Standard Model thermodynamics across the electroweak crossover

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Based on M. Laine and M. Meyer [hep-ph/1503.04935]

Motivation

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- no electroweak phase transition within the SM

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- no electroweak phase transition within the SM
- "soft point" at $T \approx 160$ GeV
- imprint on non-equilibrium BSM physics

Purpose

We want to estimate the equation of state around the electroweak crossover through perturbative 3-loop computations as well as using existing lattice data within a dimensionally reduced effective theory.

Basic observable

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$$p_B(T) = p_E(T) + p_M(T) + p_G(T)$$

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$$p_B(T) = p_E(T) + p_M(T) + p_G(T)$$

$$\begin{array}{ll} p_E & k \sim \pi T \\ p_M & k \sim gT \\ p_G & k \sim g^2 T/\pi \end{array}$$

Integration across the EW crossover

$$\frac{p(T_1)}{T_1^4}$$

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$$\frac{p(T_1)}{T_1^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T_1} \frac{dT}{T} \; T \frac{d}{dT} \left\{ \frac{p(T)}{T^4} \right\}$$

$$T_0 \ll 160 \text{ GeV} \; , \qquad T_1 \gg 160 \text{ GeV}$$

Integration across the EW crossover

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$$\frac{p(T_1)}{T_1^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T_1} \frac{dT}{T} \underbrace{T \frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\}}_{\Delta(T)}$$

in QCD: $\Delta(T)$ - trace anomaly

Master equation

$$T\frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\} = -\frac{\partial \hat{p}_R}{\partial \ln(\bar{\mu}/T)} - \frac{2\nu^2(\bar{\mu})[\mathcal{Z}_m\langle\phi^\dagger\phi\rangle]_R}{T^4} + \frac{4p_{0R}}{T^4} \ ,$$

where subscript R refers to renormalized quantities, $\bar{\mu}$ is the renormalization scale parameter and ν is the Higgs mass parameter.

Breaking of scale invariance by quantum corrections

$$T\frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\} = -\frac{\partial \hat{p}_R}{\partial \ln(\bar{\mu}/T)} - \frac{2\nu^2(\bar{\mu})[\mathcal{Z}_m\langle\phi^\dagger\phi\rangle]_R}{T^4} + \frac{4p_{0R}}{T^4}$$

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- reading off logarithms in the explicit expression
- deducing them from the running of the couplings

Explicit breaking of scale invariance

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$$T\frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\} = -\frac{\partial \hat{p}_R}{\partial \ln(\bar{\mu}/T)} - \frac{2\nu^2(\bar{\mu})[\mathcal{Z}_m\langle\phi^\dagger\phi\rangle]_R}{T^4} + \frac{4p_{0R}}{T^4}$$

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$$T\frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\} = -\frac{\partial \hat{p}_R}{\partial \ln(\bar{\mu}/T)} - \frac{2\nu^2(\bar{\mu})[\mathcal{Z}_m\langle\phi^\dagger\phi\rangle]_R}{T^4} + \frac{4p_{0R}}{T^4}$$

$$\mathcal{Z}_m\langle\phi^\dagger\phi\rangle = \frac{\partial p_E}{\partial\nu^2} + \frac{\partial p_M}{\partial\nu^2} + \frac{\partial p_G}{\partial\nu^2}$$

Kajantie, Laine, Rummukainen, Shaposhnikov [hep-ph/9508379]

D'Onofrio, Rummukainen, Tranberg [hep-ph/1404.3565] and [hep-ph/1207.0685]

Vacuum subtraction

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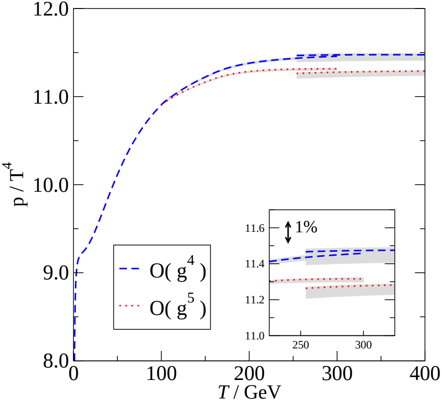
$$T\frac{d}{dT}\left\{\frac{p(T)}{T^4}\right\} = -\frac{\partial \hat{p}_R}{\partial \ln(\bar{\mu}/T)} - \frac{2\nu^2(\bar{\mu})[\mathcal{Z}_m\langle\phi^\dagger\phi\rangle]_R}{T^4} + \frac{4p_{0R}}{T^4}$$

Phenomenological results



low temperature:

Laine / Schröder

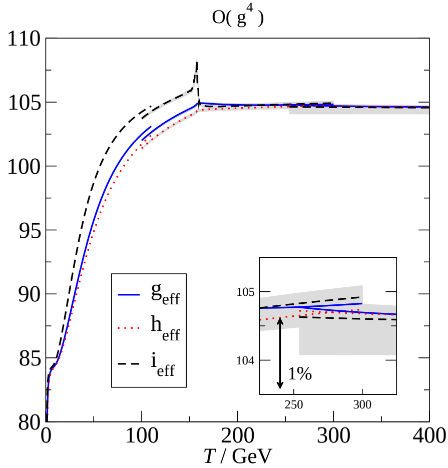
[hep-ph/0603048]

high temperature:

Gynther / Vepsäläinen

[hep-ph/0510375]

Phenomenological results



$$g_{\rm eff}(T) = \frac{e(T)}{\left[\frac{\pi^2 T^4}{30}\right]}$$

$$h_{\rm eff}(T) = \frac{s(T)}{\left[\frac{2\pi^2 T^3}{45}\right]}$$

$$i_{\rm eff}(T) = \frac{c(T)}{\left[\frac{\pi^2 T^3}{15}\right]}$$

Backup

Parametric temperature range

$$T^2 \gtrsim \frac{\nu^2}{g^2}$$

$$|\bar{m}_3^2| \sim | - \nu^2 + g^2 T^2| \lesssim \frac{g^3 T^2}{\pi}$$

$$\Rightarrow \quad \nu^2 \frac{\partial \bar{m}_3^2}{\partial \nu^2} \sim \nu^2 \sim g^2 T^2$$

Dimensional reduction

$$\SumInt_P \equiv \left(\frac{e^{\gamma}\mu^2}{4\pi}\right)^{\epsilon} T \sum_{k_0} \int \frac{d^{3-2\epsilon}k}{(2\pi)^{3-2\epsilon}} \frac{1}{\left[k_0^2 + \vec{k}^2 + m^2\right]^{\alpha}}$$

Matsubara modes $\begin{cases} k_0^b & = 2n\pi T \\ k_0^f & = (2n+1)\pi T \end{cases}$

Trace anomaly

$$\Delta(T) = \Delta_1(T) + \Delta_2(T, \bar{\mu}) + \Delta_3(T, \bar{\mu})$$

Trace anomaly

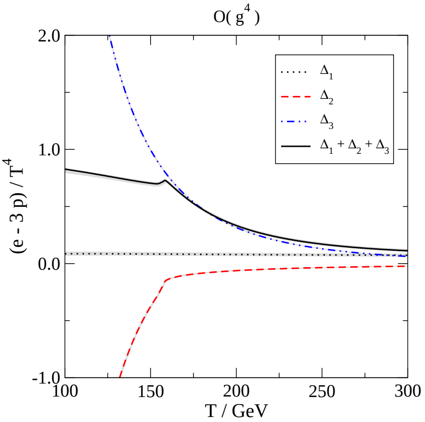
$$\Delta_1(T) = \frac{1}{(4\pi)^2}\Bigg\{\frac{198+141n_G-20n_G^2}{54}g_3^4 + \frac{266+163n_G-40n_G^2}{288}g_2^4$$
$$-\frac{144+375n_G+1000n_G^2}{7776}g_1^4 - \frac{g_2^2 g_1^2}{32} - h_t^2\left(\frac{7h_t^2}{32} - \frac{5g_3^2}{6} - \frac{15g_2^2}{64} - \frac{85g_1^2}{576}\right)$$
$$-\lambda\left(\lambda + \frac{h_t^2}{2} - \frac{g_1^2+3g_2^2}{8}\right) + \frac{\nu^2}{T^2}\left(h_t^2 + 2\lambda - \frac{g_1^2+3g_2^2}{4}\right) - \frac{2\nu^4}{T^4}\Bigg\}$$
$$-\frac{1}{(4\pi)^3}\Bigg\{32g_3^5\left(1+\frac{n_G}{3}\right)^{\frac{3}{2}}\left(\frac{11}{4}-\frac{n_G}{3}\right) + 12g_2^5\left(\frac{5}{6}+\frac{n_G}{3}\right)^{\frac{3}{2}}\left(\frac{43}{24}-\frac{n_G}{3}\right)$$
$$-4g_1^5\left(\frac{1}{6}+\frac{5n_G}{9}\right)^{\frac{3}{2}}\left(\frac{1}{24}+\frac{5n_G}{9}\right)\Bigg\} + \mathcal{O}(g^6)$$

Trace anomaly

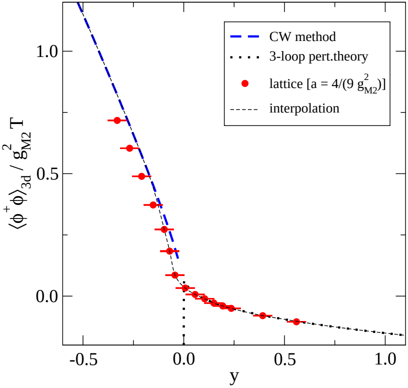
$$
\begin{aligned}
\Delta_2(T,\bar\mu) = &-\frac{2\nu^2}{T^4}\left\{1+\frac{3}{2(4\pi)^2}\left[\left(g_1^2+3g_2^2-8\lambda\right)\ln\left(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\right)-4h_t^2\ln\left(\frac{\bar\mu e^{\gamma_E}}{\pi T}\right)\right]\right\}\langle\phi^\dagger\phi\rangle_{3d}(g_{M2}^2) \\
&-\frac{\nu^2}{3T^2}\left\{1-\frac{3}{2(4\pi)^2}\left[(g_1^2+3g_2^2)\left(4\ln\frac{g_{M2}^2}{\bar\mu}+3\ln\frac{\bar\mu}{4\pi T}+\gamma_E+\frac{5}{3}+\frac{2\zeta'(-1)}{\zeta(-1)}\right)\right.\right. \\
&\left.\left.+4h_t^2\ln\left(\frac{\bar\mu e^{\gamma_E}}{8\pi T}\right)+8\lambda\ln\left(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\right)\right]\right\} \\
&+\frac{2\nu^2}{(4\pi)^3T^2}\left[\frac{g_1^2 m_{E1}+3g_2^2 m_{E2}}{T}+\frac{g_1^4 T}{16 m_{E1}}+\frac{3g_1^2g_2^2 T}{4(m_{E1}+m_{E2})}+\frac{g_2^4 T}{2m_{E2}}\left(\frac{35}{24}+\ln\frac{2m_{E2}}{g_{M2}^2}\right)\right] \\
&-\frac{8\nu^4}{(4\pi)^2T^4}\ln\left(\frac{\bar\mu e^{\gamma_E}}{4\pi T}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_3(T,\bar{\mu}) =& \frac{\nu^4}{\lambda T^4} + \frac{4\nu^4}{(4\pi)^2 T^4}\left[\ln\frac{\bar{\mu}^2}{\nu^2} + \frac{3}{2}\right] \\
&+ \frac{3\nu^4}{\lambda^2(16\pi)^2 T^4}\left\{2g_2^4\left[\ln\frac{4\lambda\bar{\mu}^2}{g_2^2\nu^2} + \frac{5}{6}\right]\right. \\
&\left. + (g_1^2+g_2^2)^2\left[\ln\frac{4\lambda\bar{\mu}^2}{(g_1^2+g_2^2)\nu^2} + \frac{5}{6}\right]\right\} \\
&- \frac{3\nu^4 h_t^4}{\lambda^2(4\pi)^2 T^4}\left[\ln\frac{2\lambda\bar{\mu}^2}{h_t^2\nu^2} + \frac{3}{2}\right] + \mathcal{O}(g^6)
\end{aligned}
$$

Trace anomaly



Higgs condensate



EoS parameter and speed of sound squared