The Embedding Theory with the Partial Gauge Fixing

Elizaveta Semenova

Department of High Energy and Elementary Particles Physics, St. Petersburg State University

Abstract

The research is focused on the problems related to the description of gravitation. The formulation of the gravity theory where the space-time is a four-dimensional surface in a flat ten-dimensional space is considered. The possibility of using the 'external' time (the time of the ambient space) in such approach is investigated. The transition to the 'external' time is realized with the help of the partial gauge fixing – the coordinate condition which equates the timelike coordinate of the surface and time of the ambient space. The application of the 'external' time could be useful for attempts to quantize the theory as well as for the Embedding Theory study when four-dimension surfaces fill all the ambient space and the coordinates on the surfaces are not introduced.

Introduction

The Einstein's General Relativity is a common theory of gravity. It works well for classical physics, but all attempts of quantum gravity theory construction in terms of metric have failed. Many problems arise from the fact that we are trying to apply the quantization procedure that works well for a field theory in a flat space to the case when the dynamic variables are the geometrical properties of space and one have to quantize them. Major problems are the Problem of Time and the Problem of formulation of Causality Principle. Roughly speaking, the problem of time is that there is none in general relativity because the Hamiltonian is a constraint. In 1975 T. Regge and C. Teitelboim proposed an alternative way of gravity description - the Embedding Theory. According to this theory the space-time is a four-dimensional surface in a flat ten-dimensional Minkovski space and the independent variable is the embedding function. This function defines four-dimensional surface in the flat ambient space $y^a(x^\mu) : R^4 \longrightarrow R^{1,9}$. In Regge and Teitelboim's approach, the standard Einstein-Hilbert's action is taken as an action of the theory.

The induced metric expressed by the embedding function $g_{\mu\nu} = \partial_\mu y^a \partial_\nu y_a$ is substituted into this action. Equations of motion of this theory are more general than the Einstein's. In order to eliminate extra decisions the imposition of the additional Einstein's constraints $G_{\mu\perp} = 0$ were suggested. The aim of this work is to study the canonical formalism of Regge-Teitelboim formulation of gravity with respect to the external time, in order to do this we need to consider the conventional Embedding Theory with additional condition (gauge fixing) $y_0 = x_0$. Since the Embedding Theory is the theory in the Minkovski ambient space, one can use its time-like coordinate as a physical time. The Problem of formulation of Causality Principle also can be solved within the framework of the modified version of the Embedding Theory that formulates gravity as a field theory in the ambient space. According to this Foliation Theory field in the ambient space describes a set of non-interacting four-dimensional surfaces. The geometry of these surfaces corresponds to the motion equations of the Embedding Theory.

Results

The addition of gauge condition into the action can lead to a loss of some motion equations. It was verified that all 10 R-T equations are realized due to the presence of the coordinate condition. Therefore, our gauge condition does not spoil the theory. After a tedious calculation the exact form of the first-class constraint algebra was obtained. We can fix the gauge by introducing it as an additional condition not in the action but in the canonical formalism. As a result, eight constraints of the Regge-Teitelboim formulation of gravity turn into seven constraints of the formulation with a partial gauge fixing, so does the constraint algebra. This suggests that in the case of the imposition of additional Einstein's constraints, the canonical formalism of the theory with respect to the time of the ambient space (ie, the partial gauge fixing in action) is equivalent to the canonical description of the theory with respect to the time of the surface.

Referenses

- *T.Regge, C.Teitelboim.* General relativity a la string: a progress report. In Proceedings of the First Marcel Grossmann Meeting, Trieste, Italy, 1975. Ed.R.Ruffini, North Holland, Amsterdam, 1977. P.77.
- *S.A.Paston.* Theor. Math. Phys., v.169, N 2, pp 1611-1619, 2011, arXiv:1111.1104.
- *S.A.Paston, A.N.Semenova.* Int. J. Theor. Phys., v.49, N 11, pp. 2648-2658, 2010, arXiv:1003.0172.