Karlsruhe Institute of Technology

Stability of the Standard Model ground state
A precision analysis

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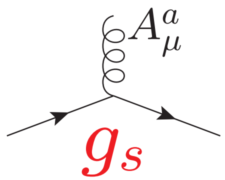
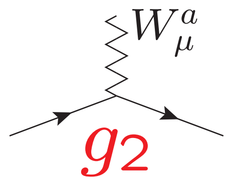
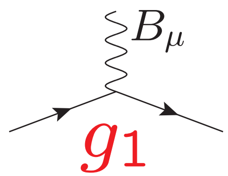
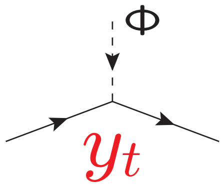
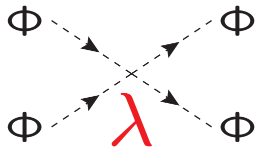
The Standard Model: $\mathbf{SU(3)_c \times SU(2)_L \times U(1)_Y}$

- gauge couplings: QCD: 

Electroweak: 



- Yukawa couplings and Higgs self-interaction: 

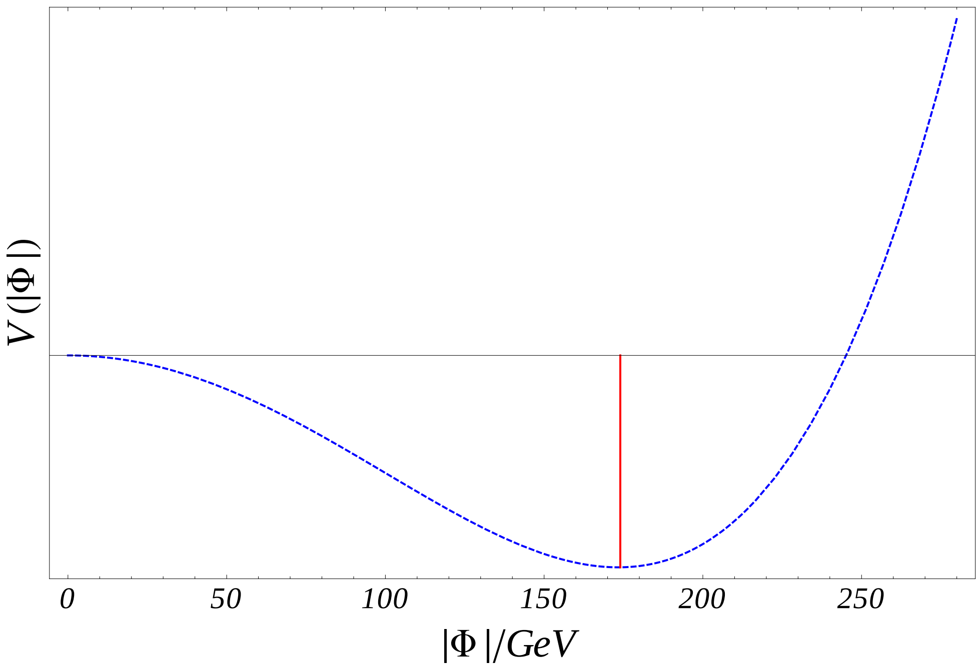


Spontaneous Symmetry Breaking

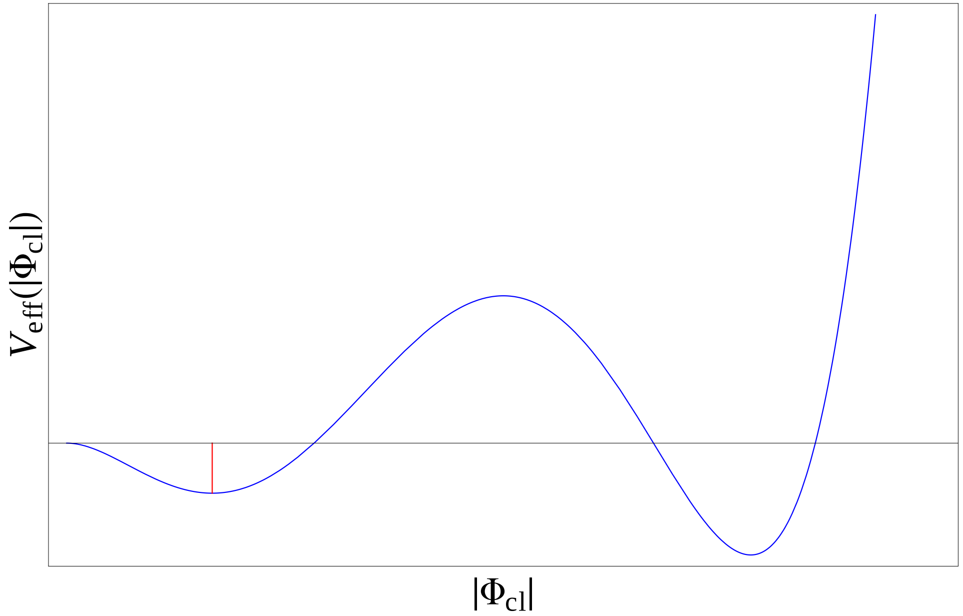
Classical Higgs potential:

$$V(|\Phi|) = m^2\, \Phi^*\Phi + \lambda\, (\Phi^*\Phi)^2$$

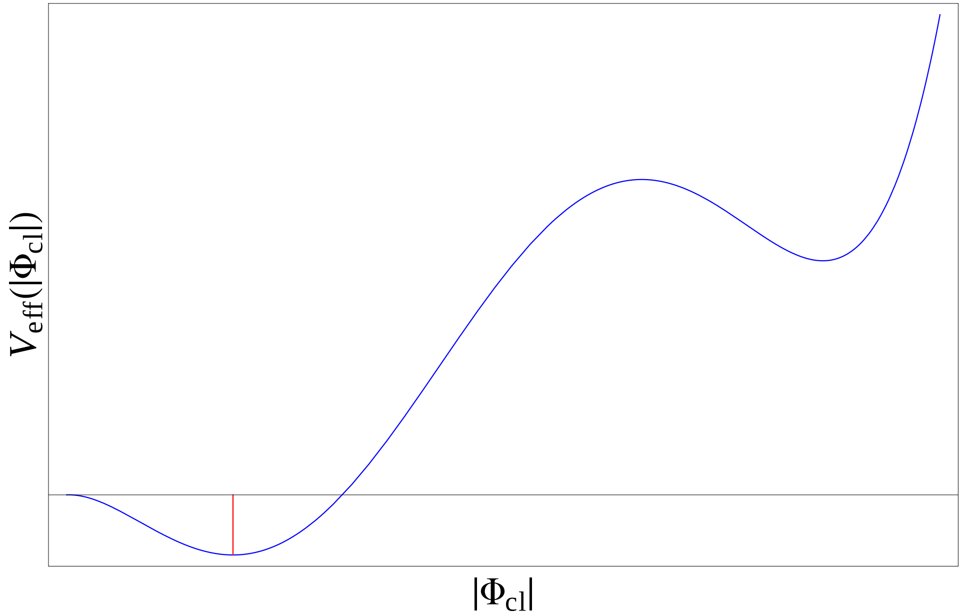
$$\begin{aligned} \Phi_{\text{cl}} &= \langle 0|\, \Phi(x)\, |0\rangle \\ |\Phi_{\text{cl}}| &= \frac{v}{\sqrt{2}} \neq 0 \\ v &\approx 246.2 \text{ GeV} \end{aligned}$$



Stability of the ground state



unstable or metastable



stable

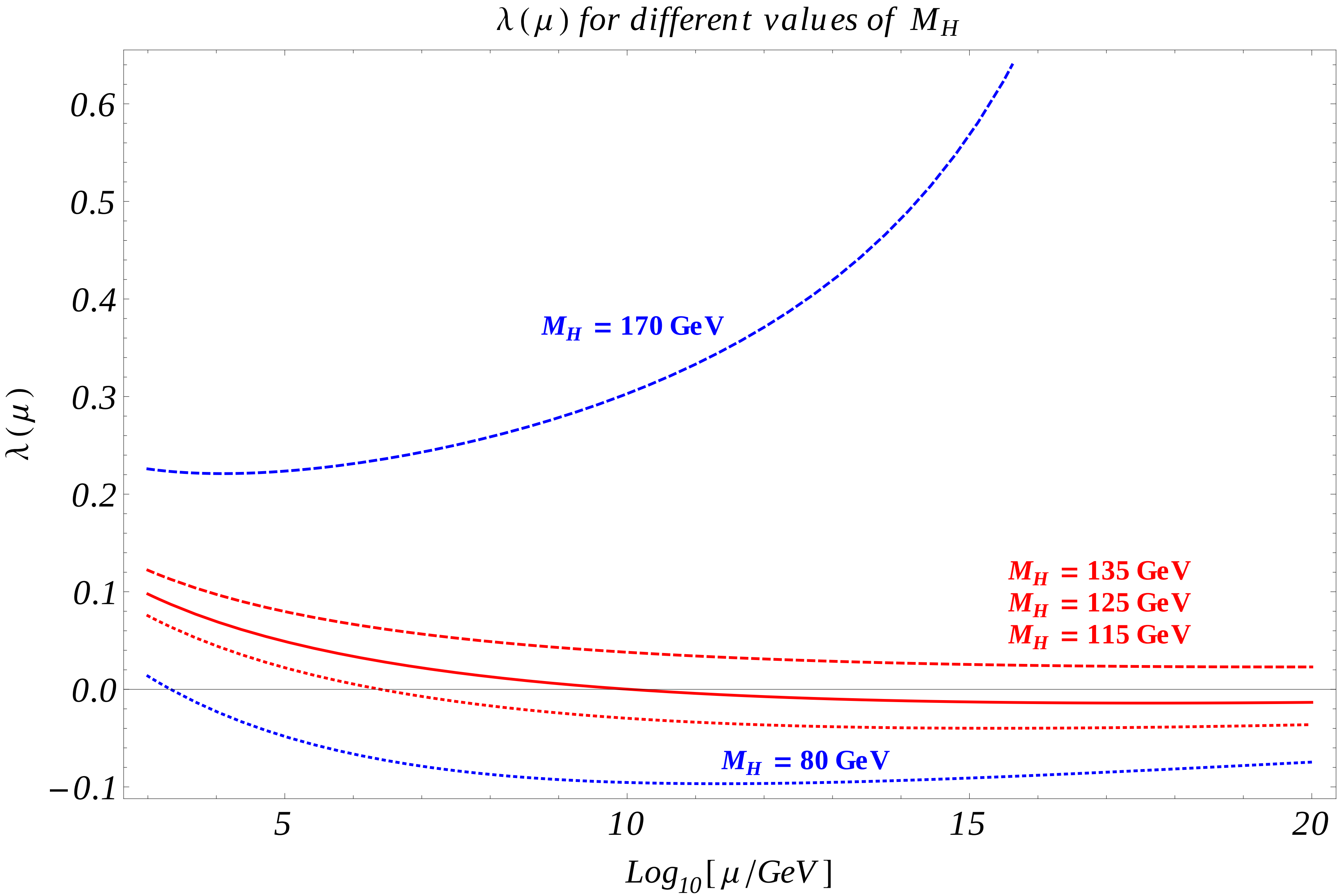
QFT: Radiative corrections → $V_{eff}\left(\lambda(\Lambda), g_i(\Lambda), y_t(\Lambda), \ldots\right)[\Phi(\Lambda)]$ [Coleman, Weinberg]

(Λ: scale up to which the SM is valid, starting scale for running e.g. $\mu_0 = M_t$)

For $\Phi \sim \Lambda \gg v$: $V_{eff}[\Phi] \approx \lambda(\Lambda)\Phi(\Lambda)^4$ [Altarelli, Isidori; Ford, Jack, Jones]

Stability of SM vacuum $\Leftrightarrow$ $\lambda(\Lambda) > 0$ [Cabibbo; Sher; Lindner; Ford]

Evolution of λ up to $\Lambda \sim M_{\text{Planck}}$



Evolution of couplings $X \in \{\lambda, g_1, g_2, g_s, y_t\}$

β-functions: $$\boxed{\mu^2 \frac{d}{d\mu^2} X(\mu^2) = \beta_X[\lambda(\mu^2), y_t(\mu^2), g_i(\mu^2), \ldots]}$$

μ: energy scale of a given physical process

$\Rightarrow$ **Coupled system of differential equations with initial conditions**

Three-loop β-functions in the SM

- for gauge couplings g_1, g_2, g_s:
  [Mihaila, Salomon, Steinhauser (2012); Bednyakov, Pikelner, Velizhanin (2012)]

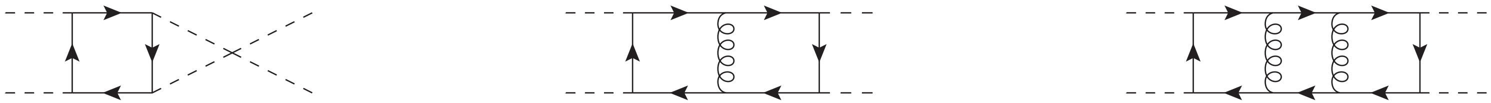
- for Yukawa couplings y_t, y_b, y_τ, etc.:
  [Chetyrkin, MZ (2012); Bednyakov, Pikelner, Velizhanin (2013)]

- for the Higgs self-coupling λ (and the mass parameter m^2):
  [Chetyrkin, MZ (2012 and 2013); Bednyakov, Pikelner, Velizhanin (2013)]

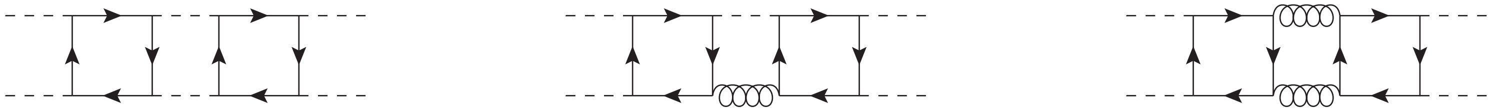
Calculation of $\beta_\lambda(\lambda, y_t, g_s, g_2, g_1)$

$\propto \frac{y_t^4}{(16\pi^2)}$ $\qquad$ $\propto \frac{\lambda^2}{(16\pi^2)}$

More loops $\Rightarrow$ more precision



$\propto \frac{y_t^4 \lambda}{(16\pi^2)^2}$ $\qquad$ $\propto \frac{y_t^4 g_s^2}{(16\pi^2)^2}$ $\qquad$ $\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$



$\propto \frac{y_t^8}{(16\pi^2)^3}$ $\qquad$ $\propto \frac{y_t^6 g_s^2}{(16\pi^2)^3}$ $\qquad$ $\propto \frac{y_t^4 g_s^4}{(16\pi^2)^3}$

Challenges:

- $\mathcal{O}(10^6)$ diagrams at 3 loops
- Treatment of γ_5 in $D = 4 - 2\varepsilon$ ['t Hooft, Veltman],
- IR divergencies [Chetyrkin, Misiak, Münz]

Results: $$\mu^2 \frac{d}{d\mu^2}\lambda(\mu) = \beta_\lambda = \sum_{n=1}^{\infty} \frac{1}{(16\pi^2)^n}\beta_\lambda^{(n)}$$ (in the $\overline{\text{MS}}$-scheme)

$$\beta_\lambda^{(1)} = -y_t^4 3 + y_t^2\lambda 6 - \lambda g_2^2\frac{9}{2} + \lambda^2 12 + g_2^4\frac{9}{16} - \lambda g_1^2\frac{3}{2} + g_1^2 g_2^2\frac{3}{8} + g_1^4\frac{3}{16} + \lambda y_b^2 6 + \lambda y_\tau^2 2 - y_b^4 3 - y_\tau^4$$

$$\beta_\lambda^{(2)} = -g_s^2 y_t^4 16 + y_t^6 15 + g_s^2 y_t^2\lambda 40 - y_t^2\lambda^2 72 + y_t^2\lambda g_2^2\frac{45}{4} + +\lambda^2 g_2^2 54 - \lambda^3 156 + \ldots$$

$$\begin{aligned}\beta_\lambda^{(3)} =& g_s^2 y_t^6\left(-38 + 240\zeta_3\right) + y_t^8\left(-\frac{1599}{8} - 36\zeta_3\right) + g_s^4 y_t^4\left(-\frac{626}{3} + 32\zeta_3 + 40N_g\right) \\ &+ g_s^2 y_t^4\lambda\left(895 - 1296\zeta_3\right) + g_s^4 y_t^2\lambda\left(\frac{1820}{3} - 48\zeta_3 - 64N_g\right) + y_t^4\lambda^2\left(\frac{1719}{2} + 756\zeta_3\right) \\ &+ y_t^6 g_2^2\left(\frac{3411}{32} - 27\zeta_3\right) + y_t^6\lambda\left(\frac{117}{8} - 198\zeta_3\right) + \ldots\end{aligned}$$

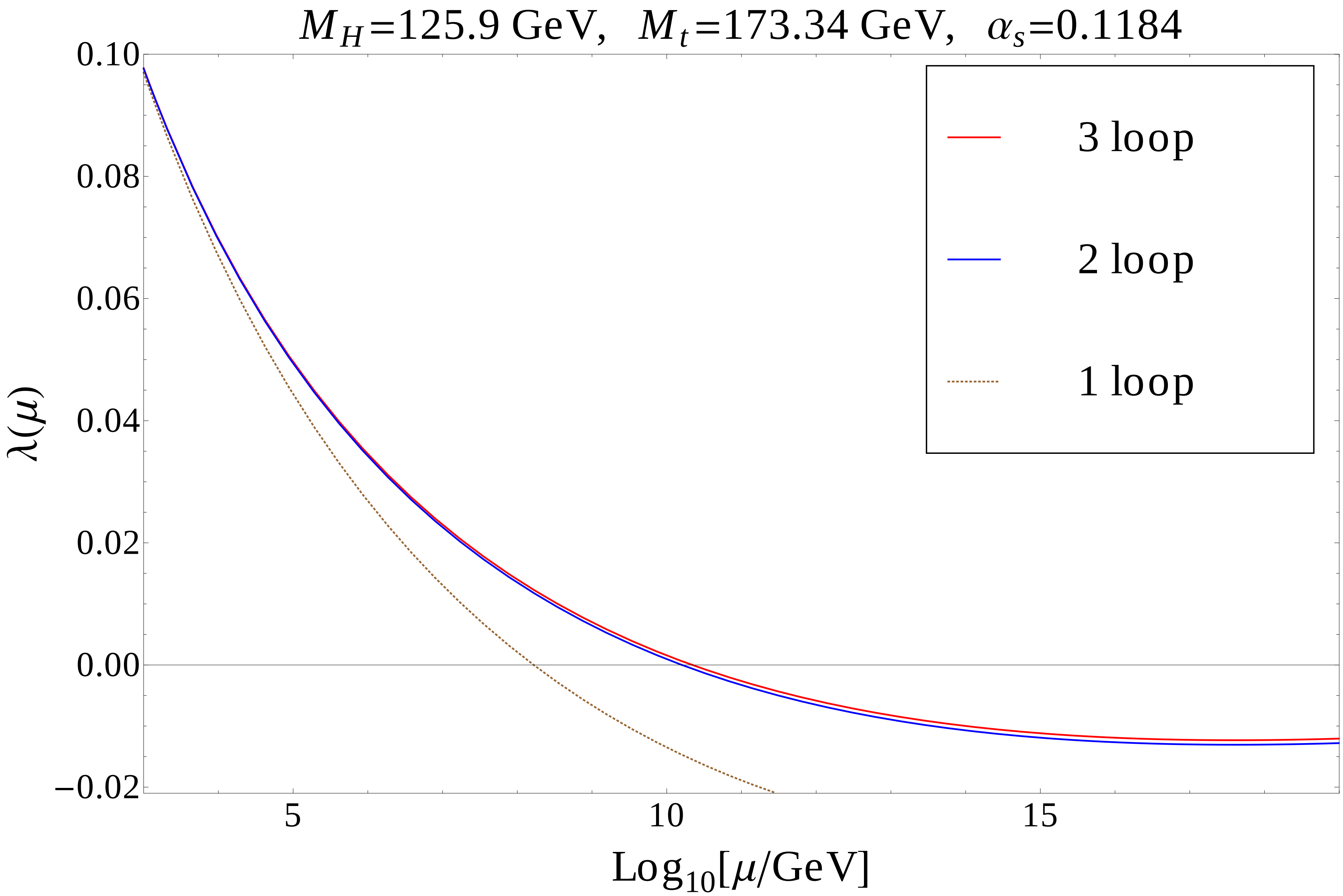
JHEP 1206 (2012) 033

JHEP 1304 (2013) 091

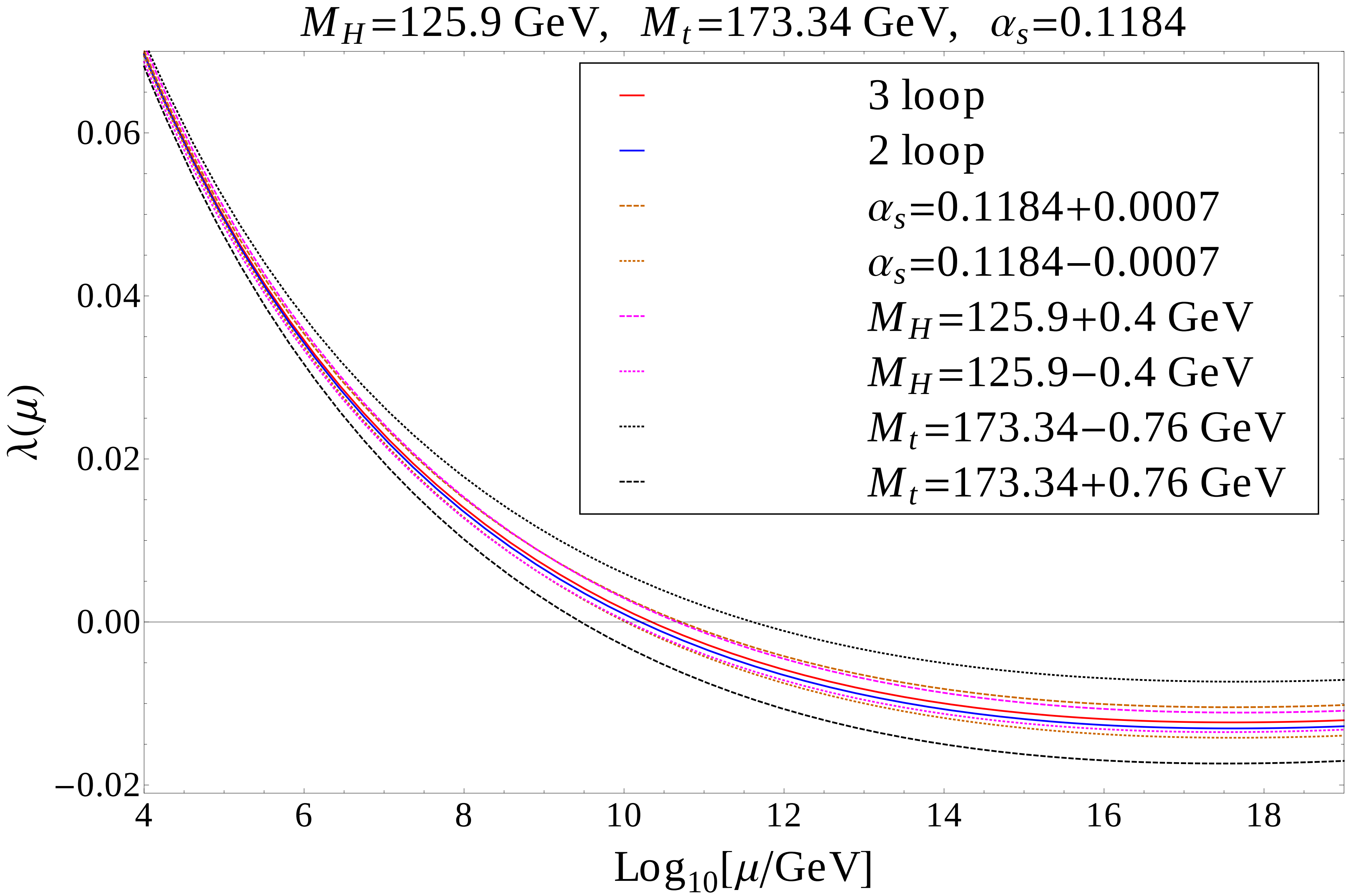
Starting values for couplings

[Sirlin, Zucchini; Hempfling, Kniehl; Jegerlehner et al; Bezrukov et al; Buttazzo et al]

From experimental data:		$\overline{\text{MS}}$ parameters:	
$M_t \approx 173.34$ GeV		$g_s(M_t) \approx 1.16$	$y_t(M_t) \approx 0.94$
$M_H \approx 125.9$ GeV	$\Rightarrow$	$g_2(M_t) \approx 0.65$	$g_1(M_t) \approx 0.36$
$\alpha_s \approx 0.1184$		$\lambda(M_t) \approx 0.13$	



M_H=125.9 GeV, M_t=173.34 GeV, α_s=0.1184
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
$\lambda(\mu)$
5
10
15
$\mathrm{Log}_{10}[\mu/\mathrm{GeV}]$
3 loop
2 loop
1 loop


M_H=125.9 GeV, M_t=173.34 GeV, α_s=0.1184
3 loop
2 loop
α_s=0.1184+0.0007
α_s=0.1184−0.0007
M_H=125.9+0.4 GeV
M_H=125.9−0.4 GeV
M_t=173.34−0.76 GeV
M_t=173.34+0.76 GeV
$\lambda(\mu)$
0.06
0.04
0.02
0.00
−0.02
4
6
8
10
12
14
16
18
Log$_{10}$[μ/GeV]

Summary

- Stability of SM vacuum $\leftrightarrow$ $\boxed{\lambda > 0}$

- 3 loop β_λ effect smaller than experimental uncertainty.

- 3 loop β_λ result improves stability.

- Vacuum state at the electroweak scale v seems not to be the absolute minimum of the effective Higgs potential for the SM up to M_{Planck}.

- However, there are large experimental uncertainties!
  Largest uncertainty: $M_t \Rightarrow$ Good reason for a linear e^+e^- collider.

$\Rightarrow$ Question of vacuum stability remains open.