Holographic study of the QCD matter under external conditions

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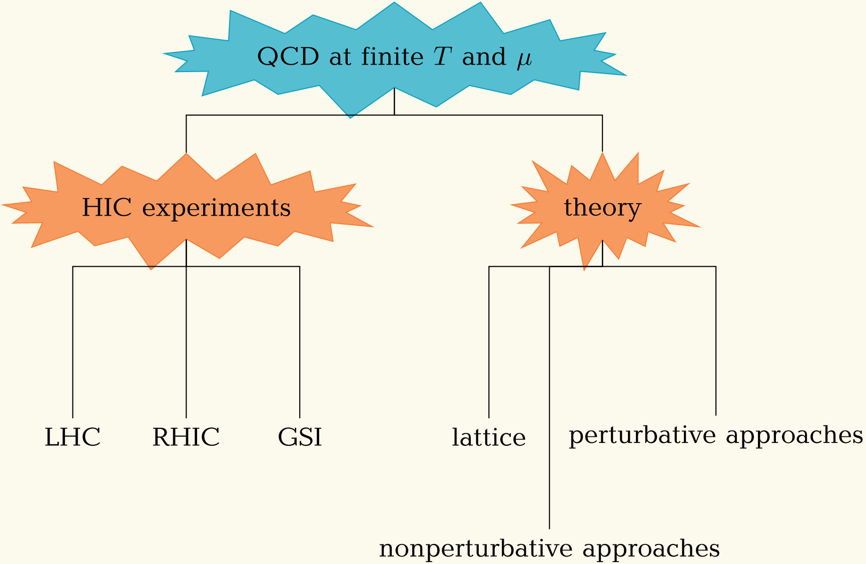
QCD at finite T and μ
HIC experiments
theory
LHC
RHIC
GSI
lattice
perturbative approaches
nonperturbative approaches

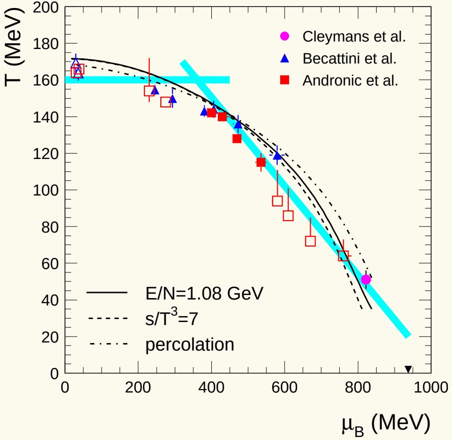

Fig. 1. The decoupling temperatures and chemical potentials extracted by Statistical Model fits to experimental data. The figure is taken from A. Andronic *et al.*, Nucl. Phys. A **837**, 65 (2010).

J.M.Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

IIB string theory on $AdS_5 \times S^5$ in low-energy approximation $\Longleftrightarrow$ $\mathcal{N} = 4$ SYM theory on ∂AdS_5 in $g_{YM} N_c \gg 1$ limit

weakly coupled theories $\Longleftrightarrow$ strongly coupled theories

QCD

AdS/QCD purpose: to describe QCD in large N_c limit by means of 5D dual theory.

Two types of AdS/QCD models:

- top-down – interpretation of the behavior of different string configurations;
- bottom-up – phenomenological approach, allows to build in real QCD properties.

Methods: S.S. Gubser, I.R. Klebanov, A.M.Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)

- $\mathcal{O}(x)$ in $4D$ theory $\Leftrightarrow$ $\phi(x,z)$ in $5D$ dual theory;
- source $\phi_{\mathcal{O}}(x)$ $\Leftrightarrow$ value on the boundary $\phi(x,\epsilon)$;
- $W_{4D}[\phi_{\mathcal{O}}(x)] = S_{5D,eff}[\phi(x,\epsilon)]$ with $\phi(x,\epsilon) = \phi_{\mathcal{O}}(x)$;
- differentiating $S_{5D,eff}$ with respect to $\phi_{\mathcal{O}}$ $\Rightarrow$ QCD Green's functions;
- the poles of the 2-pt correlators $\Rightarrow$ the mass spectrum.

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The action of the theory in the bulk:

$$S = \int d^4x dz \sqrt{-g} e^{-az^2} U^2(b, 0; az^2) \left(-\frac{1}{4g_5^2} F_{MN} F^{MN}\right)$$

- $g = \det g_{MN}$, the anti-de Sitter metric with the radius L is parametrized as: $g_{MN}dx^M dx^N = \frac{L^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$;
- $F_{MN} = \partial_M V_N - \partial_N V_M, \; M, N, = 0, 1, 2, 3, 4$;
- U – the Tricomi confluent hypergeometric function;
- the 5D gauge coupling $g_5^2 = 12\pi^2 L/N_c$

The spectral representation for the two-point correlator of vector currents $\int d^4x e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(q^2)$:

$$\Pi_V(q^2) = -\sum_{n=0}^{\infty} \frac{F_n^2}{q^2 - 4a(n+1+b)}$$

the poles: $m_n^2 = 4a(n+1+b)$ – the mass spectrum with an arbitrary intercept
the residues: $F_n^2 = \frac{2aL}{g_5^2}\left(1 - \frac{b}{n+1+b}\right)$

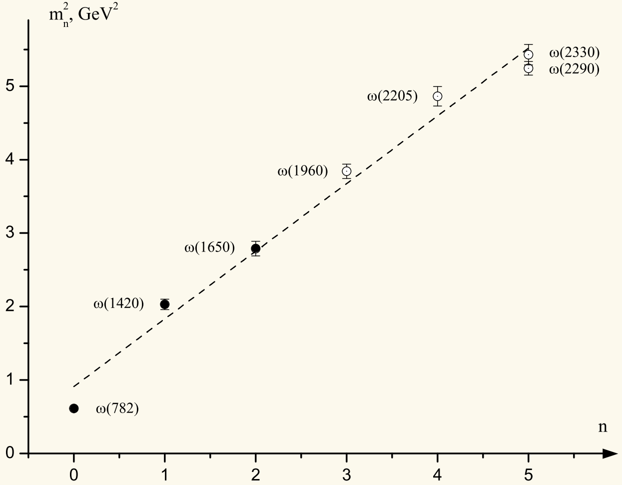

FIG. 2. Matching the ω mesons with radial number n. The well-established states are filled.

Start with adding the Euclidean gravitational part of the action:

$$S = \int d^4x dz \sqrt{g} e^{-az^2} U^2(b, 0; az^2) \left(-\frac{1}{2\kappa^2}(\mathcal{R} + 12/L^2) + \frac{1}{4g_5^2} F_{MN} F^{MN} \right),$$

κ – the gravitational constant, $\mathcal{R}$ – the scalar curvature.

Solution of Einstein and Maxwell equations:

$\Rightarrow$ $A_0 = i(\mu - Qz^2)$, with μ – the quark chemical potential, Q – the quark number density;

$\Rightarrow$ 2 different geometries corresponding to different phases:

(1) thermal charged AdS – confining phase
$ds^2 = \frac{L^2}{z^2}\left(f_{tc}(z) dt^2 + d\vec{x}^2 + \frac{1}{f_{tc}(z)} dz^2 \right)$, where $f_{tc}(z) = 1 + q'^2 z^6$

(2) Reissner-Nordstrem black hole – deconfined phase
$ds^2 = \frac{L^2}{z^2}\left(f_{RN}(z) dt^2 + d\vec{x}^2 + \frac{1}{f_{RN}(z)} dz^2 \right)$, where
$f_{RN}(z) = 1 - (1/z_h^4 + q^2 z_h^2) z^4 + q^2 z^6$

The BH charge and quark number density are connected via:

$$Q = \sqrt{\frac{3g_5^2 L^2}{2\kappa^2}} q.$$

the 1st order Hawking-Page phase transition between different geometries

$\Longleftrightarrow$ $\Delta S = 0$ $\Longleftrightarrow$ (De)confinement transition

The order parameter for the phase transition $\Delta S = \bar{S}_{RN} - \bar{S}_{tc}$:

$$\Delta S = \frac{L^3 V_3}{\kappa^2} \frac{1}{T_{RN}} \left[\frac{a z_h^2 - 1}{(z_h)^4} e^{-a z_h^2} U^2(b, 0; a z_h^2) + \frac{2ab}{z_h^2} e^{-a z_h^2} U(b, 0; a z_h^2) U(b+1, 1; a z_h^2) + \right.$$

$$+ \left(\frac{1}{z_h^4} + q^2 z_h^2 \right) \frac{1}{2\Gamma^2(1+b)} - a^2 F(z_h, \infty) - \frac{q^2}{a} \int_0^{a z_h^2} dt e^{-t} U^2(b, 0; t) +$$

$$\left. + \frac{q'^2}{a} \frac{1 - b + 2b^2 \psi'(1+b)}{\Gamma^2(1+b)} \right],$$

where we call the integral

$$F(x, y) = \int_{a x^2}^{a y^2} \frac{dt}{t} e^{-t} [U^2(b, 0; t) + 4b U(1+b, 1; t) U(b, 0; t) +$$

$$+ 2b^2 U^2(1+b, 1; t) + 2b(1+b) U(2+b, 2; t) U(b, 0; t)],$$

and charges are defined as:

$$q = \sqrt{\frac{2\kappa^2}{3 g_5^2 L^2}} \frac{\mu}{z_h^2},$$

$$q' = \sqrt{\frac{3\kappa^2}{2 g_5^2 L^2}} \frac{a\mu}{1 - b + 2b^2 \psi'(1+b)}.$$

Setting $a,\ b\ \Rightarrow$ position of $z_h \Rightarrow$ curve on the $(T,\,\mu)$ plane.

$$T = -\frac{1}{4\pi}\,\frac{\partial f_{RN}}{\partial z}\bigg|_{z=z_h} = \frac{1}{\pi z_h} - \frac{1}{3\pi}\frac{\kappa^2}{g_5^2 L^2}\mu^2 z_h$$

$$\frac{\kappa^2}{g_5^2 L^2}\,-?$$

The basic concept of AdS/QCD:

duality between the gravitational part of the action and gluodynamics.

$\Rightarrow$ It is natural to fix κ by matching high energy asymptotes of the two-point correlators of glueball currents in the bottom-up model and in QCD: $\kappa^2 = \pi^2 L^3/N_c^2$.

$$\Rightarrow\ \frac{\kappa^2}{g_5^2 L^2} = \frac{1}{12 N_c}$$

If we consider quarks in the adjoint representation of $SU(3)$ (as for supersymmetric QCD) $\Rightarrow\ \frac{\kappa^2}{g_5^2 L^2}\Big|_{adj} = \frac{1}{12}$, it is analogous to having the Veneziano limit $\frac{N_f}{N_c} = \frac{3}{3}$.

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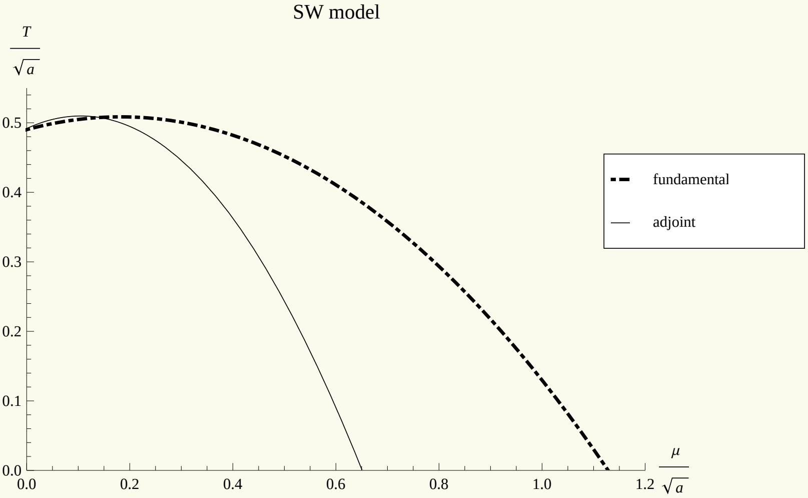

Fig. 3. The phase diagram for dimensionless T and μ.

Consider the case $\mu = 0$. Extracting input parameters from experimental vector spectra, we can predict the deconfinement temperature T_c:

Particle	Radial states	m_n^2, GeV^2	T_c, MeV
ρ	$n = 0, 1, 2$	$1.18(n + 0.61)$	143
ω	$n = 0, 1, 2$	$1.09(n + 0.66)$	149
ρ	$n = 0, 1, 2, 3, 4$	$0.99(n + 0.89)$	207
ω	$n = 0, 1, 2, 3, 4$	$1.03(n + 0.74)$	166
ρ	$n = 0, 1, 2, 4, 5$	$0.88(n + 1.12)$	270
ω	$n = 1, 2, 3, 4$	$0.95(n + 1.04)$	255
-	mean slope[1]	$1.14(n + 1)$	263

Lattice with physical quarks[2]: $150 - 170$ MeV.
Lattice with non-dynamical quarks and $N_c \to \infty$[3]: ~ 250 MeV.
Lattice for $SU(3)$ theory[4]: $260 - 270$ MeV.

[1] D. V. Bugg, Phys. Rept. **397**, 257 (2004).
[2] S. Borsanyi *et al.* [Wuppertal-Budapest Collab.], JHEP **1009**, 073 (2010).
[3] B. Lucini, A. Rago and E. Rinaldi, Phys. Lett. B **712**, 279 (2012)
[4] G. Boyd *et al.* Nucl. Phys. B **469**, 419 (1996); Y. Iwasaki, K. Kanaya, T. Kaneko and T. Yoshie, Nucl. Phys. Proc. Suppl. **53**, 429 (1997)

The phase transition at $T = 0$ is expected for values of the baryon chemical potential $\mu_B = 3\mu_q \approx 1$ GeV.

- about the nucleon mass;
- experimental data converge to $\mu_B \simeq 1.1 \div 1.2$ GeV;
- the Nambu–Jona-Lasinio model[5]: $\mu_B \simeq 1.05$ GeV.

In the SW model

Taking the universal slope $4a = 1.14$ GeV^2 (and $b = 0$), we get at $T = 0$ the position of the critical baryon chemical potential for different quark representations:

$$\text{fundamental representation:} \quad \mu_B \simeq 1.8 \text{ GeV},$$
$$\text{adjoint representation:} \quad \mu_B \simeq 1.0 \text{ GeV}.$$

Matching the experimental ρ meson mass $4a = 776^2$ MeV^2:

$$\text{fundamental representation:} \quad \mu_B \simeq 1.3 \text{ GeV},$$
$$\text{adjoint representation:} \quad \mu_B \simeq 0.75 \text{ GeV}.$$

[5] S. P. Klevansky, Rev. Mod. Phys. **64**, no. 3 (1992).

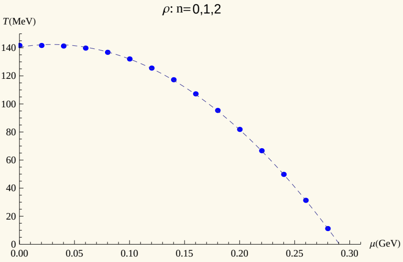

Fig. 4. The phase diagram for the first radial excitations of the ρ meson.

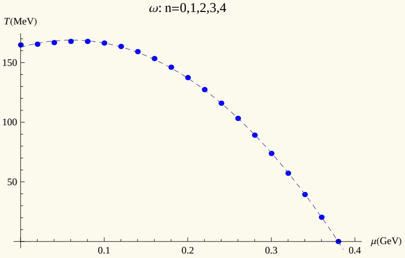

Fig. 5. The phase diagram for the first five radial excitations of the ω meson.

Endpoints of the phase diagrams on the $(T,\ \mu_B)$ plane:

ρ **meson**, $n = 0, 1, 2$ – $(T_c = 143$ MeV, $\mu_B = 0)$ and $(T = 0,\ \mu_B \simeq 0.9$ GeV).

ω **meson**, $n = 0, 1, 2, 3, 4$ – $(T_c = 166$ MeV, $\mu_B = 0)$ and $(T = 0,\ \mu_B \simeq 1.1$ **GeV**).

This work is devoted to the determination of the QCD phase diagram on the $(T,\ \mu_B)$ plane within the bottom-up holographic approach.

- We have analyzed the transition of the hadron matter to the deconfined phase at $T_c \Rightarrow$ agreement with lattice results for pure gluodynamics;
- We have shown that predictions of critical T_c and μ_B become ambiguous because of lack of reliable experimental data on the radially excited light mesons;
- We have considered the effect of the external quark medium suppressed as $1/N_c$. Way out: the Veneziano limit or quarks in the adjoint representation. Nevertheless at $N_c = 3$ we get a rather realistic phase diagram.
- AdS/QCD passes one more test as it gives predictions in good agreement with lattice and experimental ones.