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New physics is needed to understand black holes

or

Renormalization of the gravitational force in analogy with the electroweak theory

Gerard 't Hooft

Erice, School of Subnuclear Physics June 26, 27 2015

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or more generally,

to understand the small-distance structure of quantized general relativity when we approach the Planckian regime.

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to understand the small-distance structure of quantized general relativity when we approach the Planckian regime.

This new physics is

- exact, spontaneously broken, local, conformal invariance $(++)$
- advanced approaches to indefinite metric fields $(-)$
- deterministic theory of quantum mechanics $(+)$

Intro

Physics as a science could not exist without **symmetries**

e.g. Galileï invariance.

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$\rightarrow$ Lorentz invariance

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Poincaré invariance:

If you know Nature's laws at one space-time point, you know them everywhere.

Then why is physics still so difficult?
answer:

If we know how two particles collide, we still do not know how they collide at higher relative energies.

This is because a symmetry is missing:

How do we perform scale transformations?

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Suppose scale transformations were an exact symmetry ...

if we knew how the **vacuum** transforms under a scale transformation;

scale transformation symmetry is *spontaneously broken*

However, we also have gravity. Therefore
we need *local* spontaneously broken scale invariance.
That is:

local, spontaneously broken, conformal invariance:

$$g_{\mu\nu} \rightarrow \Omega(\vec{x}, t)\, g_{\mu\nu}\,.$$

See an essay submitted to the Gravity Research Foundation

The electroweak theory (summary)

- The Fermi interaction (= the large distance limit)

$\longrightarrow$ Dirac monopoles are singular

- Auxiliary fields
- Divergences and counter terms
- Adding the kinetic terms
- The local symmetry algebra

- Result: **A renormalizable theory**

$\longrightarrow$ Magetic monopoles are now regular solitons

(in extended, grand unified theories)

Gravity (summary)

- The Einstein Hilbert action (= the large distance limit)

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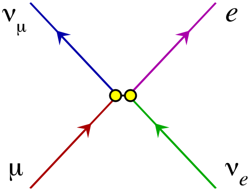
The electroweak theory



The fundamental Fermi interaction:

$\mathcal{L}^F = \frac{1}{2} g_F^2 \, J_\mu J_\mu \; ; \qquad J_\mu = \overline{\psi}\gamma_\mu\psi \; .$

Auxiliary field: A_μ:

$\mathcal{L} = -\frac{1}{2} A_\mu^2 + g_F A_\mu J_\mu \; . \qquad$ Solve: $A_\mu = g_F \, J_\mu \; \rightarrow \; \mathcal{L} = \mathcal{L}^F$

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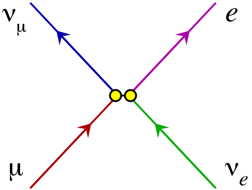

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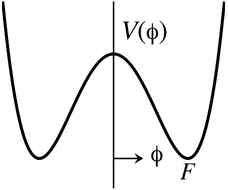
Second auxilary field: ϕ:

$$V(\phi) = \tfrac{1}{2}\lambda_\phi(\phi^2 - F^2)^2$$

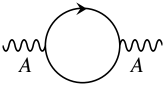
$$\tfrac{1}{2}A_\mu^2 \rightarrow \tfrac{1}{2}A_\mu^2\phi^2$$

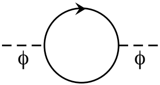


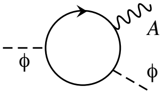
Fermion mass terms: $-\lambda_Y \, \overline{\psi} \, \phi \, \psi$

Divergences and counter terms

The only dynamical fields are the ψ fields. Divergent diagrams:







$\sim\; k^2\, A_\mu^2$ $\qquad$ $\sim k^2\,\phi^2$ $\qquad$ $\sim k\,\phi^2\,A_\mu$

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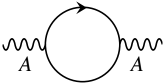

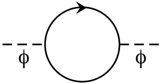

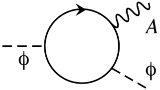

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$$-\tfrac{1}{4} F_{\mu\nu} F_{\mu\nu} \qquad\qquad -\tfrac{1}{2}(\partial_\mu \phi)^2 \qquad - A_\mu\,\phi\,\partial_\mu\phi$$

or: $\qquad -\tfrac{1}{2}(D_\mu\phi)^2\ , \qquad D_\mu = \partial_\mu + g_F A_\mu$

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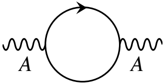

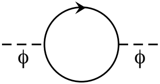

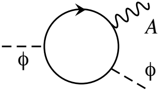

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All terms that can arise as divergences must be added to $\mathcal{L}$.

The new terms all vanish as $k \to 0$

One has to be aware of all possible local symmetries.

This leads to the Yang-Mills theory.

Exactly the same program can be applied to the gravitational force!

But it is not trivial. One has to choose the right auxiliary fields. And the right algebra must still be guessed.

The following attempt appears to be successful.

$$\mathcal{L} \;=\; \mathcal{L}^{EH} \;+\; \mathcal{L}^{\mathrm{matter}} \qquad + \mathcal{L}^{\mathrm{kin}}$$

$$\mathcal{L}^{EH} \quad = \; \tfrac{1}{16\pi G}\sqrt{-g}\,(R-2\Lambda)$$

$$\mathcal{L}^{\mathrm{matter}} = \; \mathcal{L}^{YM}(A) + \mathcal{L}^{\mathrm{bos}}(A,\,\phi,\,g_{\mu\nu}) + \mathcal{L}^{\mathrm{ferm}}(A,\,\psi,\,\phi,\,g_{\mu\nu})$$

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Now, introduce auxiliary fields. Write:

$$g_{\mu\nu} = \omega^2(\vec x, t)\,\hat g_{\mu\nu}\ . \qquad \mathcal{L} = \mathcal{L}(\omega,\,\hat g_{\mu\nu},\,A_\mu,\psi,\phi)$$

"Trivial" local symmetry:

$$\hat g_{\mu\nu} \rightarrow \Omega(\vec x, t) g_{\mu\nu}\ , \quad \omega \rightarrow \Omega^{-1}\omega\ , \quad \phi \rightarrow \Omega^{-1}\phi\ , \quad \psi \rightarrow \Omega^{-3/2}\psi\ .$$

this is a local, conformal ivariance.

$$\mathcal{L}^{EH} = \sqrt{-\hat{g}}\left(\frac{1}{16\pi G}(\omega^2\,\hat{R} + 6\hat{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega) \;-\; \frac{\Lambda}{8\pi G}\,\omega^4\right)$$

$$\mathcal{L}^{\rm matter} \;=\; -\,\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \,+$$

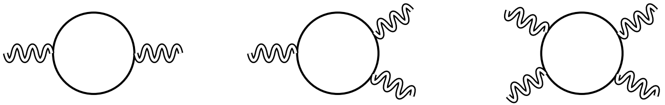
$$\sqrt{\hat{g}}\left(-\frac{1}{2}\hat{g}^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{2}m^2\omega^2\phi^2 - \frac{1}{12}\hat{R}\phi^2 - \frac{\lambda}{8}\phi^4\right) - \mathcal{L}^{\rm ferm}$$

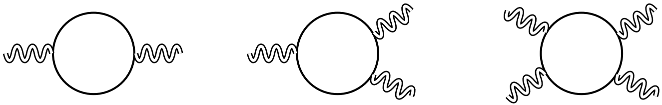
This seems to be a perfectly renormalizable Lagrangian!

Except, there is no kinetic term for the field $\hat{g}_{\mu\nu}$ ...

(The Einstein-Hilbert term entirely morphed
into the kinetic terms for our new ω field)

Indeed, there are still divergences in the pure gravity diagrams, for which no counter terms are to be found in the above Lagrangian:



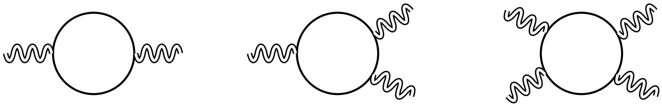


All these diagrams diverge quartically: $\sim k^4\,(\delta\hat{g}_{\mu\nu})^n$

Only one term exists that obeys local conformal invariance: the Weyl term:

$$\mathcal{L}^{\rm kin} = -\frac{\lambda^W}{2}\, C_{\mu\nu\alpha\beta}\, C_{\mu\nu\alpha\beta} \quad \rightarrow \quad -\frac{\lambda^W}{4}(\partial^2 \hat{g}^{\rm transverse}_{\mu\nu})^2$$

(contracted using $\hat{g}^{\mu\nu}$, while ω drops out)



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According to the rule, which was perfectly valid for the SM, every counter term that is needed in the Lagrangian, must also be put in the "bare" Lagrangian. That certainly also holds for kinetic terms. If we accept $\mathcal{L}^W$ as a kinetic term for gravity, the theory becomes renormalizable.

The $1/\lambda^W$ expansion is the usual perturbation expansion

However,

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this kinetic term is not of the usual form. It has $(k)^4$ terms, *including terms of order* $(k_0)^4$.

Consequence:

negative metric states

This we postpone for a moment

Good News:

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Black holes

With the kinetic terms in place, solutions that used to have singularities, now may become regular.
This can also be due to the *enhanced local symmetry* (here: conformal symmetry)

A decaying black hole has an information problem: where do the microstates go when it decays?

An observer falling in experiences a horizon when the black hole still has its original mass M.
An outside observer sees the mass M shrink to 0 while the ingoing observer is still lingering on the horizon.

Who is right?

Answer: they are both right and both wrong:
the mass is not conformally invariant, or,
they may be using *different vacuum expectation values* for the ω field, because they define *the vacuum state* differently.

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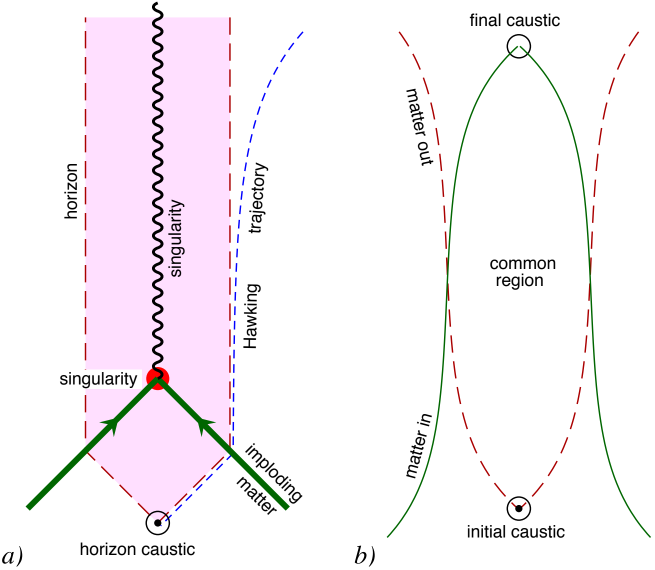
The $\hat{g}_{\mu\nu}$ metric is

$$\mathrm{d}s^2 = M^2(\tilde{t})\left(-\mathrm{d}t^2(1-\tfrac{2}{r}) + \frac{\mathrm{d}r^2}{1-2/r} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)\right)$$

$M^2(\tilde{t})$ is different for different observers: they use a different $\hat{g}_{\mu\nu}$.
They are allowed to, if they also use a different ω field.

For the outside observer, there is no horizon. Of course (s)he sees no singularity.

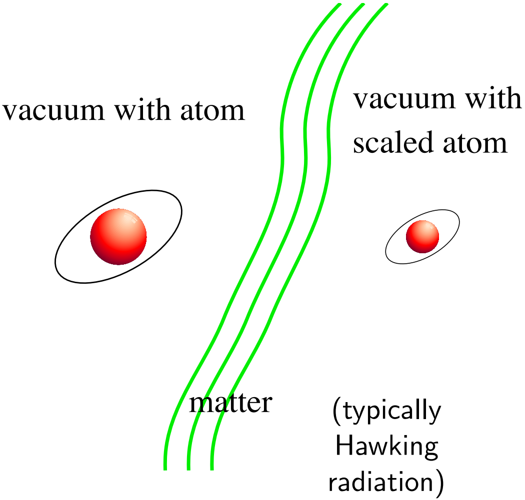
conformally symmetric black hole is a *locally regular soliton*:



Actually, the common region does not exist – shrinks to zero by a conformal transformation that becomes extreme in the limit of classical black holes.

Important ne feature:
the ω field can go to zero without prducing physical singularities

Local conformal symmetry:



More good news: Gravity has become renormalizable

But that is not all . . . !

The local symmetry only works if the anomalies cancel out.

Here, *all conformal anomalies must cancel out*

All couplings must be scale-invariant: We must be at a fixed point of the ren. group, or:

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All β functions must be zero

There are as many β functions as there are physical constants. So: as many equs. as there are "unknown" constants.

<u>All constants</u>, including all masses, cosmological constant, and Weyl constant, must be <u>computable</u>.

The only unknown is the SM algebra.

Bad news:

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Most algebras, such as today's SM, have no physically acceptable fixed point(s)

Resolution? Interesting algebras, such as $SU(N)$ and $SO(N)$ at sufficiently large N may have fixed points in the perturb. regime.

The coupling constant expansion then coincides with a $1/N$ expansion.

Technically complex.

Also not solved:

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Technically complex.

Also not solved: **the hierarchy problem**

Other bad news:

Other bad news: *negative metric*

Weyl^2 action, together with *EH* action, with $\lambda^W \equiv 1/M^2$, gives:

• $1/(k^2 - i\varepsilon)$ pole: 2 degrees of freedom, helicity ± 2:
the spin 2 massless graviton;

• $1/(k^2 + M^2 - i\varepsilon)$ pole: 5 d.o.f.: helicities ± 2, ± 1, and 0.
gravitello: *a massive spin 2 particle, with negative metric:*

$$1/k^2 - 1/(k^2 + M^2) = 1/(k^2 + \lambda^W k^4)$$

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Note, that conformal gravity differs in an essential way from ordinary field theories: *Energy is not conformally invariant*

And the metric $\hat{g}_{\mu\nu}$ is not locally observable . . . .

Therefore, stability, and unitarity, have to be approached differently.

In perturbation theory: (*back to original $g_{\mu\nu}$ field*)

$$\mathcal{L}^{\rm kin} = \mathcal{L}^{EH} - \frac{\lambda^W}{2}\, C_{\mu\nu\alpha\beta}\, C_{\mu\nu\alpha\beta} \rightarrow$$

$$-\frac{1}{4}(\partial g_{\mu\nu}^{\rm trtr})^2 - \frac{\lambda^W}{4}(\partial^2 g_{\mu\nu}^{\rm trtr})^2 \quad \text{(trtr = traceless, transverse)}$$

Field theory of "wrong metric" theories:

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Field theory of "wrong metric" theories: $(M^2 \equiv 1/\lambda)$

$$\begin{aligned}
\mathcal{L} &= -\tfrac{1}{2}(\partial g)^2 - \tfrac{1}{2}\lambda(\partial^2 g)^2 + J\, g\ . \\
&= -\tfrac{1}{2}(\partial g)^2 - \tfrac{1}{2}{\phi_2}^2/\lambda + J\, g \qquad\qquad + \eta(\phi_2 - \lambda\partial^2 g)\ . \\
&\quad \eta = \phi_2/\lambda\ , \qquad g \equiv \phi_1 + \phi_2\ , \qquad \phi_2 \to \lambda\partial^2 g\ . \\
\mathcal{L} &= -\tfrac{1}{2}(\partial\phi_1)^2 + \tfrac{1}{2}(\partial\phi_2)^2 + \tfrac{1}{2}M^2\phi_2^2 + J\,(\phi_1 + \phi_2)\ .
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\end{aligned}$$

The gravitello field ϕ_2 has wrong metric.

Also the field ω would have wrong metric, but that is a ghost particle for the local conf. symm.

Identify: $\phi_2 = \lambda^W C_{\alpha\beta\mu\nu}\ ,\qquad \phi_1 = (g - \phi_2)^{\rm trtr}$

Negative metric in the propagator would give contributions of the form

$$S\sum(\,|\mathrm{light}\rangle\langle\mathrm{light}| - |\mathrm{heavy}\rangle\langle\mathrm{heavy}|\,)S^\dagger = \mathbb{I}\ ,$$

if we use the wrong normalization of states.

This can be done better, in principle. Look at the essentials:

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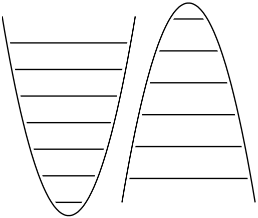
$$H = |\vec{k}|\,(p_1^2 + x_1^2) - \sqrt{\vec{k}^{\,2} + m^2}\,(p_2^2 + x_2^2) \quad \equiv \quad A\,a^\dagger a - B\,b^\dagger b\ + C$$



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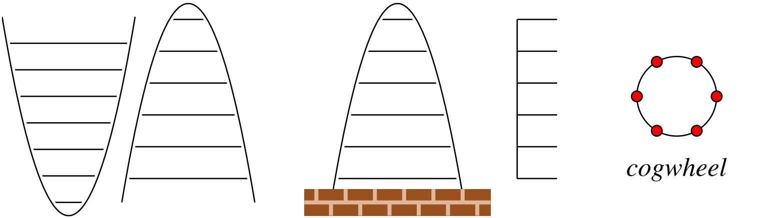


See my Erice lectures last year

Turning an harmonic oscillator upside-down:

replace $b \leftrightarrow b^\dagger$

This has the effect of making both x and p purely imaginary:

$x \rightarrow ix\,,\;\; p \rightarrow ip$

Or: the field of the gravitello (high frequency modes ϕ_2 of $\hat{g}_{\mu\nu}$) must be chosen purely imaginary

just as the field ω describing the conformal part of $g_{\mu\nu}$

The *cogwheel model* (with both upper and lower bounds on the energy spectrum) has x and p lie on the unit circle in the complex plane.

This is the case where quantum mechanics becomes *deterministic*.

See my Erice lectures last year

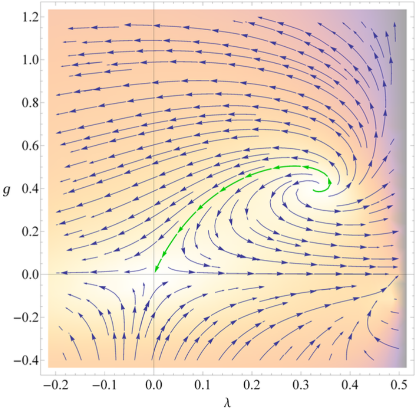
However, the metric problem has not yet been solved this way.

The ϕ_2 field couples with imaginary coupling constant to the matter-sources.

This still gives violation of unitarity.

Conjecture: conformally invariant matter might allow us to handle this situation.

Connection to the *asymptotic safety* idea (Weinberg)



At the non-trivial fixed point, this theory is also controlled by effective operators with dimension 4.

The Landau ghost in this theory is equivalent to our gravitello.

What we added is the possibility to do perturbation expansions.

But beware, we also have to go to the conformal fixed point!

Conclusions

Gravity should be handled as a candidate – renormalizable theory.

Local conformal symmetry then appears to be an exact, but spontaneously broken, symmetry.

Atoms, and all other particles obtain their physical sizes only *relative to a field* ω .

$(m \to m/\omega \ , x \to x\omega, \quad$ small distance limit now is $\omega \to 0$).

Normally, ω has expectation value $\langle\,\omega\,\rangle = 1$.

Black holes are now ordinary solitons, just as magnetic monopoles.

Price: theory must be at its conformal fixed point: all β functions vanish.

For that, the SM algebra must be modified, e.g. in a GUT.

The indefinite metric problem can be addressed, but has not yet been solved in a satisfactory way.

It is suspected that local conformal symmetry is essential in the resolution of this problem.

$\sim$ *The End* $\sim$