Axial anomaly, vector meson dominance and mixing

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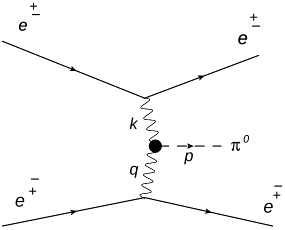
Outline

Transition form factors



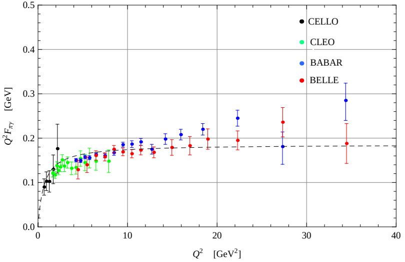
Form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \to M$ (M=π^0, η, η'):

$$\int d^4x e^{ikx} \langle M(p)| T\{J_\mu(x) J_\nu(0)\}|0\rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \quad (1)$$

Kinematics: $k^2 = 0$, $-q^2 \equiv Q^2 > 0$ (space-like region), $q^2 > 0$ (time-like region)

π^0 TFF: experimental status

Pion transition form factor: available data



- The current experimental status of the pion transition form factor (TFF) $F_{\pi\gamma}$ is rather controversial:
  The measurements of the BABAR collaboration [Aubert et al. '09] show a steady rise of $Q^2 F_{\pi\gamma}$, surpassing the pQCD predicted asymptote $Q^2 F_{\pi\gamma} \to \sqrt{2} f_\pi$, $f_\pi = 130.7$ MeV at $Q^2 \simeq 10$ GeV^2 and questioning the collinear factorization. Later BELLE collaboration data [2012] are more consistent with pQCD.

Axial anomaly

In QCD, for a given flavor q, the divergence of the axial current $J^{(q)}_{\mu 5} = \bar{q}\gamma_\mu\gamma_5 q$ acquires both electromagnetic and gluonic anomalous terms:

$$\partial_\mu J^{(q)}_{\mu 5} = m_q \bar{q}\gamma_5 q + \frac{e^2}{8\pi^2} e_q^2 N_c F\tilde{F} + \frac{\alpha_s}{4\pi} N_c G\tilde{G}, \quad (2)$$

An octet of axial currents

$$J^{(a)}_{\mu 5} = \sum_q \bar{q}\gamma_5\gamma_\mu \frac{\lambda^a}{\sqrt{2}} q$$

Singlet axial current $J^{(0)}_{\mu 5} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$:

$$\partial^\mu J^{(0)}_{\mu 5} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi} C^{(0)} N_c F\tilde{F} + \frac{\sqrt{3}\alpha_s}{4\pi} N_c G\widetilde{G}, \quad (3)$$

The diagonal components of the octet of axial currents
$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$,
$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$
acquire an electromagnetic anomalous term only:

$$\partial^\mu J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(m_u\overline{u}\gamma_5 u - m_d\overline{d}\gamma_5 d) + \frac{\alpha_{em}}{2\pi}C^{(3)}N_c F\tilde{F}, \quad (4)$$

$$\partial^\mu J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d - 2m_s\overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi}C^{(8)}N_c F\tilde{F}. \quad (5)$$

The electromagnetic charge factors $C^{(a)}$ are

$$C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}},$$
$$C^{(8)} = \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}},$$
$$C^{(0)} = \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \quad (6)$$

Axial anomaly: real and virtual photons

- Axial anomaly determines the $\pi^0 \to \gamma\gamma$ decay width: a unique example of a low-energy process, precisely predicted from QCD.
- The dispersive approach to axial anomaly leads to the *anomaly sum rule* (ASR) providing a handy tool to study the meson transition form factors – $M \to \gamma\gamma^*$ (even beyond the factorization hypothesis).

The VVA triangle graph amplitude $T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0|T\{J_{\alpha 5}(0)J_\mu(x)J_\nu(y)\}|0\rangle$ can be presented as a tensor decomposition

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) &= F_1\ \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\ \varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+ F_3\ k_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\ q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\ &+ F_5\ k_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\ q_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma , \end{aligned} \tag{7}$$

$F_j = F_j(p^2, k^2, q^2; m^2)$, $p = k + q$.

Dispersive approach to axial anomaly leads to [Hořejší, Teryaev'95]:

$$\int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)} , \tag{8}$$

$$A_3 \equiv \frac{1}{2} Im(F_3 - F_6),\ \ N_c = 3;$$

ASR and meson contributions

Saturating the l.h.s. of the 3-point correlation function with the resonances and singling out their contributions to ASR we get the (infinite) sum of resonances with appropriate quantum numbers:

$$\pi \sum f_M^a F_{M\gamma} = \int_{4m^2}^{\infty} A_3(s, Q^2; m^2) ds = \frac{1}{2\pi} N_c C^{(a)}, \qquad (9)$$

where the coupling (decay) constants f_M^a:

$$\langle 0 | J_{\alpha 5}^{(a)}(0) | M(p) \rangle = i p_\alpha f_M^a \ , \qquad (10)$$

and form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \to M$ are:

$$\int d^4 x e^{ikx} \langle M(p) | T\{J_\mu(x) J_\nu(0)\} | 0 \rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \qquad (11)$$

- Sum of finite number of resonances decreasing $F_{M\gamma}^{\mathrm{asymp}}(Q^2) \propto \frac{f_M}{Q^2}$- infinite number of states are needed to saturate the ASR (collective effect). [Y.K.,A.Oganesian,O.Teryaev, PLB 695 (2011) 130]

Isovector channel: π^0

$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$, $C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}}$.

- π^0 + higher contributions ("continuum"):

$$\pi f_\pi F_{\pi\gamma}(Q^2) + \int_{s_0}^{\infty} A_3(s, Q^2) = \frac{1}{2\pi} N_c C^{(3)}. \tag{12}$$

The spectral density $A_3(s, Q^2)$ can be calculated from VVA triangle diagram:

$$A_3(s, Q^2) = \frac{1}{2\sqrt{2}\pi} \frac{Q^2}{(Q^2+s)^2}. \tag{13}$$

The pion TFF:

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2}. \tag{14}$$

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2} \tag{15}$$

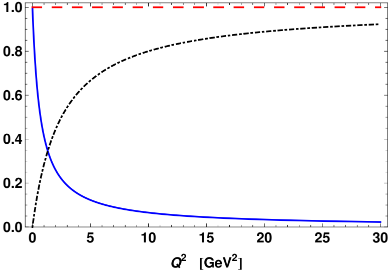
The limit $Q^2 \to \infty$ + pQCD prediction $Q^2 F_{\pi\gamma} = \sqrt{2} f_\pi$ gives

$$s_0 = 4\pi^2 f_\pi^2 = 0.67\, GeV^2$$

– fits perfectly the value extracted from SVZ (two-point) QCD sum rules $s_0 = 0.7\, GeV^2$ [Shifman,Vainshtein,Zakharov'79].
– proves BL interpolation formula[Brodsky,Lepage'81]:

$$F_{\pi\gamma}^{\mathrm{BL}}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{1}{1 + Q^2/(4\pi^2 f_\pi^2)}. \tag{16}$$

Corrections interplay



- The full integral is exact

$$\frac{1}{2\pi} = \int_0^{\infty} A_3(s, Q^2)ds = I_\pi + I_{cont}$$

- The continuum contribution $I_{cont} = \int_{s_0}^{\infty} A_3(s, Q^2)ds$ may have perturbative as well as power corrections.
- $\delta I_\pi = -\delta I_{cont}$: small relative correction to continuum – due to exactness of ASR – must be compensated by large relative correction to the pion contribution!

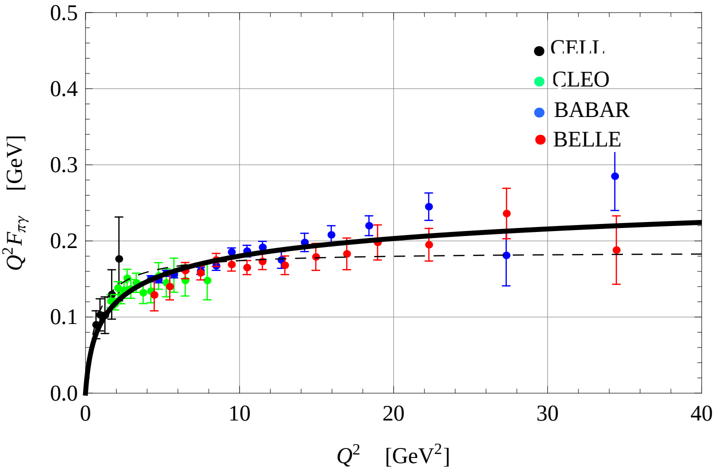
Possible corrections to A_3

- Perturbative two-loop corrections to spectral density A_3 are zero [Jegerlehner&Tarasov'06]
- Nonperturbative corrections to A_3 are possible: vacuum condensates, instantons, short strings.
- General requirements for the correction $\delta I = \int_{s_0}^{\infty} \delta A_3(s, Q^2) ds$: $\delta I = 0$
  - at $s_0 \to \infty$ (the continuum contribution vanishes),
  - at $s_0 \to 0$ (the full integral has no corrections),
  - at $Q^2 \to \infty$ (the perturbative theory works at large Q^2),
  - at $Q^2 \to 0$ (anomaly perfectly describes pion decay width).

$$\delta I = \frac{1}{2\sqrt{2}\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma), \tag{17}$$

$$\delta F_{\pi\gamma} = \frac{1}{\pi f_\pi} \delta I_\pi = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{\lambda s_0 Q^2}{(s_0 + Q^2)^2} (\ln \frac{Q^2}{s_0} + \sigma). \tag{18}$$

Correction vs. experimental data



CELLO+CLEO+BABAR: $\lambda = 0.14$, $\sigma = -2.43$, $\chi^2/n.d.f. = 1.08$

Time-like region: $q^2 > 0 (Q^2 < 0)$ and VMD

The ASR for time-like q^2 is given by the double dispersive integral:

$$\int_0^\infty ds \int_0^\infty dy \frac{\rho^{(a)}(s,y)}{y - q^2 + i\epsilon} = N_c C^{(a)}, \; a = 3, 8. \tag{19}$$

The real and imaginary parts of the ASR read:

$$p.v. \int_0^\infty ds \int_0^\infty dy \frac{\rho^{(a)}(s,y)}{y - q^2} = N_c C^{(a)}, \tag{20}$$

$$\int_0^\infty ds \rho^{(a)}(s, q^2) = 0, \; a = 3, 8. \tag{21}$$

$$ReF_{\pi\gamma}(q^2) = \frac{N_c C^{(3)}}{2\pi^2 f_\pi} \left[p.v. \int_0^{s_3} ds \int_0^\infty dy \frac{\rho^{(a)}(s,y)}{y - q^2} \right] = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 - q^2}.$$

The TFF in the time-like region at $q^2 = s_0 = 0.67$ GeV^2 has a pole, which is numerically close to the ρ meson mass squared, $m_\rho^2 \simeq 0.59$ GeV^2 — **VMD model**.

Octet channel of ASR

ASR in the octet channel:

$$J^{(8)}_{\alpha 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s),$$

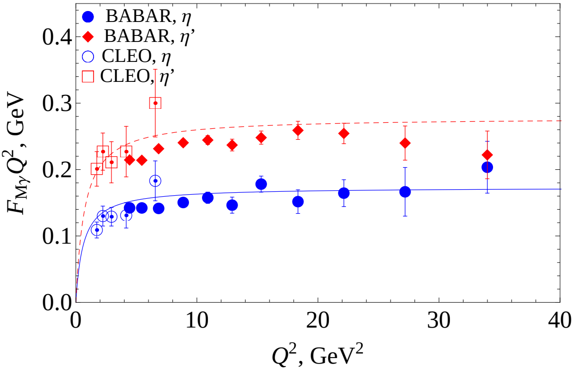
$$f^8_\eta F_{\eta\gamma}(Q^2) + f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\sqrt{6}\pi^2} \frac{s_0^{(8)}}{s_0^{(8)} + Q^2}. \quad (22)$$

- Significant mixing.
- η' decays into two real photons, so it should be taken into account explicitly along with η meson.

ASR at $Q^2 \to \infty$ -continuum threshold $s_0^{(8)}$:

$$4\pi^2((f^8_\eta)^2 + (f^8_{\eta'})^2 + 2\sqrt{2}[f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'}]) = s_0^{(8)} \quad (23)$$

η, η' TFF in the space-like region ($Q^2 > 0$ ($q^2 < 0$))

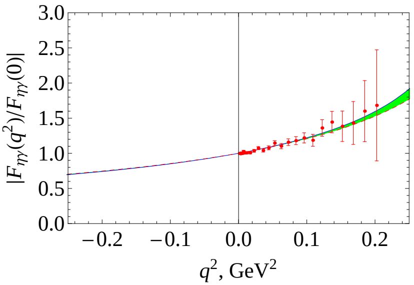


$$F_{\eta\gamma}(Q^2) = \frac{5}{12\pi^2 f_s f_\pi}\frac{s_0^{(3)}(\sqrt{2}f_s\cos\phi - f_q\sin\phi)}{s_0^{(3)}+Q^2} + \frac{1}{4\pi^2 f_s}\frac{s_0^{(8)}\sin\phi}{s_0^{(8)}+Q^2}, \quad (24)$$

$$F_{\eta'\gamma}(Q^2) = \frac{5}{12\pi^2 f_s f_\pi}\frac{s_0^{(3)}(\sqrt{2}f_s\sin\phi + f_q\cos\phi)}{s_0^{(3)}+Q^2} - \frac{1}{4\pi^2 f_s}\frac{s_0^{(8)}\cos\phi}{s_0^{(8)}+Q^2}, \quad (25)$$

where $s_0^{(3)} = 4\pi^2 f_\pi^2$, $s_0^{(8)} = (4/3)\pi^2(5f_q^2 - 2f_s^2)$. [YK, Oganesian, Teryaev Phys.Rev. D87 (2013) 3, 036013]

η TFF in the time-like region vs. data



$$F_{\eta\gamma}(q^2) = \frac{5}{12\pi^2 f_s f_\pi} \frac{s_3(\sqrt{2} f_s \cos\phi - f_q \sin\phi)}{s_3 - q^2} + \frac{1}{4\pi^2 f_s} \frac{s_8 \sin\phi}{s_8 - q^2}, \qquad (26)$$

$s_3 = 4\pi^2 f_\pi^2$, $s_8 = (4/3)\pi^2(5f_q^2 - 2f_s^2)$.

[YK, Oganesian, Teryaev JETP Lett. 99 (2014) 679]

Summary

- Meson TFFs are unique quantities which link (seemingly different) physics concepts: **axial anomaly, mixing and VMD model**.
- The ASR in the isovector channel gives ground for the VMD model in the time-like region.
- A possible new non-OPE correction (e.g. due to short strings) to spectral density is supported by BABAR and not excluded by BELLE data. More accurate data is required for definite conclusions.
- Due to mixing in the $\eta - \eta'$ system, ASR results in "shifted" intervals of dualities of η and η' and gives ground for VMD model for the processes involving η and η' mesons in time-like region.

Thank you for your attention!