Quark deconfinement and Gluon condensate in a weak magnetic field from QCD Sum Rules

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Introduction.

The behaviour of strongly interacting matter in the presence of external magnetic fields is a very active research field. It has a strong impact on experiments at the LHC (peripherial collisions of heavy nuclei at high energy), as well as on astronomical objects like neutron stars, magnetars, and the early universe. In addittion, lattice QCD (LQCD) has shown that the critical temperature for deconfinement/chiral symmetry restoration decreases with increasing field strength. This behaviour is dubbed *inverse magnetic catalysis*, and it reveals an unexpected, non-trivial phenomenon.

Given the dual nature of the QCD phase transition, a pertinent question is to what extent inverse magnetic catalysis is due to the mechanisms of either chiral symmetry restoration and/or of deconfinement. One way to address this question is to find a relation between deconfinement and chiral symmetry restoration parameters as a function of the magnetic field. Since the transition happens for temperatures in the realm of non-perturbative phenomena, the relation searched for needs to carry non-perturbative information. A non-perturbative tool that does not rely on effective models is that of QCD Finite Energy Sum Rules (FESR).

A key parameter that emerges from this analysis signalling quark-gluon deconfinement is the squared energy threshold, s_0, above which the hadronic spectral function is well approximated by perturbative QCD (pQCD).

Here, we present FESR in the axial-vector channel, and in the presence of an external magnetic field, to explore the relation between (i) the deconfinement and chiral symmetry restoration parameters, s_0 and $\langle\bar{q}q\rangle$, and (ii) obtain the behaviour of the gluon condensate as a function of the magnetic field intensity at zero temperature.

Finite Energy QCD Sum Rules with magnetic fields.

The charged axial-vector current correlator in the absence of a magnetic field can be written as

$$\begin{aligned}\Pi_{\mu\nu}(q^2) &= i\int d^4x\, e^{iqx} \ <0|T(A_\mu(x)\ ,\ A_\nu^\dagger(0))|0> \\ &= (-g_{\mu\nu}q^2 + q_\mu q_\nu)\,\Pi_A(q^2) + q_\mu q_\nu\,\Pi_0(q^2), \qquad (1)\end{aligned}$$

where $A_\mu(x) =: \bar{d}(x)\gamma_\mu\gamma_5 u(x) :$ is the charged axial-vector current, with $s \equiv q^2 > 0$ the squared energy. Concentrating on e.g. $\Pi_0(q^2)$ and invoking the Operator Product Expansion (OPE) of current correlators at short distances beyond perturbation theory, one of the two pillars of the QCD sum rule method, one has

$$\Pi_0^{\text{QCD}}(Q^2) = C_0\,\hat{I} + \sum_{N=1} \frac{C_{2N}(Q^2,\mu^2)}{Q^{2N}}\langle\hat{\mathcal{O}}_{2N}(\mu^2)\rangle\ , \qquad (2)$$

The Wilson coefficients C_N depend on the Lorentz indexes and quantum numbers of the currents, and on the local gauge invariant operators $\hat{\mathcal{O}}_{2N}$ built from the quark and gluon fields in the QCD Lagrangian.

The second pillar of the QCD sum rule method is to consider an integration contour in the complex square energy plane, and invoke Cauchy's theorem assuming that QCD can be used on the circle of radius $|s_0|$. Since there are no further singularities this leads to the FESR

$$-\frac{1}{2\pi i}\oint_{C(|s_0|)} ds\, s^{N-1}\Pi_0^{\text{QCD}}(s) = \frac{1}{\pi}\int_0^{s_0} ds\, s^{N-1}\text{Im}\,\Pi_0^{\text{HAD}}(s), \qquad (3)$$

with $N \geq 1$, and $\Pi_0^{\text{QCD}}(s)$ given by Eq. (2). In the presence of a magnetic field, and in the weak field limit $eB < s_0$, the Wilson coefficients acquire themselves a B-field dependence.

$$\begin{aligned} C_0 \ln\left(\frac{-s}{\mu^2}\right) &\to \frac{1}{4\pi}\ln\left(\frac{-s}{\mu^2}\right) + \sum_{n=1} C_0^{(n)}\frac{(eB)^n}{s^n} \\ C_{2N} &\to \sum_{m=0} C_{2N}^{(m)}\frac{(eB)^m}{s^m} \qquad (4)\end{aligned}$$

Substituting Eqs. (4) and (2) into Eq. (3), the first two sum rules ($N = 1,\ 2$) become

$$0 = \frac{1}{\pi}\int_0^{s_0} ds\ \text{Im}\Pi_0^{\text{HAD}}(s) - \frac{1}{4\pi}s_0 + C_0^{(1)}(eB)\ , \qquad (5)$$

$$\begin{aligned}-C_4^{(0)}\langle O_4\rangle + C_2^{(1)}(eB)\langle O_2\rangle &= \frac{1}{\pi}\int_0^{s_0} ds\, s\ \text{Im}\,\Pi_0^{\text{HAD}}(s) \\ &\quad - \frac{1}{8\pi}s_0^2 + C_0^{(2)}(eB)^2. \qquad (6)\end{aligned}$$

S_0 and $C_4\langle O_4\rangle$ from FESR.

Hadronic Sector.

The axial-vector current in the presence of a magnetic field can be interpolated by the charged pion current

$$A_\mu = -f_\pi D_\mu \pi^+ = -f_\pi(\partial_\mu - ie\mathcal{A}_\mu)\pi^+, \qquad (7)$$

where f_π is the pion decay constant, π^+ the pion field, and $\mathcal{A}_\mu = (B/2)(0,-y,x,0)$ the vector potential in the symmetric gauge. Therefore, the axial-vector correlator in the hadronic sector can be written as

$$\Pi_0^{\text{HAD}}(q^2) = i f_\pi^2 \widetilde{G}_\pi(q^2), \qquad (8)$$

where $\widetilde{G}_\pi(q^2)$ is the charged pion propagator in presence of magentic fields, when it is expressed in terms of a sum over Landau levels and setting $q_\perp^2 = 0$

$$\Pi_0^{\text{HAD}}(q_\parallel^2 = s) = -2f_\pi^2\sum_{l=0}^{\infty}\frac{(-1)^l}{s-(2l+1)eB}. \qquad (9)$$

The imaginary part of Eq. (9) in the weak field limit $eB < s_0$ is given by

$$\text{Im}\Pi_0^{\text{HAD}}(s) = f_\pi^2\pi\delta(s-eB). \qquad (10)$$

Substituting Eq. (10) into the QCD sum rules Eqs. (5)-(6) gives

$$\begin{aligned} 0 &= f_\pi^2 - \frac{1}{4\pi}s_0 + C_0^{(1)}(eB) \\ -C_4\langle O_4\rangle &= f_\pi^2(eB) - \frac{1}{8\pi}s_0^2 + C_0^{(2)}(eB)^2. \qquad (11)\end{aligned}$$

pQCD Sector.

To perform the perturbative calculation of the coefficients $C_0^{(1)}$ and $C_0^{(2)}$ we use the weak field expansion of the quark propagator in the presence of a constant magnetic field, and in the chiral limit, up to order $\mathcal{O}(B^2)$

$$\begin{aligned} iS_B(k) &= i\frac{\not{k}}{k^2} - (e_qB)\frac{\gamma_1\gamma_2(\gamma\cdot k)_\parallel}{k^4} \\ &\quad - 2i(e_qB)^2\frac{\left[k_\perp^2(\gamma\cdot k)_\parallel - k_\parallel^2(\gamma\cdot k)_\perp\right]}{k^8}, \qquad (12)\end{aligned}$$

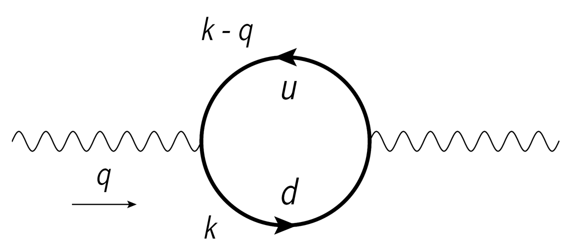

Figure 1

The pQCD contribution to the axial-vector current correlator in the presence of a magnetic field is depicted in Fig. 1, where we also define the kinematics. The thick internal lines represent the full quark propagators in the magnetic field background.

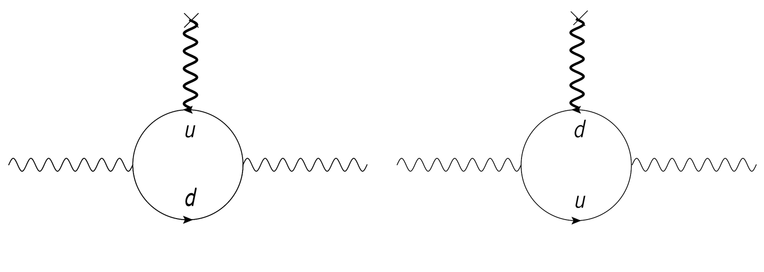

Figure 2

To first order in e_qB only one of the two quark propagators carries the magnetic effects. This is depicted in Fig. 2 where the wavy line starting from a cross represents the external magnetic field. The two diagrams in Fig. 2 that determine the coefficient $C_0^{(1)}$, vanish identically when contracted with the momenta carried by the axial-vector currents.

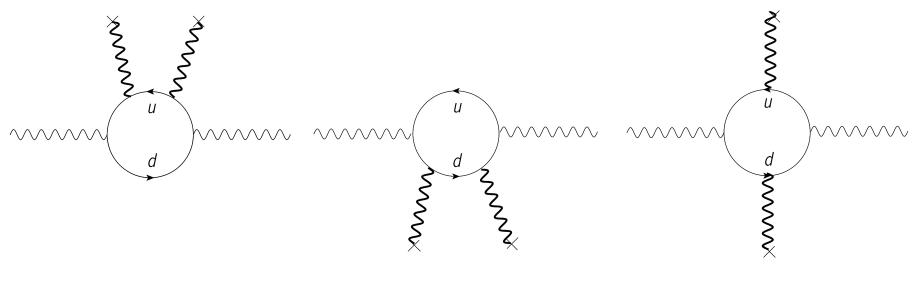

Figure 3

The first non-trivial magnetic contribution to the pQCD axial-vector current correlator is of order $(e_qB)^2$. The relevant diagrams are shown in Fig. 3. We obtain the coefficient of the axial-vector current correlator to second order in the magnetic field

$$\Pi_0^{B^2} = -\left(\frac{17}{18}\right)\frac{(eB)^2}{4\pi^2}. \qquad (13)$$

Using this result together with the first equation in Eq. (4), we obtain the Wilson coefficient of the pQCD contribution to second order in the magnetic field

$$C_0^{(2)} = -\left(\frac{17}{18}\right)\frac{1}{4\pi^2}. \qquad (14)$$

GMOR relation.

The last ingredient needed to find s_0 and $C_4\langle O_4\rangle$ is the magnetic field dependence of f_π. Invoking the Gell-Mann-Oakes-Renner (GMOR) relation f_π is related to the light quark condensate $\langle\bar{q}q\rangle$

$$m_\pi^2 f_\pi^2 = -2\,(m_u + m_d)\langle\bar{q}q\rangle. \qquad (15)$$

The light-quark condensate in the presence of magnetic field has been computed by LQCD. We make use of this result, and parametrize the magnetic field dependence of the light-quark condensate.

Results.

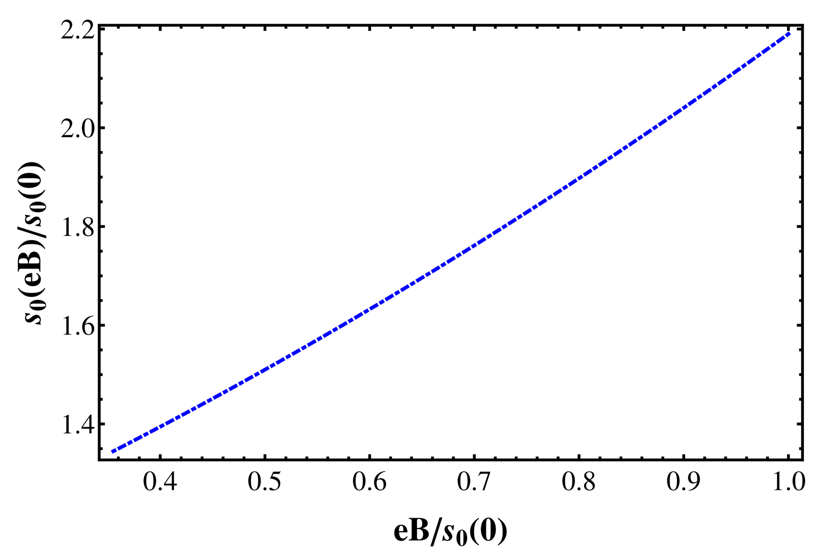

Figure 4

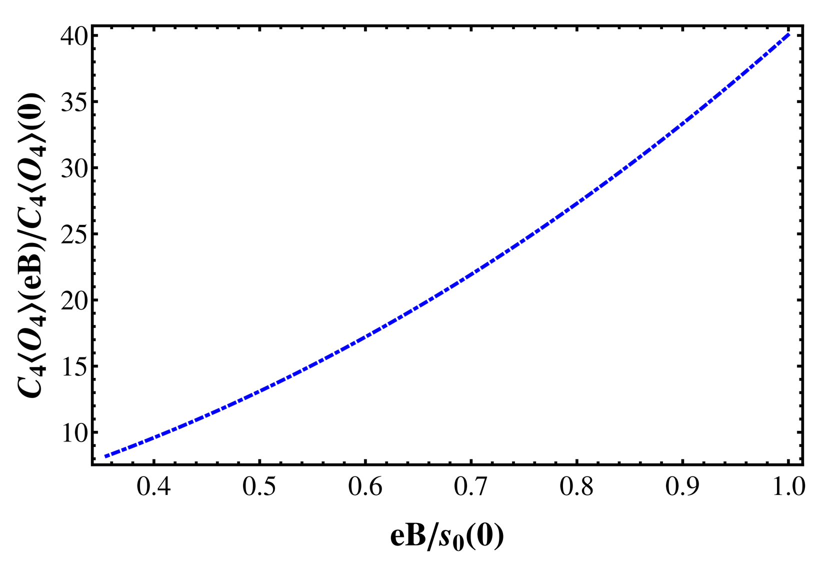

Figure 5

The solutions for s_0 and for $C_4\langle O_4\rangle$ as functions of eB are plotted in Figs. 4 and 5, respectively. Note that s_0 is proportional to the absolute value of the light-quark condensate, and that together with $C_4\langle O_4\rangle$ it increases with increasing magnetic field.

Conclusions.

We studied QCD FESR for the axial-vector current correlator in the presence of a magnetic field in the weak field limit $eB < s_0$. We have shown that the presence of the field modifies both the pQCD as well as the hadronic sectors of the FESR. The magnetic field dependence of s_0 is thus proportional to the magnetic field dependence of the absolute value of the light-quark condensate. Therefore the magnetic field both helps the formation of the condensate and acts against deconfinement. The gluon condensate also grows as a function of the field strength which goes hand in hand with the behavior of the magnetic field, both as a catalyst of chiral symmetry breaking and confinement.

Forthcoming Research.

The results obtained here should serve as a basis for studies at finite temperature in an external magnetic field.

References

[1] A. Ayala, C. A. Dominguez, L. A. Hernandez, M. Loewe, J. C. Rojas and C. Villavicencio. arXiv:1504.01308 [hep-ph].