac{1}{\phi}\frac{d\widetilde{V}_{\rm eff}}{d\phi} \equiv \Delta_G^{-1}(\phi)\Big|_{k=0}$$

An example of something I won't talk about...

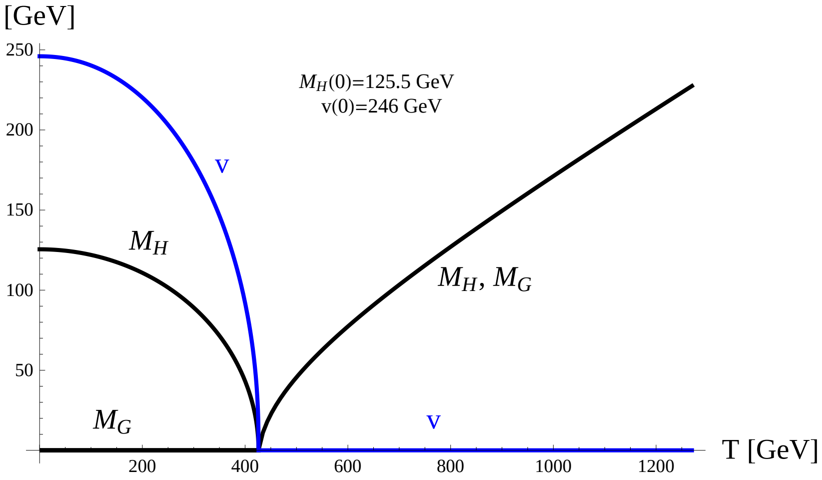
$$\delta\lambda_2^B \;=\; \delta\lambda_1^B \;=\; \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{2\lambda}{16\pi^2\epsilon}}$$

$$=\; -\,\lambda \;-\; \frac{16\pi^2\epsilon}{2} \;-\; \frac{(16\pi^2\epsilon)^2}{4\lambda} \;+\; O(\epsilon^3)$$

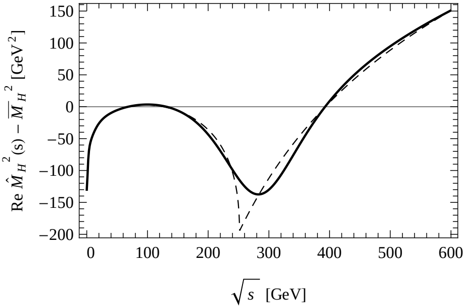
$$\delta\lambda_2^A \;=\; \delta\lambda_1^A \;=\; -\,\lambda \;+\; \frac{(16\pi^2\epsilon)^2}{8\lambda} \;+\; O(\epsilon^3)$$

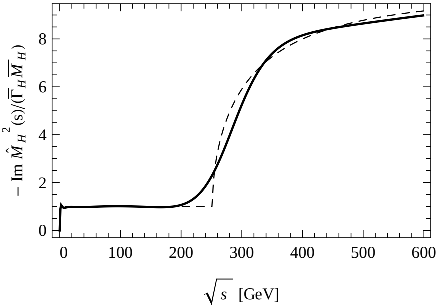
$$\delta m_1^2 \;=\; -\,m^2 \;-\; m^2\,\frac{16\pi^2\epsilon}{4\lambda} \;+\; O(\epsilon^2)$$

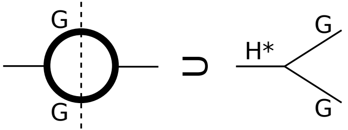
Second-order phase transition in scalar $\mathbb{O}(2)$ model



Higgs self-energy in symmetry-improved 2PI (scalar sector)

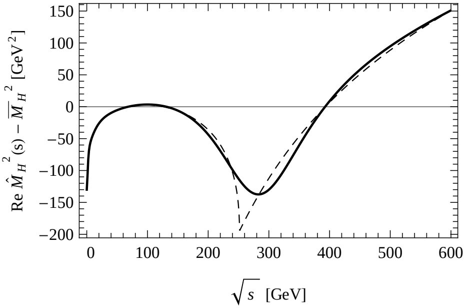


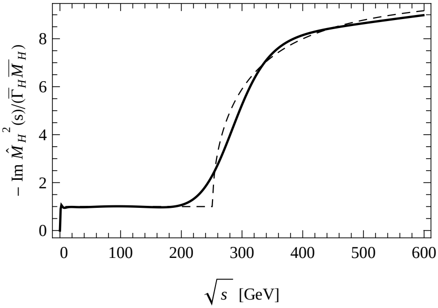


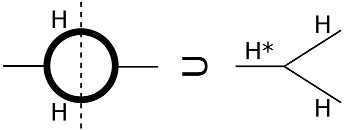


for $\sqrt{s} > 0$

Higgs self-energy in symmetry-improved 2PI (scalar sector)

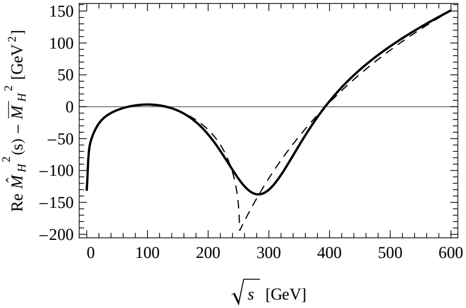


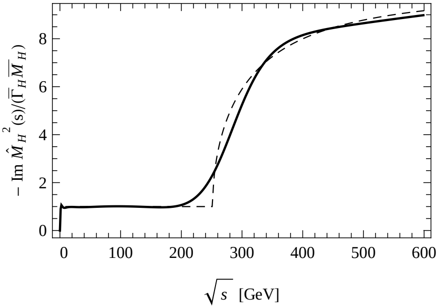


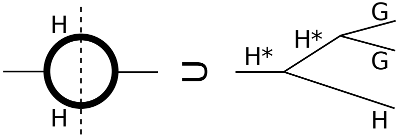


for $\sqrt{s} > 2\,\overline{M}_H$

Higgs self-energy in symmetry-improved 2PI (scalar sector)



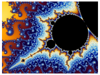




also for $\sqrt{s} \lesssim 2\,\overline{M}_H$

Conclusions

- The **CJT formalism** is a powerful **first-principles** formulation of QFT **beyond perturbation theory**, which performs automatically "fractal" resummations



- Its applicability was limited by "bad" description of symmetries
- Novel approach for global symmetries:

  Symmetry-improved CJT [Pilaftsis and Teresi, Nucl. Phys. **B874** (2013) 594-619]
- IR divergences of the SM effective potential [Pilaftsis and Teresi, 1502.07986]
- Include fermions [Pilaftsis and Teresi, in preparation]
- Study impact on the stability analyses of the Standard Model
- RG running
- Extend to gauge symmetries
- ...

Backup slides

Goldstone theorem

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi^i)^2 + \frac{m^2}{2}(\varphi^i)^2 - \frac{\lambda}{4}[(\varphi^i)^2]^2 \qquad \phi^1 = v + H,\ \phi^2 = G$$

Standard 1PI Ward Identities

$$\Gamma[\mathcal{O}\phi] = \Gamma[\phi] \quad \Longrightarrow \quad \frac{\delta\Gamma}{\delta\phi_x^i} T_{ij}^a \phi_x^j = 0$$

Goldstone theorem: $v \int_x \frac{\delta^2\Gamma}{\delta G_x \delta G_y} = 0$ at the extremum of $\Gamma[\phi]$

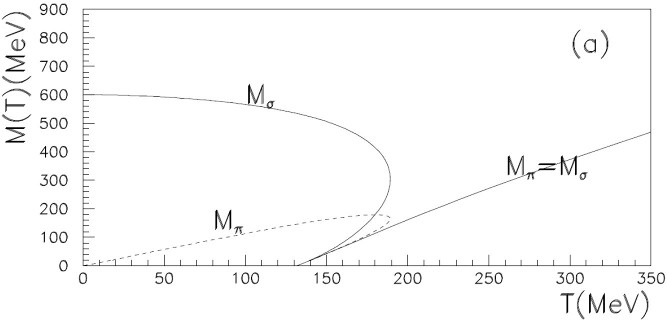
2PI Ward Identity

$$\Gamma[\mathcal{O}\phi, \mathcal{O}\Delta\mathcal{O}^T] = \Gamma[\phi, \Delta]$$

$$\Longrightarrow \quad v \int_x \frac{\delta^2\Gamma}{\delta G_x \delta G_y} + \frac{\delta^2\Gamma}{\delta G_y \delta \Delta_{xz}^{GH}} (\Delta_{xz}^H - \Delta_{xz}^G) = 0$$

No Goldstone theorem for the truncated 2PI effective action

Massive Goldstone bosons



[Petropoulos, 1998]

Symmetry-improved equations of motion

Constraint: $v \int_x \Delta^{G\,-1}_{xy} = 0$

Introduce Lagrange multiplier L:

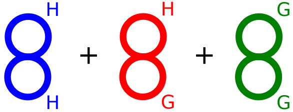
$$\frac{\partial \Gamma_{\rm tr}[v,\Delta]}{\partial v} = L\, m^2$$
$$\frac{\delta \Gamma_{\rm tr}[v,\Delta]}{\delta \Delta_i(k)} = 0$$
$$v\,\Delta_G^{-1}\Big|_{k=0} = 0$$

A reducible singularity has been IR regulated

Effectively $\dfrac{\delta \Gamma_{\rm tr}[\phi,\Delta]}{\delta \phi} = 0$ has been replaced by $\underbrace{\dfrac{\delta \Gamma_{\rm tr}[\phi]}{\delta \phi}}_{= v\,\Delta_G^{-1}\big|_{k=0}} = 0$

$\mathbb{O}(2)$ Hartree-Fock approximation

Hartree-Fock:



$$\Delta_{H/G} = \frac{1}{k^2 - M^2_{H/G} + i\epsilon}$$

$$M_H^2 = 3\lambda v^2 - m^2 + (\delta\lambda_1^A + 2\delta\lambda_1^B)v^2 - \delta m_1^2 +$$

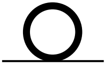
$$+ (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B) \int_k i\Delta_H(k) + (\lambda + \delta\lambda_2^A) \int_k i\Delta_G(k)$$

$$M_G^2 = \lambda v^2 - m^2 + \delta\lambda_1^A v^2 - \delta m_1^2 +$$

$$+ (\lambda + \delta\lambda_2^A) \int_k i\Delta_H(k) + (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B) \int_k i\Delta_G(k)$$

$$v\, M_G^2 = 0$$

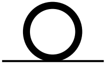
Naive renormalization



$$= \int i\Delta(k) \sim M^2 \frac{1}{\epsilon} \qquad \delta m^2 \stackrel{?}{=} M^2 \frac{1}{\epsilon}$$

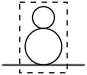
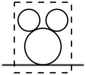
but $M^2 = M^2(T) \quad \Longrightarrow$ T-dependent counterterms?!

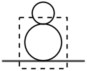
Naive renormalization

 $= \int i\Delta(k) \sim M^2 \frac{1}{\epsilon}$ $\qquad \delta m^2 \overset{?}{=} M^2 \frac{1}{\epsilon}$

but $M^2 = M^2(T) \quad \Longrightarrow$ T-dependent counterterms?!

Why?

$\delta m^2 \quad \sim$  $=$  $+$  $+ \quad \ldots$

 $\supset$  $\sim \quad \delta\lambda$

Standard BPHZ analysis misses subdivergences

[Blaizot, Iancu and Reinosa, 2003; Berges et al., 2005]

CJT counterterms

Special counterterms δm_0^2, δm_1^2 to renormalize:

$$\frac{\delta^2\Gamma}{\delta\phi\,\delta\phi}\,, \quad \frac{\delta\Gamma}{\delta\Delta}$$

and $\delta\lambda_0$, $\delta\lambda_1$, $\delta\lambda_2$ for:

$$\frac{\delta^4\Gamma}{\delta\phi\,\delta\phi\,\delta\phi\,\delta\phi}\,, \quad \frac{\delta^3\Gamma}{\delta\phi\,\delta\phi\,\delta\Delta}\,, \quad \frac{\delta^2\Gamma}{\delta\Delta\,\delta\Delta}$$

Also, different $\mathbb{O}(N)$ invariants receive different counterterms

Renormalizability

Only standard counterterm involved in higher-order operators

[Berges et al., 2005]

Hands-on renormalization

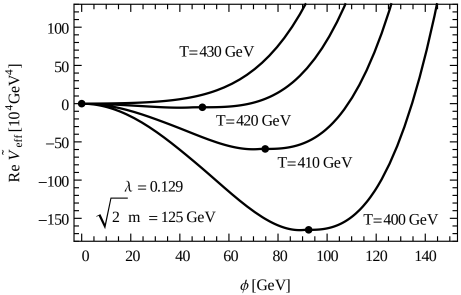
- Separate UV-finite and UV-divergent parts of the EoMs
- Use the UV-finite eqns to rewrite the divergent parts as:

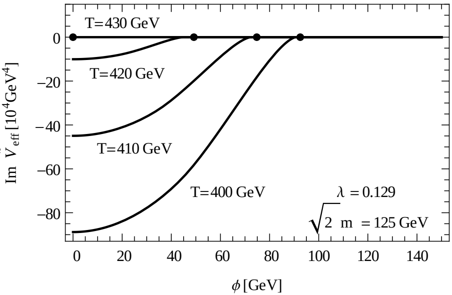
$$(\ldots)\,\mathcal{T}_H^{\text{fin}} \;+\; (\ldots)\,\mathcal{T}_G^{\text{fin}} \;+\; (\ldots)\,v^2 \;+\; (\ldots)\,1 \;=\; 0$$

$\mathcal{T}^{\text{fin}} \equiv$  $\Big|_{\text{finite}}$

- All the T-dependence is in $\mathcal{T}_H^{\text{fin}}$, $\mathcal{T}_H^{\text{fin}}$, v^2
  $\Longrightarrow$ cancel separately the $(\ldots)$ subdivergences
- $4+4$ equations for 5 counterterms δm_1^2, $\delta\lambda_1^A$, $\delta\lambda_1^B$, $\delta\lambda_2^A$, $\delta\lambda_2^B$
  $\Longrightarrow$ non-trivial check for the renormalization procedure

$\widetilde{V}_{\rm eff}(\phi)$ in the Hartree-Fock approximation





- Second-order phase transition
- $\mathrm{Im}\widetilde{V}_{\rm eff}(\phi)$ for $\phi < v$: *physical* instability of the homogeneous vacuum

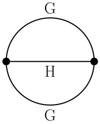
[Weinberg and Wu, 1987]

(in standard 2PI and in perturbation theory, threshold at $\phi_{\rm thres} \neq v$)

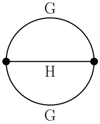
The Goldstone-boson catastrophe in the Standard Model

[Martin, 2013]

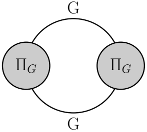
Contributions to the SM effective potential:



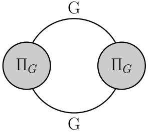
$\sim \quad m_G^2 \log m_G^2$ ✓

$\frac{d}{d\phi}$ 

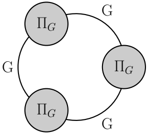
$\sim \quad \log m_G^2$ ✗



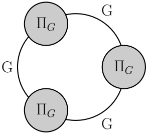
$\sim \quad \log m_G^2$ ✗

$\frac{d}{d\phi}$ 

$\sim \quad \frac{1}{m_G^2}$ ✗



$\sim \quad \frac{1}{m_G^2}$ ✗

$\frac{d}{d\phi}$ 

$\sim \quad \left(\frac{1}{m_G^2}\right)^2$ ✗

$$m_G^2 = \lambda\phi^2 - m^2 \qquad M_G^2 = m_G^2 + \Pi_G^{(1)}|_{k=0} + \ldots = 0 \quad \text{at } \phi = v$$

Partial-resummation approach

Why do we care?

- conceptually: V_{eff} should be well-defined for all ϕ
  unphysical instability at $\phi = v$
- quantitatively: IR div. at $\phi \neq v$ can have a large impact at $\phi = v$, and therefore to the extrapolated high-energy V_{eff}

Partial resummation [Martin, 2014; Elias-Miro, Espinosa, Kostandin, 2014]

- approximately resum ring diagrams:

$$V^{(1)} \to \frac{3}{4(16\pi^2)}\left(m_G^2 + \Pi_G(0)\right)^2 \left[\log\left(\frac{m_G^2 + \Pi_G(0)}{\mu^2}\right) - \frac{3}{2}\right]$$

- $\frac{d}{d\phi}V^{(1)}$ still divergent, because so is $\frac{d}{d\,m_G^2}\Pi_G(0)$

$$\Longrightarrow \quad \Pi_G(0) \to \Pi_g \equiv \Pi_G(0) - \frac{3\lambda}{(16\pi^2)}\, m_G^2 \left(\log(m_G^2/\mu^2) - 1\right)$$

- subtract double-counted diagrams

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Partial resummation [Martin, 2014; Elias-Miro, Espinosa, Kostandin, 2014]

- approximately resum ring diagrams:

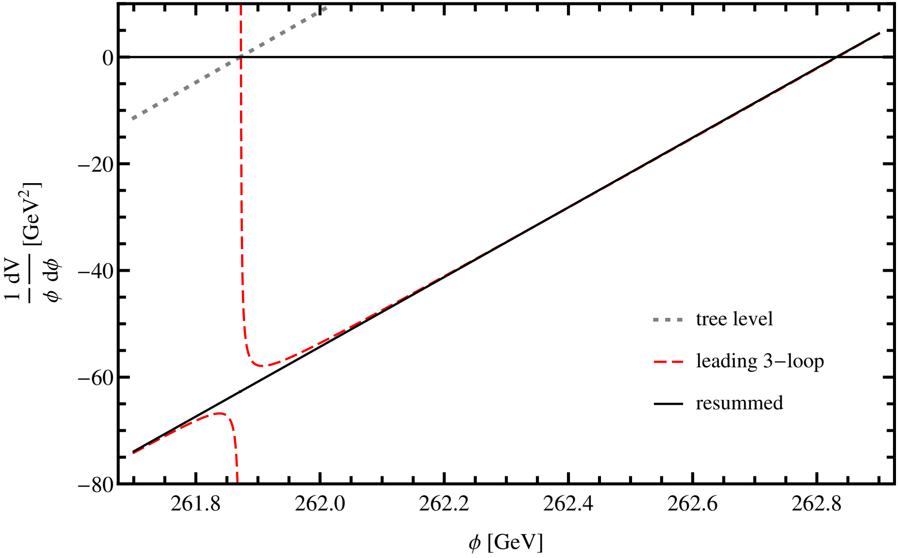
$$V^{(1)} \to \frac{3}{4(16\pi^2)} \left(m_G^2 + \Pi_G(0)\right)^2 \left[\log\left(\frac{m_G^2 + \Pi_G(0)}{\mu^2}\right) - \frac{3}{2}\right]$$

- $\dfrac{d}{d\phi} V^{(1)}$ still divergent, because so is $\dfrac{d}{d\, m_G^2} \Pi_G(0)$

$$\Longrightarrow \quad \Pi_G(0) \to \Pi_g \equiv \Pi_G(0) - \frac{3\lambda}{(16\pi^2)}\, m_G^2 \left(\log(m_G^2/\mu^2) - 1\right)$$

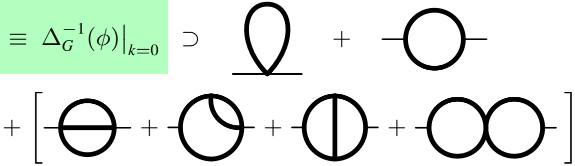
- subtract double-counted diagrams

Partial-resummation approach: results



Symmetry-improved 2PI approach

- More complete resummation:
  - first-principle approach
  - it takes into account the momentum-dependence of self-energy insertions
  - more topologies
  - no ad-hoc subtraction of $m_G^2 \log m_G^2$ contributions in Π_G
- $$-\frac{1}{\phi}\frac{d\widetilde{V}_{\rm eff}}{d\phi} \equiv \Delta_G^{-1}(\phi)\big|_{k=0} \supset$$



$\Big|_{\Delta\approx\Delta_0(\phi)}$

- No IR divergences: the would-be divergent self-energies are 2PR
- Correct threshold properties:
  - at the minimum Goldstone bosons really massless inside loops
  - IR divergences really resummed, not hidden in truncation artifacts
  - no unphysical instabilities

Symmetry-improved 2PI: scalar sector