External time canonical formalism for gravity in terms of the Embedding Theory

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The Embedding Theory

The Einstein's General Relativity works well for classical physics.
1975 T. Regge and C. Teitelboim - the Embedding Theory is an alternative way of gravity description.

$$y^a(x^\mu) : \mathbb{R}^4 \longrightarrow \mathbb{R}^{1,9} \tag{1}$$

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2\varkappa} R + \mathcal{L}_m \right) \tag{2}$$

$$g_{\mu\nu}(x) = \eta_{ab} \partial_\mu y^a(x) \partial_\nu y^b(x) \tag{3}$$

Regge-Teitelboim equations:

$$\left(G^{\mu\nu} - \varkappa T^{\mu\nu} \right) b^a_{\mu\nu} = 0 \tag{4}$$

Canonical formalism for the Embedding Theory

$$S = -\frac{1}{2\varkappa}\int d^4x\,\sqrt{-g}\;R, \quad \text{R-T equations:} \quad G^{\mu\nu}\,b^a_{\mu\nu} = 0 \tag{5}$$

$$R = g^{\mu\nu}R_{\mu\nu}, \quad R_{\mu\nu} = \partial_\rho\Gamma^\rho{}_{\mu\nu} - \partial_\nu\Gamma^\rho{}_{\mu\rho} + \Gamma^\rho{}_{\mu\nu}\Gamma^\sigma{}_{\rho\sigma} - \Gamma^\sigma{}_{\mu\rho}\Gamma^\rho{}_{\nu\sigma} \tag{6}$$

$$\Gamma^\mu{}_{\rho\sigma} = \frac{1}{2}g^{\mu\nu}\left(\partial_\sigma g_{\nu\rho} + \partial_\rho g_{\nu\sigma} - \partial_\nu g_{\rho\sigma}\right), \quad g_{\mu\nu} = \partial_\mu y^a\partial_\nu y_a \tag{7}$$

$$S = \int dx^0 L(y^a,\dot{y}^a), \quad L = \int d^3x\frac{1}{2}\left(\frac{\dot{y}^a B_{ab}\,\dot{y}^b}{\sqrt{\dot{y}^a\overset{3}{\Pi}_{\perp ab}\,\dot{y}^b}} + \sqrt{\dot{y}^a\overset{3}{\Pi}_{\perp ab}\,\dot{y}^b}\;B^c_c\right) \tag{8}$$

Einstein's constraints

$$\pi_a = \frac{\delta L}{\delta \dot{y}^a} = B_{ab} n^b - \frac{1}{2} n_a \left(n_c B^{cd} n_d - B^c_c \right) \tag{9}$$

Here $n_a \parallel W^4,\ n_a \perp W^3$

$$n^a = \sqrt{g^{00}}\ \overset{3}{\Pi}_{\perp}{}^a_b\, \partial_0 y^b = \frac{\overset{3}{\Pi}_{\perp}{}^a_b\, \partial_0 y^b}{\sqrt{\partial_0 y^c\, \overset{3}{\Pi}_{\perp cd}\, \partial_0 y^d}} \tag{10}$$

$$n_\mu G^{\mu\nu} \approx 0 \ <=>$$

$$\mathcal{H}^0 = \frac{1}{2} \left(n_c B^{cd} n_d - B^c_c \right) \approx 0, \quad \mathcal{H}^i = -2\sqrt{-\overset{3}{g}}\ \overset{3}{D}_k \left(L^{ik,lm}\, \overset{3}{b}{}^a_{lm}\, n_a \right) \approx 0 \tag{11}$$

$$\pi_a \approx B_{ab} n^b \tag{12}$$

Partial gauge fixing

It is helpful to consider conventional embedding theory with an additional coordinate condition (gauge fixing)

$$y^0(x^\mu) = x^0 \tag{13}$$

$$S = \int dx\, \frac{1}{2} \left(\frac{\dot{y}^A\ B_{AB}\ \dot{y}^B}{\sqrt{1 + \dot{y}^A\ \overset{3}{\Pi}_{\perp AB}\ \dot{y}^B}} + \sqrt{1 + \dot{y}^A\ \overset{3}{\Pi}_{\perp AB}\ \dot{y}^B}\ B^D_D \right), \tag{14}$$

hereinafter A,B... indexes take values 1,2...9. We have a set of 7 constraints:

$$\Phi_i = \pi_A \overset{3}{e}{}^A_i,$$

$$\mathcal{H}^0 = \frac{1}{4\sqrt{-\overset{3}{g}}} \overset{3}{b}{}^{-1}{}^{ik}_{A} \pi^A L^{-1}_{ik,lm} \overset{3}{b}{}^{-1}{}^{lm}_{B} \pi^B - \sqrt{-\overset{3}{g}}\ \overset{3}{b}{}^A_{ik} \overset{3}{b}{}^D_{lm} \eta_{DA} L^{ik,lm},$$

$$\mathcal{H}^i = -\sqrt{-\overset{3}{g}}\ \overset{3}{D}_k \left(\frac{1}{\sqrt{-\overset{3}{g}}} \overset{3}{b}{}^{-1}{}^{ik}_{A}\ \pi^A \right)$$

Constraints

It is convenient to deal with convolution of constraints and arbitrary functions and to use a linear combination $\Psi^k = \mathcal{H}^k + \Phi_i \overset{3}{g}{}^{ik}$ instead of the constraints $\mathcal{H}^k$.

$$\left\{ \Psi_\xi, \overset{3}{g}_{ik} \right\} = 0, \qquad \Psi_\xi = \int d^3x (\mathcal{H}^k + \Phi_i \overset{3}{g}{}^{ik}) \xi_k \tag{15}$$

The constraints Ψ^k generate transformations which are an isometric bending of the surface W^3.

- Now we have a set of constraints: Φ_ξ, $\mathcal{H}^0_\xi$, Ψ_ξ

$$H = \int d^3x \left(\pi_A \dot{y}^A - L \right) =$$

$$= \int d^3x \frac{1}{2\sqrt{1 + \dot{y}^A \overset{3}{\Pi}_{\perp AB} \dot{y}^B}} \left(\frac{\dot{y}^A B_{AB} \dot{y}^B}{\left(\sqrt{1 + \dot{y}^A \overset{3}{\Pi}_{\perp AB} \dot{y}^B} \right)^2} - B^D_D \right) \approx 0 \quad (16$$

Constraint algebra

After a tedious calculation we get the exact form of the first-class constraint algebra:

$$\{\Phi_\xi, \mathcal{H}^0_\zeta\} = -\int d^3x \mathcal{H}^0 \xi^i \partial_i \zeta$$

$$\{\Phi_\xi, \Phi_\zeta\} = -\int d^3x \; \Phi_k \left(\xi^i \overset{3}{D}_i \zeta^k - \zeta^i \overset{3}{D}_i \xi^k \right)$$

$$\{\Phi_\xi, \Psi_\zeta\} = -\int d^3x \; \Psi^k \left(\xi^i \overset{3}{D}_i \zeta_k + \zeta_i \overset{3}{D}_k \xi^i \right)$$

$$\{\Psi_\zeta, \Psi_\xi\} = \int d^3x \left(-\Psi^l r^\xi_A \overset{3}{D}_l r^A_\zeta \right) - (\zeta \leftrightarrow \xi)$$

$$\{\Psi_\xi, \mathcal{H}^0_\zeta\} = \int d^3x \; \Psi^p \overset{3}{D}_p \left(\pi^B \hat{B}^C_B r^\xi_C \zeta \right) - 2 \int d^3x \hat{B}^C_B \pi^B \Psi^t \zeta \left(\overset{3}{D}_t r^\xi_C \right)$$

$$\{\mathcal{H}^0_\xi, \mathcal{H}^0_\zeta\} = -\int d^3x \mathcal{H}^l \xi \overset{3}{D}_l \zeta - \int d^3x \Psi^k \hat{B}_{BA} \pi^B \xi \overset{3}{D}_k \left(\hat{B}^A_C \pi^C \xi \right) - (\xi \leftrightarrow \zeta)$$

Another approach to the problem

Fix the gauge by introducing it as an additional condition $\chi = y^0 - x^0 \approx 0$.

$$\{\Psi^4, \chi\} = \int d^3x \frac{\delta \pi_a w^a}{\delta \pi_b} \frac{\delta}{\delta y^b}(y^0 - x^0) = w^0 \neq 0. \tag{18}$$

We express a pair of variables $y^0 = x^0$, π_0.

- Eight constraints of Regge-Teitelboim formulation of gravity turn into seven constraints of the formulation with a partial gauge fixing, so does the constraint algebra.

This suggests that in the case of the imposition of additional Einstein's constraints, canonical formalism of the theory with respect to the time of the ambient space (i.e., the partial gauge fixing in action) is equivalent to the canonical description of the theory with respect to the time of the surface.

Results

- Canonical formalism Regge-Teitelboim formulation of gravity with partial gauge fixing that matches time of the surface and time of the space has been build.
- The exact form of the first-class constraint algebra for embedding theory with partial gauge fixing has been obtained.
- The constraint algebra for embedding theory with partial gauge fixing has been compared with one obtained without it.

Plans: The Splitting Theory

The Splitting Theory formulates gravity as a theory of the field $z^M(y^a)$ in the flat ambient space, M=1…6.

$$z^M(y^a) = const \tag{19}$$

Motivation: other interactions was successfully quantized as a field theories in a flat space.

- Many 4-dimensional surfaces, each describes dynamics of 3-dimensional surfaces in the flat ambient space $R^{1,N-1}$.
- The surfaces pass through each point of $R^{1,N-1}$ space.
- The surfaces neither intersect nor interact.
- All disturbances propagate along the surfaces.
- Geometry of the surfaces corresponds to the solution of R-T equations $G^{ab}\, b^M_{ab} = 0$.

For more detailes see: arXiv:1111.1104, arXiv:1003.0172

Thank you for your attention!