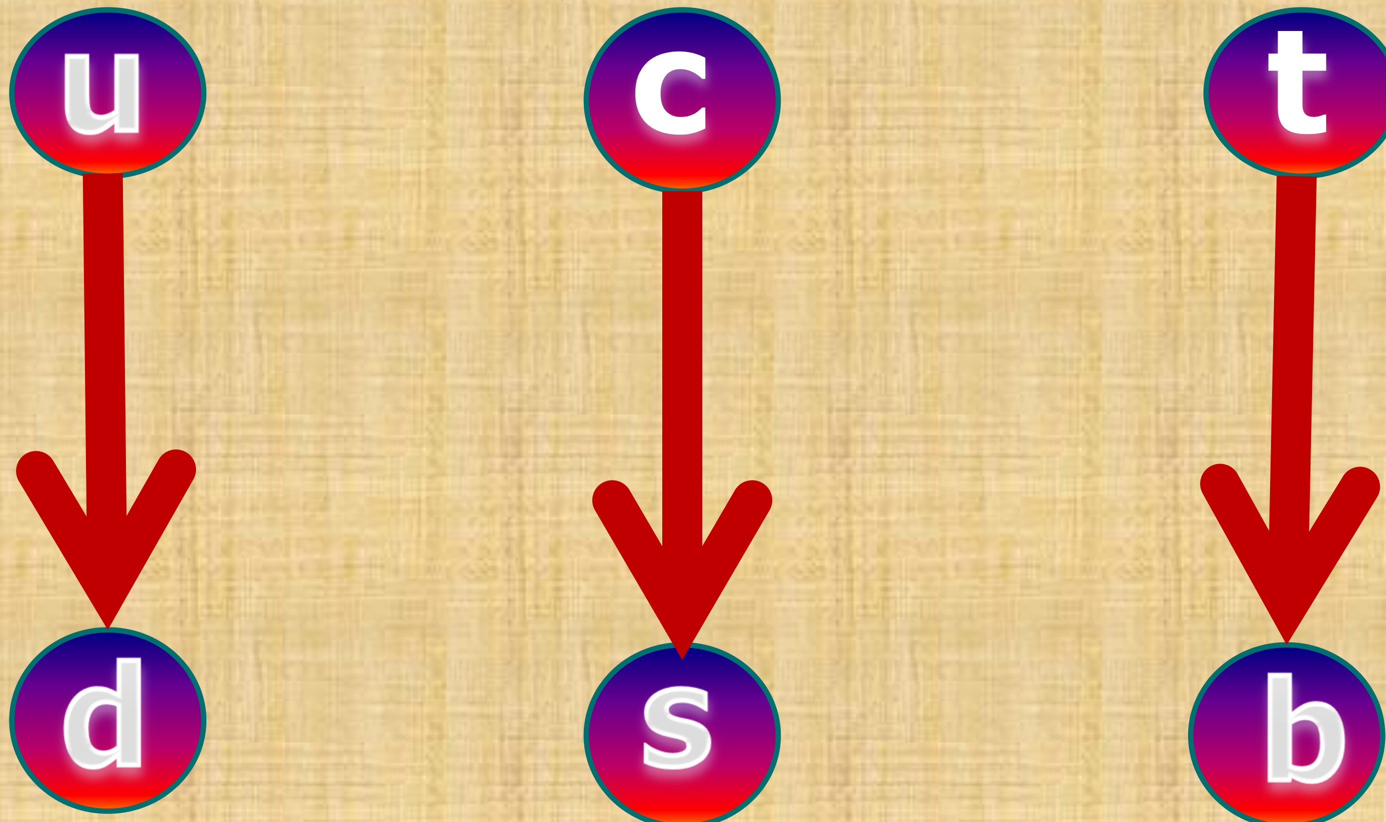
FLAVOR MIXING and NEUTRINO MASSES

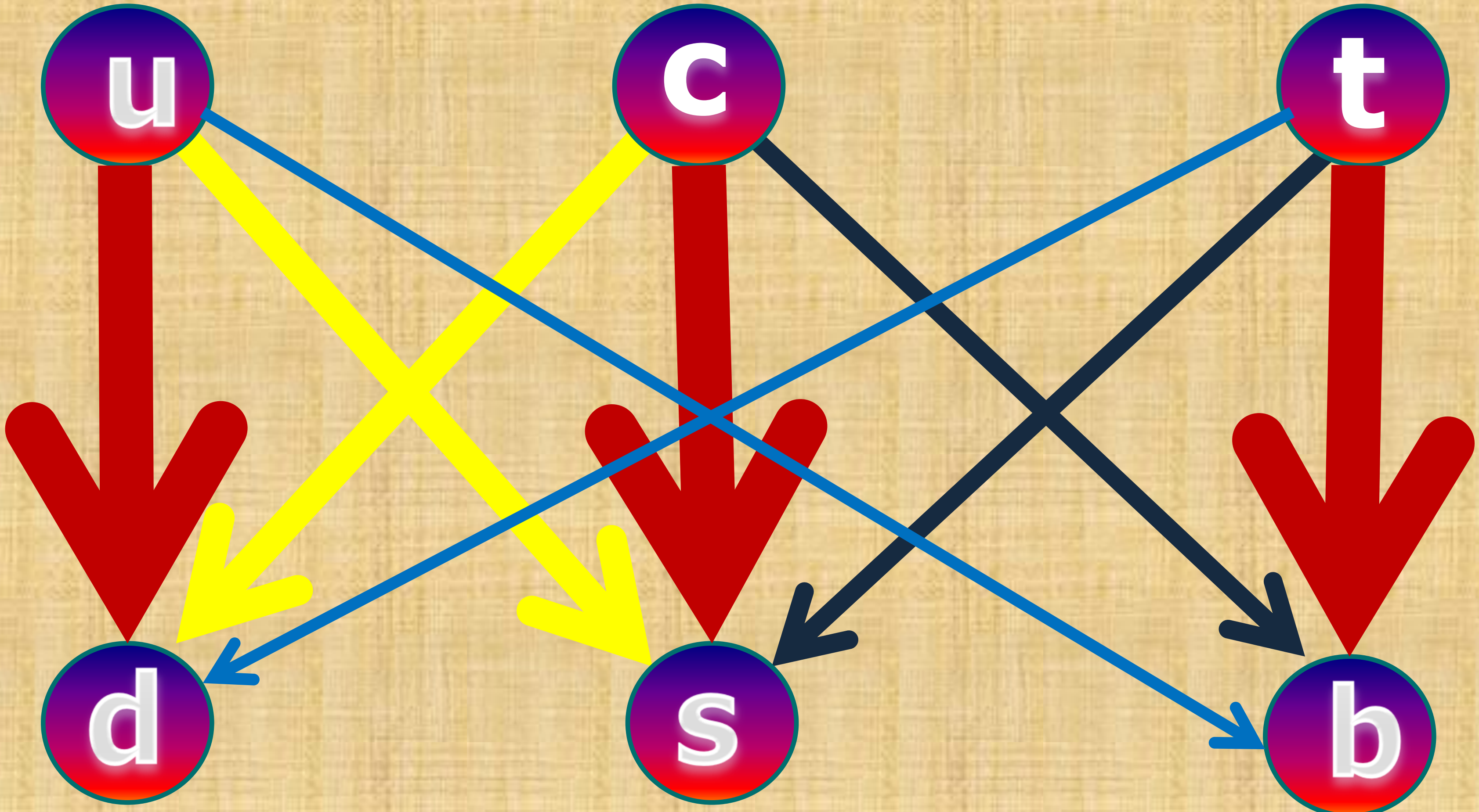
H. FRITZSCH

SU(2) X U(1)

QUARKS



WEAK TRANSITIONS



FLAVOR MIXING

CKM - matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

EXPERIMENTS

$$\left(|V_{CKM}|\right) \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{pmatrix}$$

H. Fritzsch – Z. Xing

$$V = \begin{bmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \bullet \begin{bmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

θ_u θ θ_d

experiments

$$\theta_d \cong 11.7^o$$

$$\theta_u \cong 5.4^0$$

$$\theta \cong 2.4^0$$

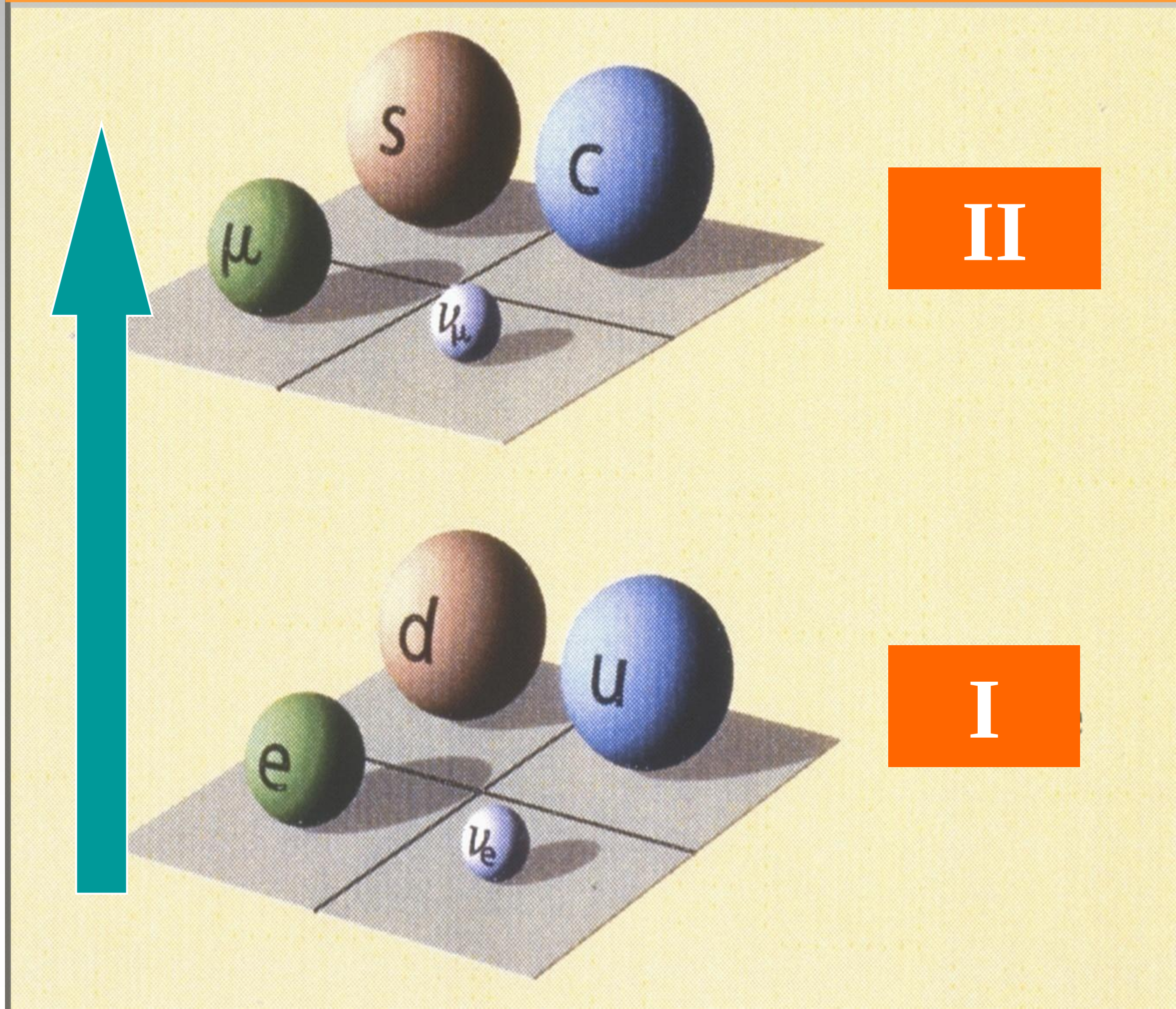
flavor mixing angles

$$\Updownarrow$$

fermion masses

flavor mixing

2 families



mass matrices

texture 0

$$\begin{pmatrix} 0 & a \\ a^{\otimes} & b \end{pmatrix}$$

H. Fritzsch - S. Weinberg / 1978

$$SO(10) \rightarrow \begin{pmatrix} 0 & a \\ a^{\otimes} & b \end{pmatrix}$$

mixing angles <=> masses

$$\begin{pmatrix} 0 & a \\ a^{\otimes} & b \end{pmatrix} \succ \begin{pmatrix} -m_u & 0 \\ 0 & m_c \end{pmatrix}$$

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}} \qquad \theta_u \approx \sqrt{\frac{m_u}{m_c}}$$

OBSERVED QUARK MASSES

$$\sqrt{\frac{m_d}{m_s}} \approx 0.23 \qquad \sqrt{\frac{m_u}{m_c}} \approx 0.09$$

$$\theta_d \cong 13.0^o \qquad \theta_u \cong 5.4^0$$

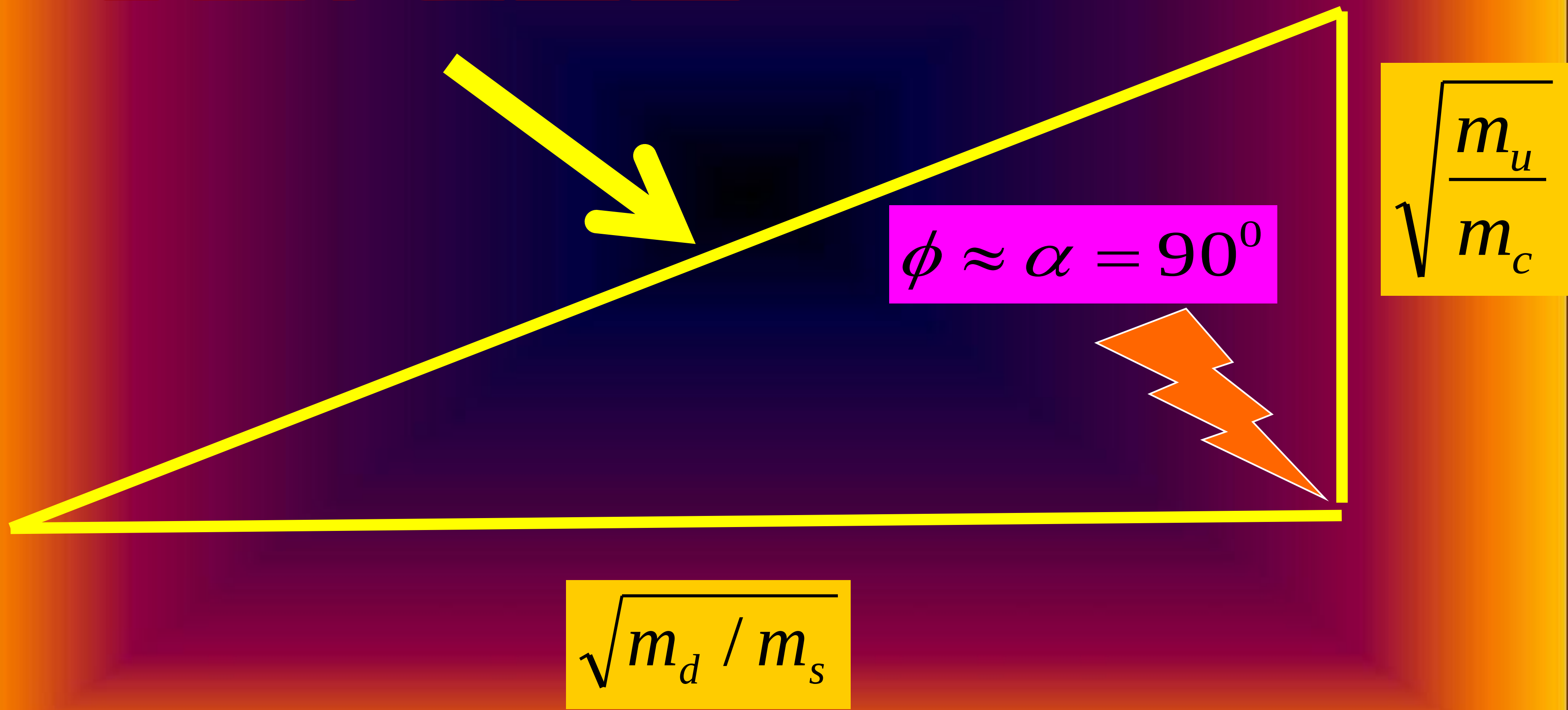
$$\boxed{\sqrt{\frac{m_d}{m_s}} \approx 0.23}$$

$$\boxed{\sqrt{\frac{m_u}{m_c}} \approx 0.09}$$

Cabibbo angle

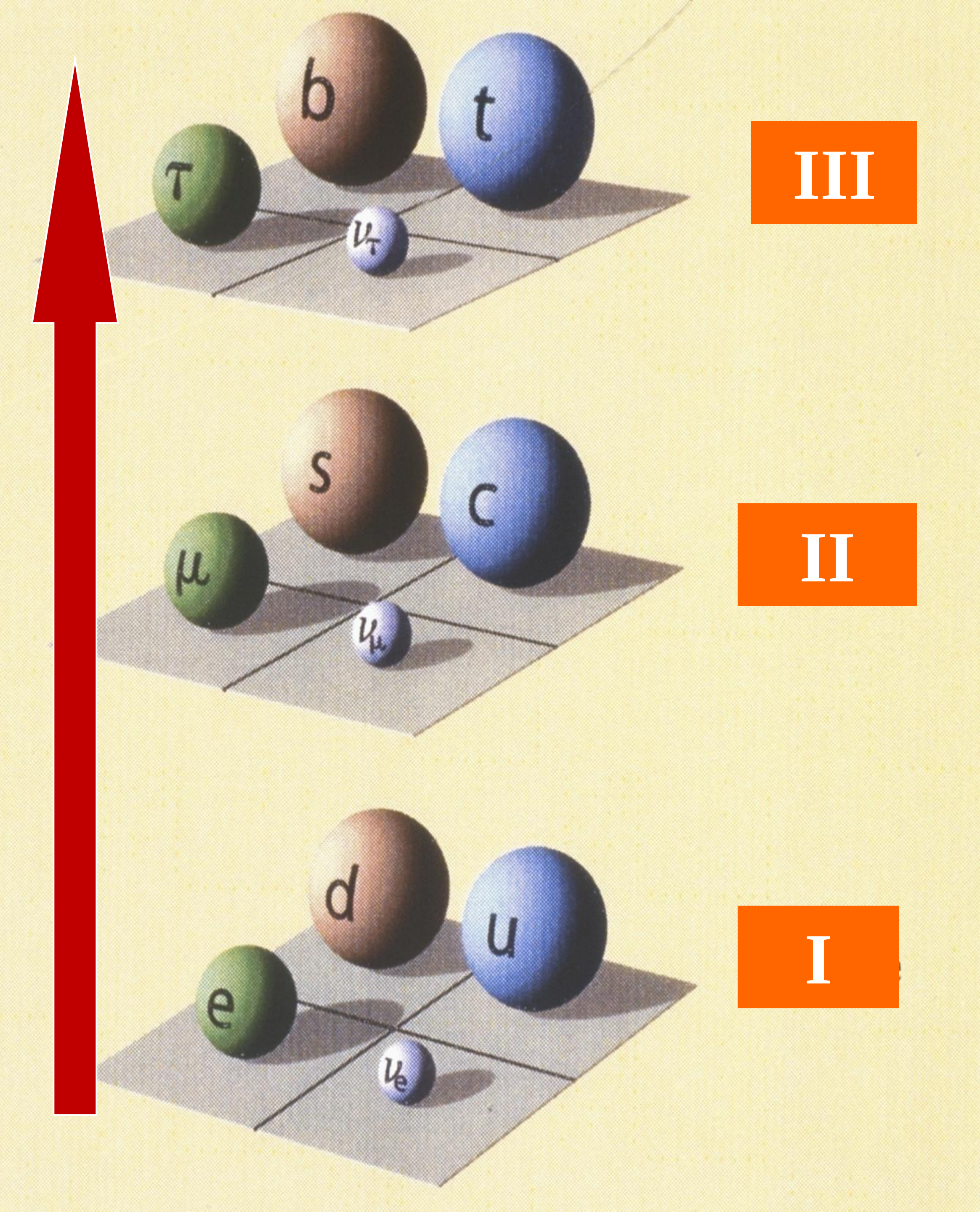
$$\boxed{\theta_c \cong \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \cong 13^0}$$

CABIBBO ANGLE



3 families

flavor mixing



texture zeros

$$\begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}$$

$$V = \begin{bmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \bullet \begin{bmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix} \Rightarrow$$

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}}$$

$$\tan\theta_d = \sqrt{\frac{m_d}{m_s}}$$

$$\tan\theta_d = \sqrt{\frac{m_d}{m_s}}$$

$$\theta_d \approx 13.0 \pm 0.4^o$$

$$Exp: 11.7^o \pm 2.6^o$$

$$\boxed{\tan\theta_u = \sqrt{\frac{m_u}{m_c}}}$$

$$\theta_u \approx 5.0^o \pm 0.7^o$$

$$\boxed{Exp: 5.4^o \pm 1.1^o}$$

$$Experiment:$$

$$V_{cb} = 0.0398 \ldots 0.0424$$

$$\boxed{V_{cb} \cong \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}}}$$

$$V_{cb} \cong \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}} \approx 0.04$$

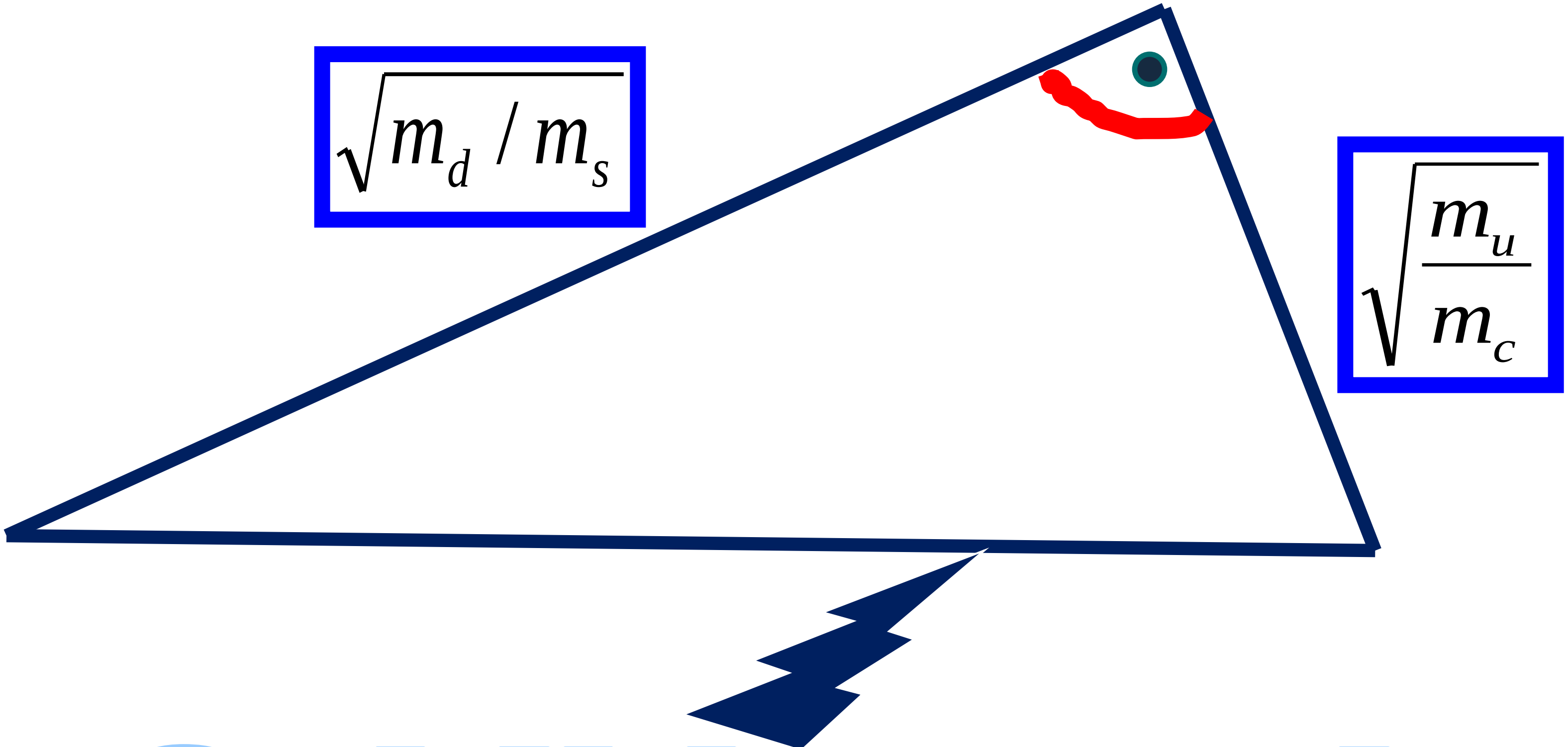
$$m_s \approx 0.075 \quad m_b \approx 4.2$$

$$m_c \approx 1.3 \quad m_t \approx 172$$

m: GeV

unitarity triangle

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cu} & V_{cs} & V_{cb} \\ V_{tu} & V_{ts} & V_{tb} \end{pmatrix}$$

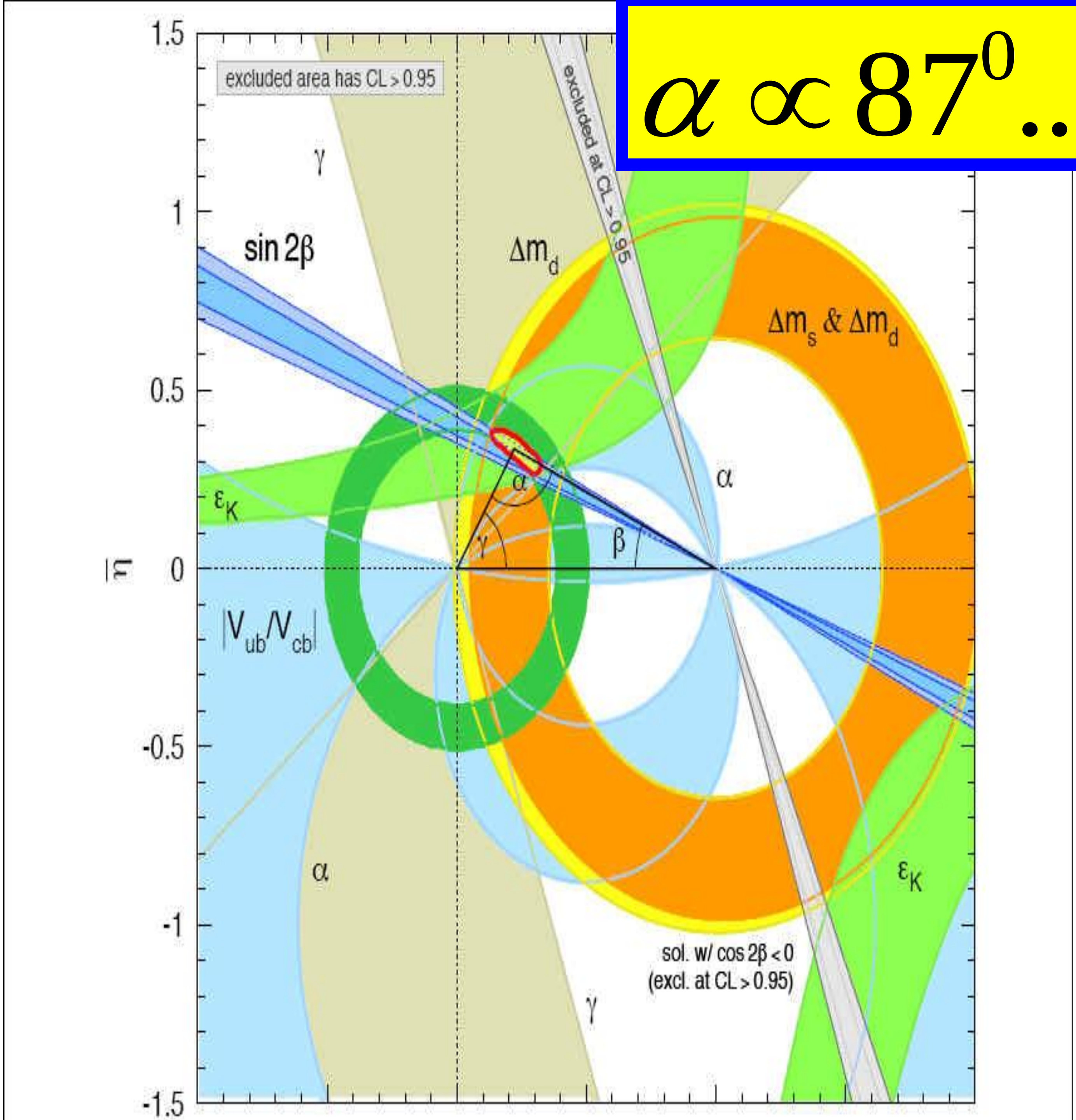


Cabibbo angle

unitarity triangle

(rectangular)

$\alpha \propto 87^0 \ldots 95^0$



$$\phi \approx \alpha = 90^0$$

$$V = \begin{bmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} i & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \bullet \begin{bmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Maximal CP-violation

Experiments
(Maximal CP-Violation)

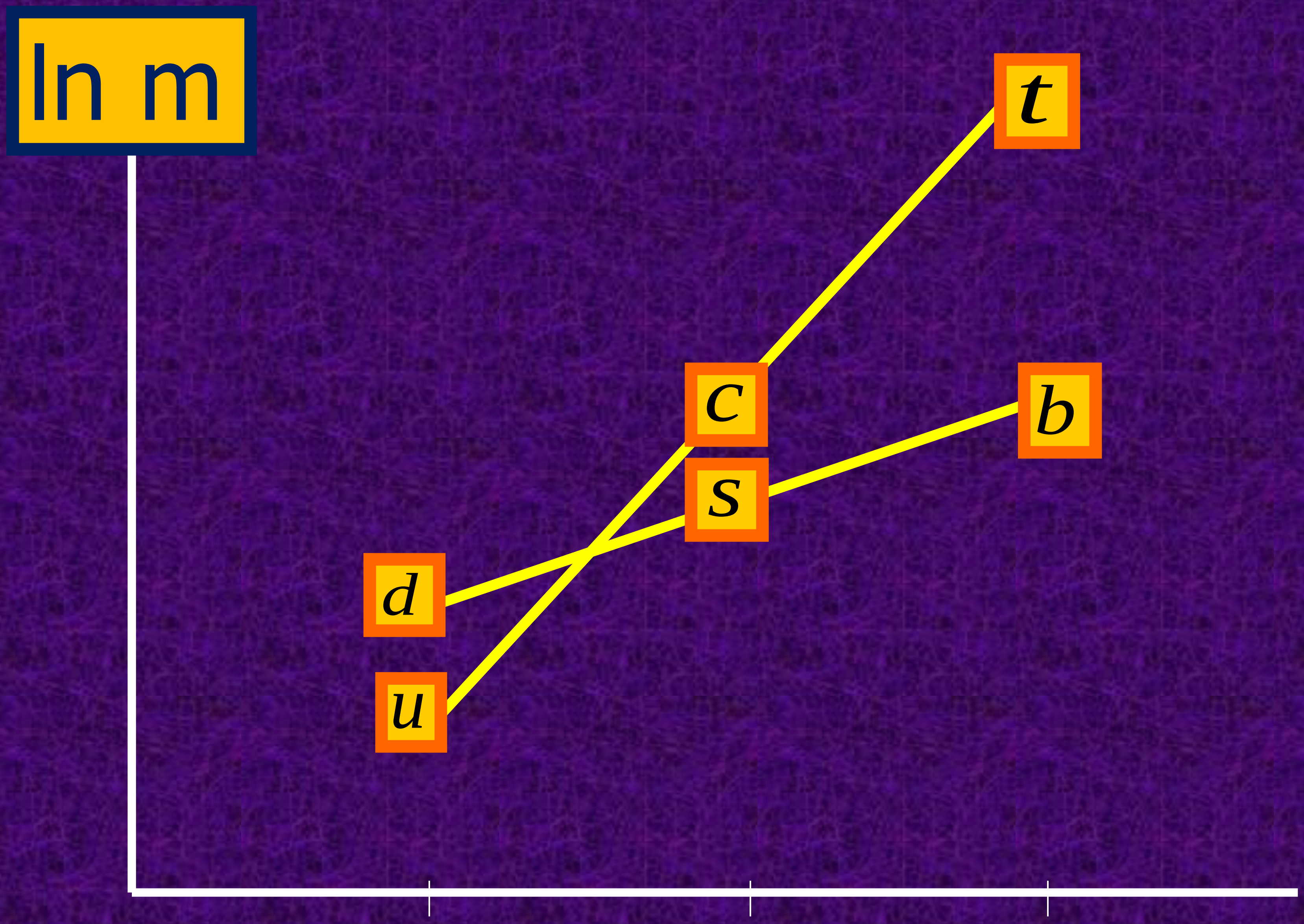
$$V_{CKM} \approx \begin{pmatrix} 0.97 \cdot i + 0.02 & -0.20 \cdot i + 0.09 & 0.004 \\ -0.09 \cdot i + 0.20 & 0.02 \cdot i + 0.97 & 0.04 \\ -0.01 & -0.04 & 1 \end{pmatrix}$$

$$m(c) : m(t) = m(u) : m(c)$$

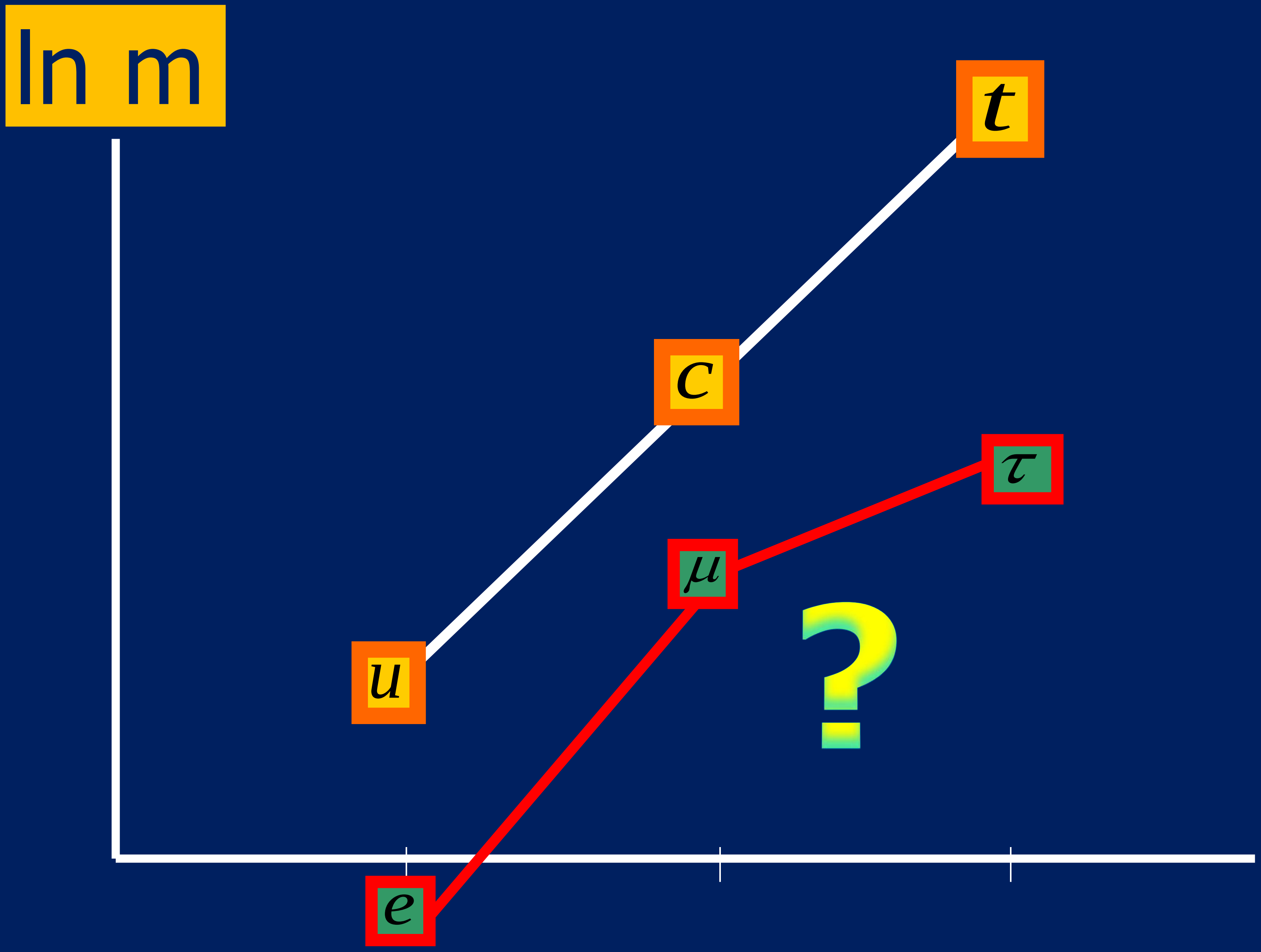
1/207 **1/207**

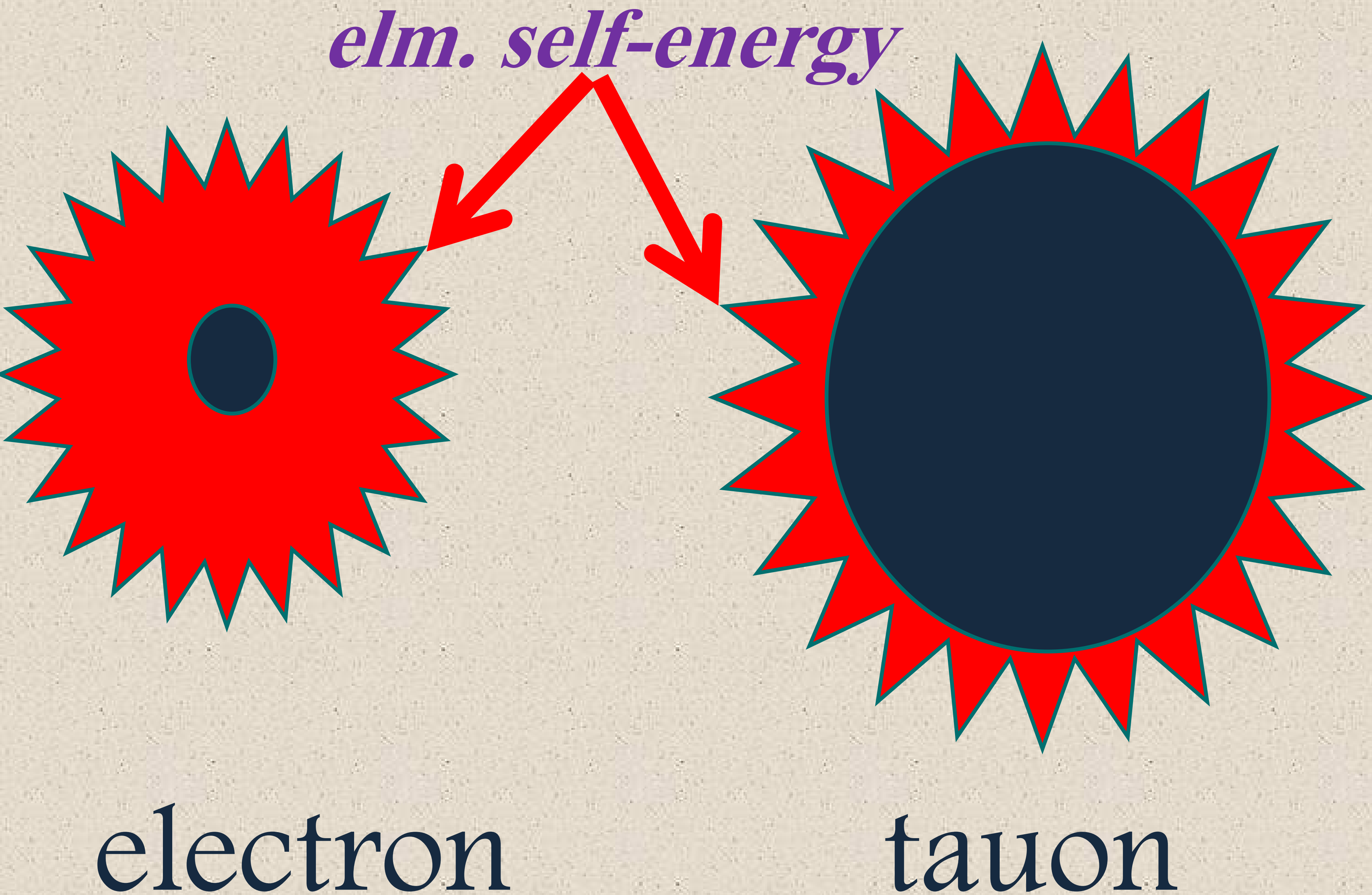
$$m(s) : m(b) = m(d) : m(s)$$

1/23 **1/23**



ln m
t
c
b
s
d
u


ln m
t
c
u
τ
μ
e
?


elm. self-energy
electron
tauon

Radiative Corrections

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix} + \left(\frac{\alpha}{2\pi}\right) \begin{pmatrix} \sim & \sim & \sim \\ \sim & \sim & \sim \\ \sim & \sim & \sim \end{pmatrix} + \ldots$$

$$m_{q,l} = m_{q,l}^{bare} \pm const. \cdot \left(\frac{\alpha}{2\pi}\right) m_{t,b,\tau}$$

radiative corrections

$$m_e = m_e^o + const.\left(\frac{\alpha}{\pi}\right)m(\tau) + \ldots$$

$$\approx 6.0 \quad MeV - 5.5 \quad MeV \approx 0.5 \quad MeV$$

$$m_e^0 \approx 6.0 \quad MeV$$

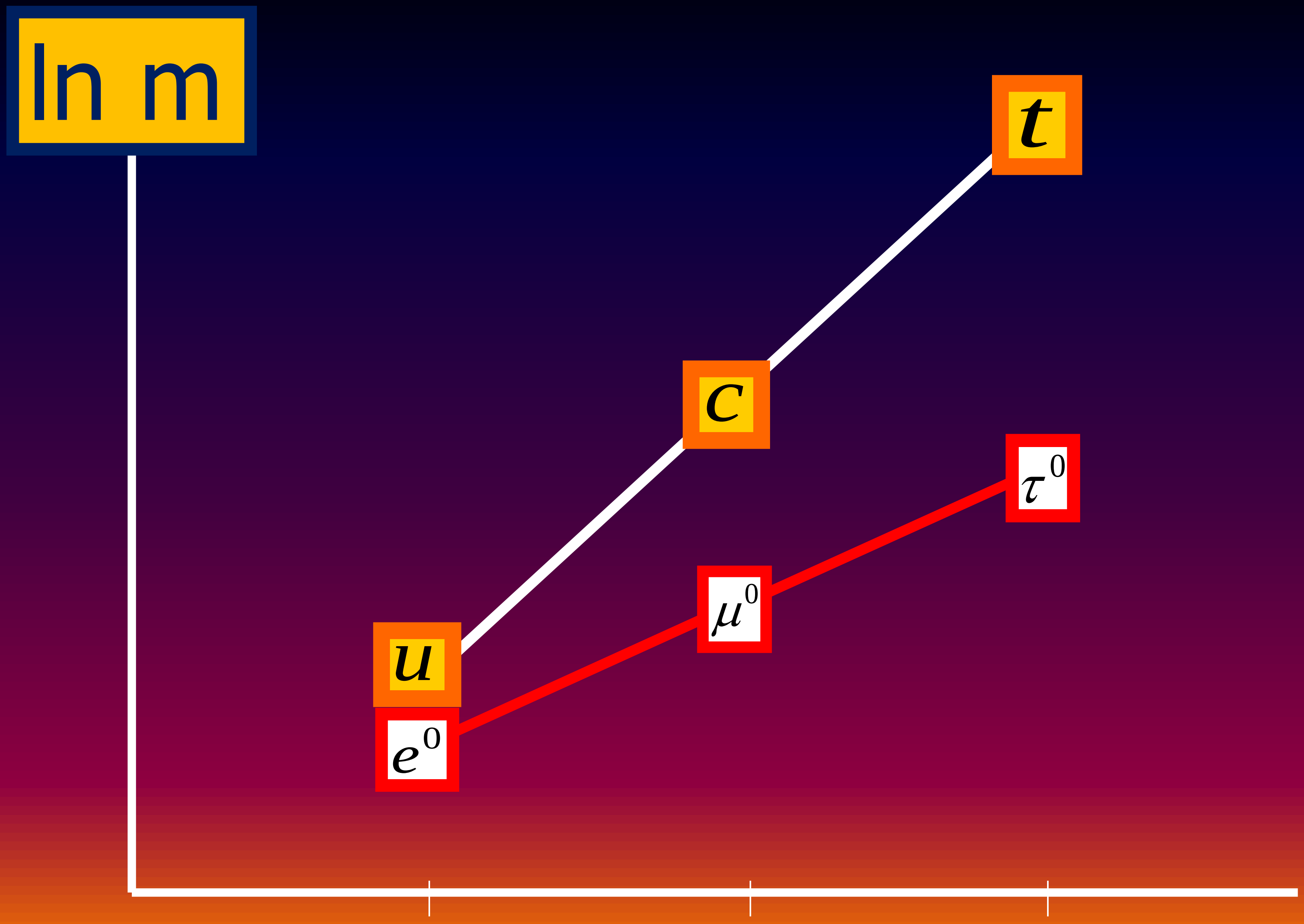
$$m_\mu = m_\mu^0 + const.\left(\frac{\alpha}{\pi}\right)m(\tau) + \ldots$$

$$\approx 111.2 \quad MeV - 5.5 \quad MeV \approx 105.7 \quad MeV$$

$$m_\mu^0 \approx 111.2 \quad MeV$$

$$\frac{m_{\mu}^{0}}{m_{\tau}^{0}} \cong 0.05$$

$$\frac{m_{e}^{o}}{m_{\mu}^{0}} \cong 0.05$$



ln m
t
c
u
τ0
μ0
e0

mixing of leptons

Bruno Pontecorvo

1913 - 1993

$$\boxed{\nu_e = \cos\theta \cdot \nu_1 + \sin\theta \cdot \nu_2}$$

neutrino mixing

B. PONTECORVO

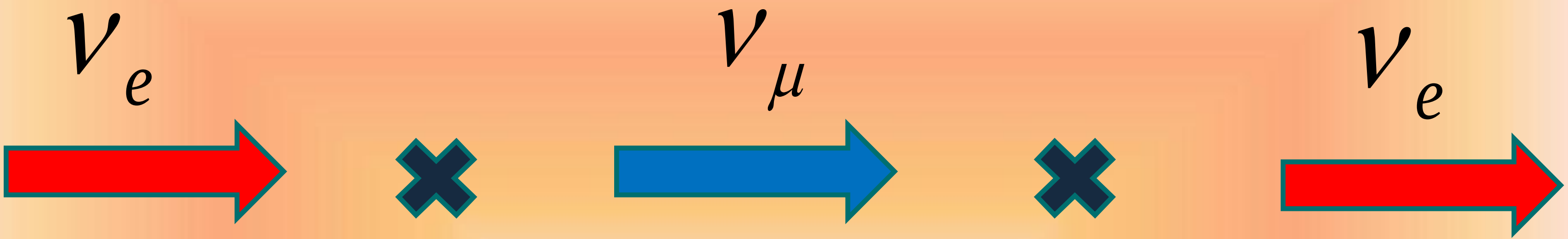
PHYS. JETP 6, 429

(1957)

H. FRITZSCH - P. MINKOWSKI

PHYSICS LETTERS 62B, 76

(1976)

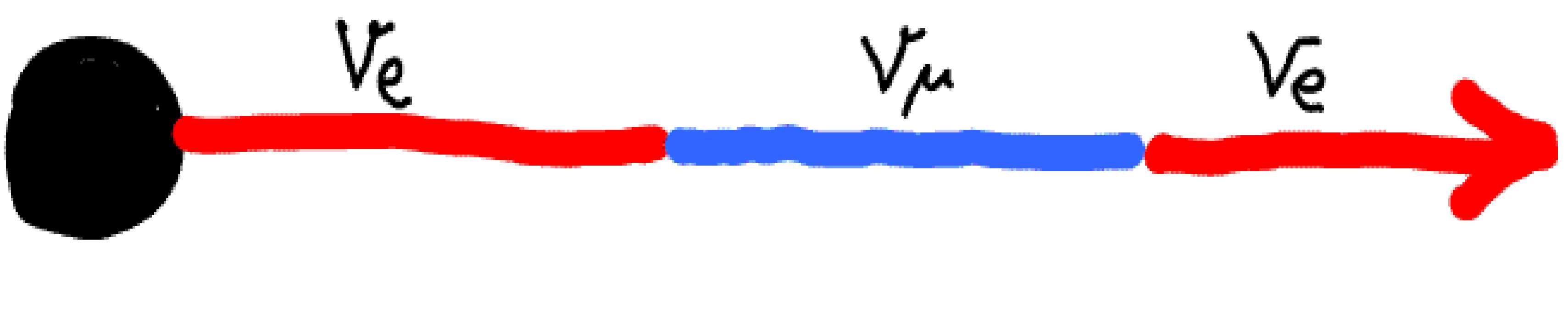


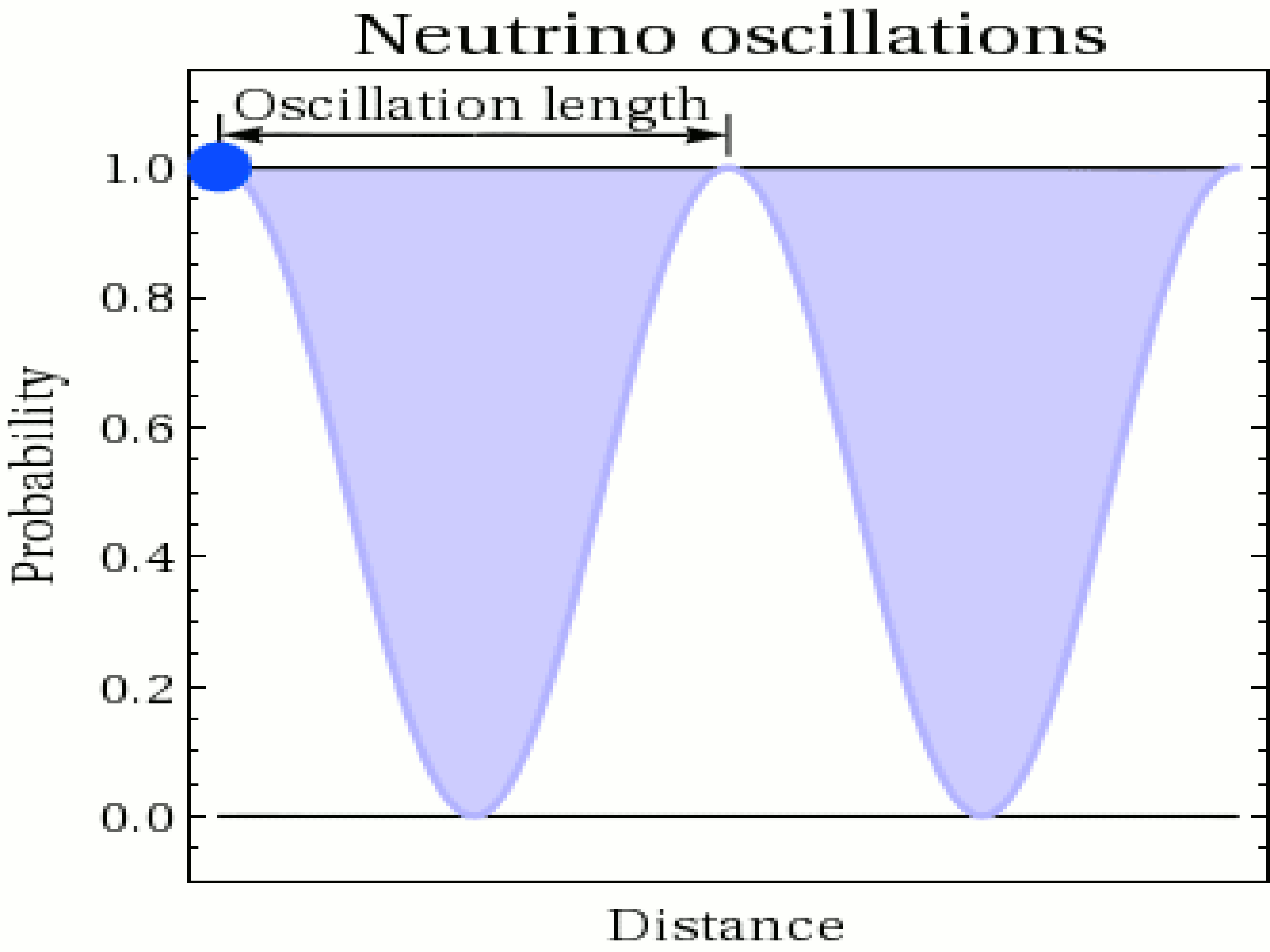
NEUTRINO OSCILLATIONS

$$\nu_e = \frac{1}{\sqrt{2}}\nu_1 + \frac{1}{\sqrt{2}}\nu_2$$

$$\nu_\mu = -\frac{1}{\sqrt{2}}\nu_1 + \frac{1}{\sqrt{2}}\nu_2$$

$$\Theta = \pi/2$$




Neutrino oscillations
Oscillation length
1.0
0.8
0.6
0.4
0.2
0.0
Probability
Distance

Fritzsch - Minkowski

2 - neutrino case

(ν_e, ν_μ)

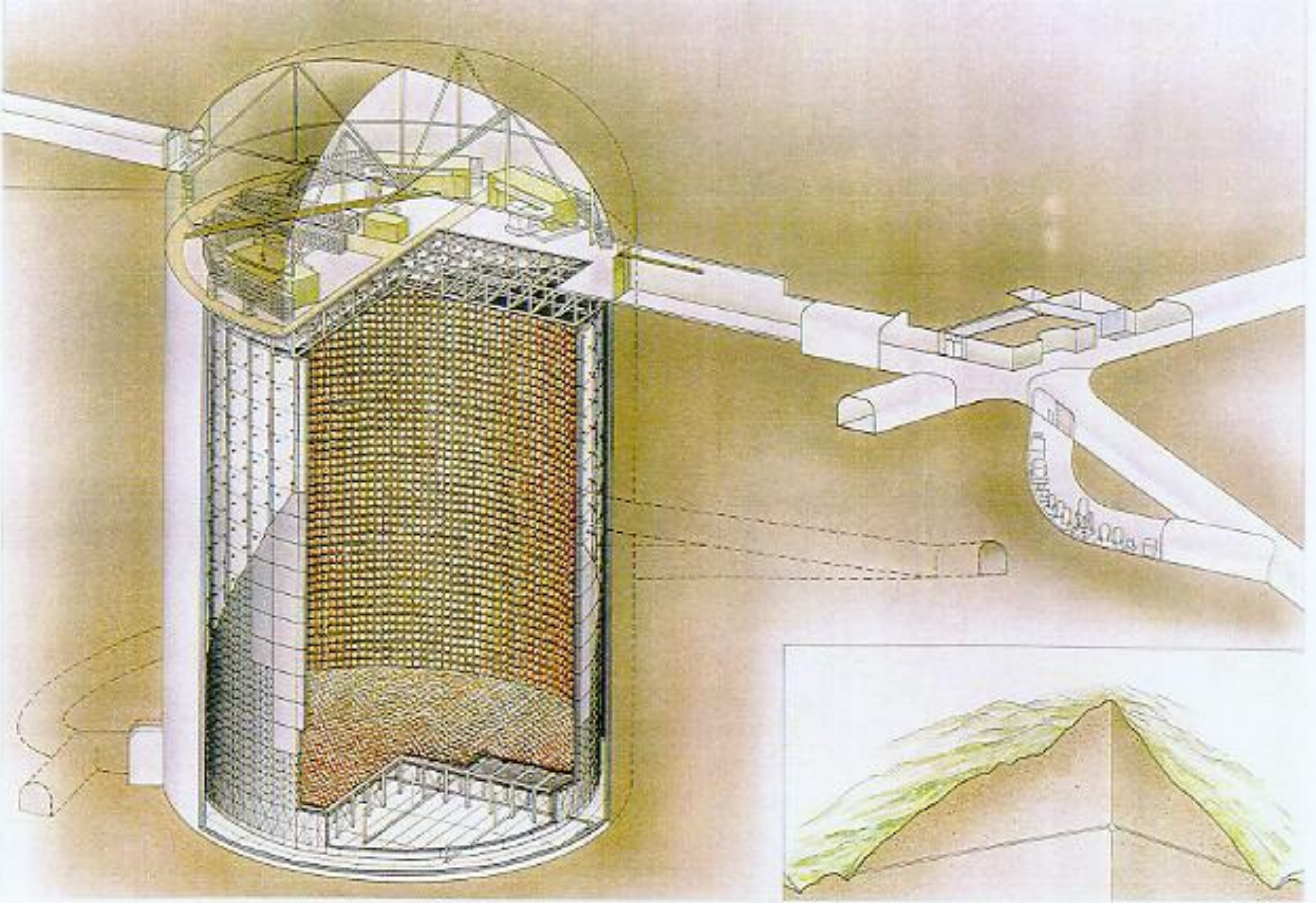
$$U \longrightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P_{\alpha\to\beta}(\alpha \neq \beta) =$$

$$\sin^2(2\theta) \cdot \sin^2 \frac{\Delta m^2 \cdot L}{4E}$$

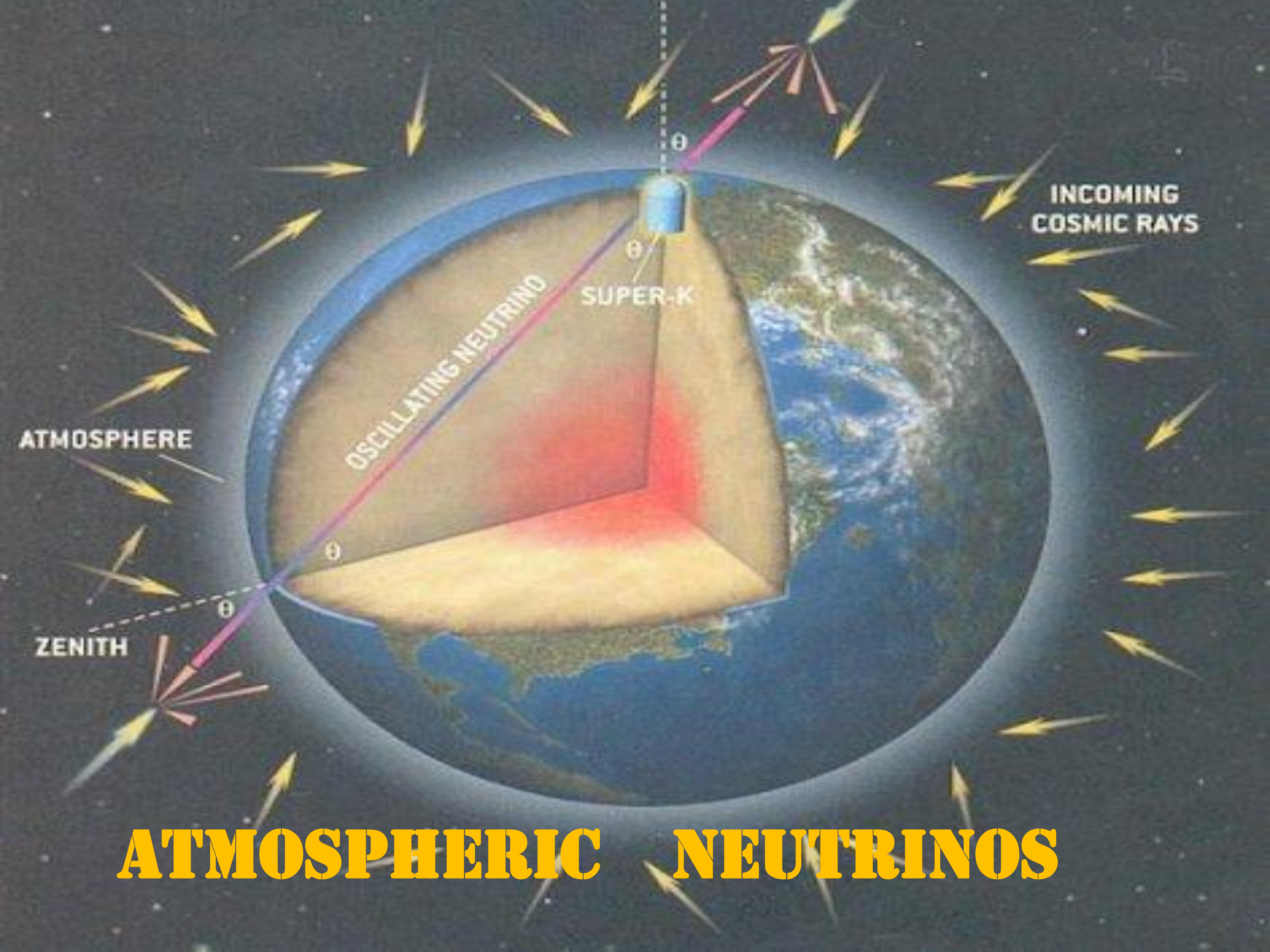
$$= \sin^2(2\theta) \cdot \sin^2\left(1.267 \cdot \frac{\Delta m^2 L}{E} \frac{\mathrm{GeV}}{\mathrm{eV}^2\mathrm{km}}\right)$$


Kamioka
Nemuro
Kushiro
AKITA
Hiraizumi
Sea of Japan
Sado isl.
YAMAGATA
NIIGATA
Mt. Zao
FUKUSHIMA
Mt. Bandai
L. Inawashiro
Wajima
Notto pen.
Itoigawa
TOYAMA
KANAZAWA
L. Chuzenji
Nikko
NAGANO
Shirakawa-go
Matsumoto
Tojimbo
UTSUNOMIYA
MITO
FUKUI
Takayama
MAEBASHI
Kamikochi
Tango pen
Wakasa Bay
HONSHU
URAWA
KOFU
Hikawa
TOKYO
GIFU
Magome
Izushi
L. Biwa
Fuji Five Lakes
CHIBA
Maibara
KYOTO
NAGOYA
Mt. Fuji
YOKOHAMA
KOBE
OTSU
Himeji
SHIZUOKA
Hakone
Atami
Ito
Awaji isl.
OSAKA
NARA
TSU
Asuka
Ise
Toba
Hamamatsu
Izu pen.
WAKAYAMA
Hongu
Shirahama
Shingu
Pacific Ocean
Naze

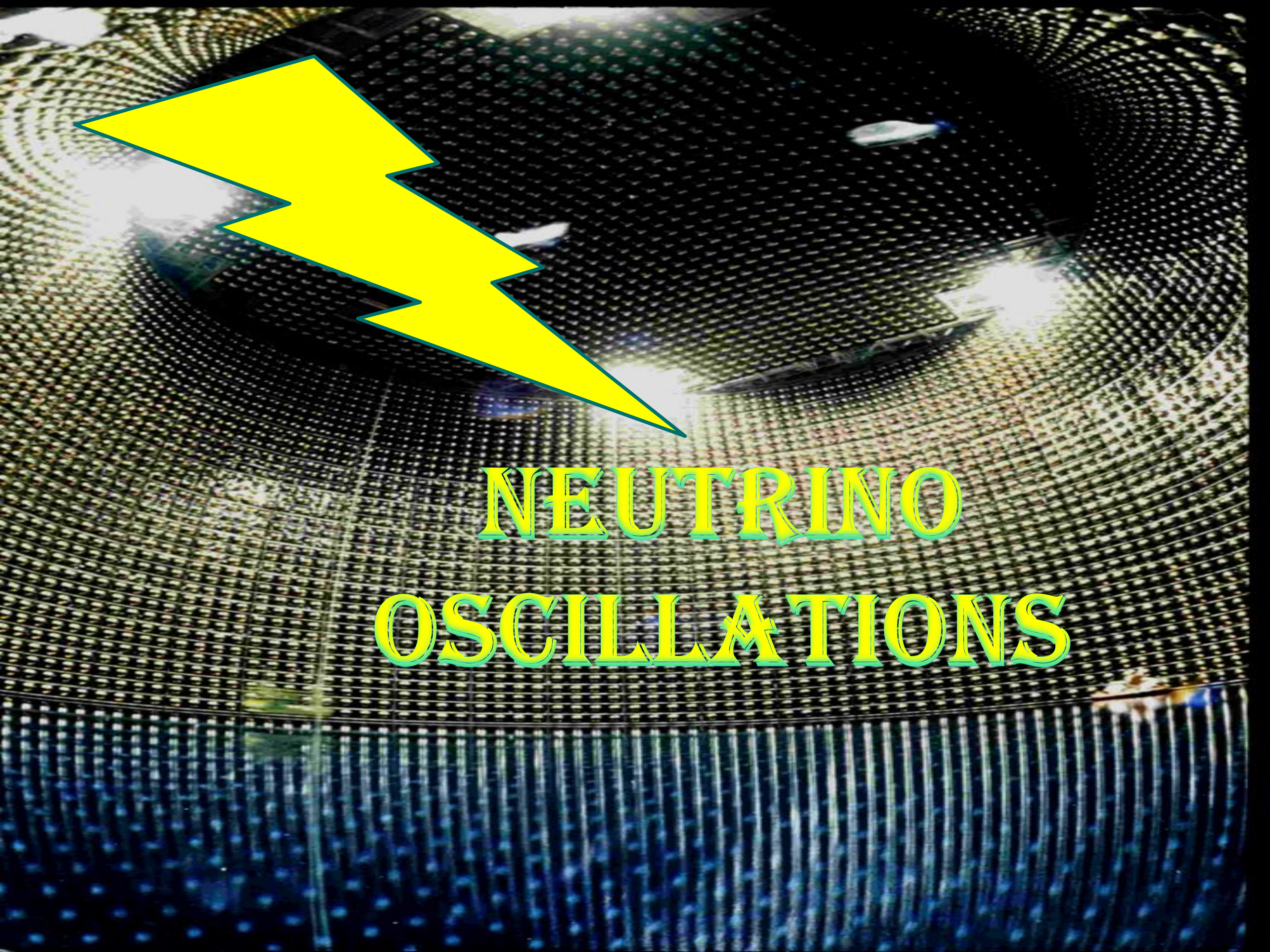


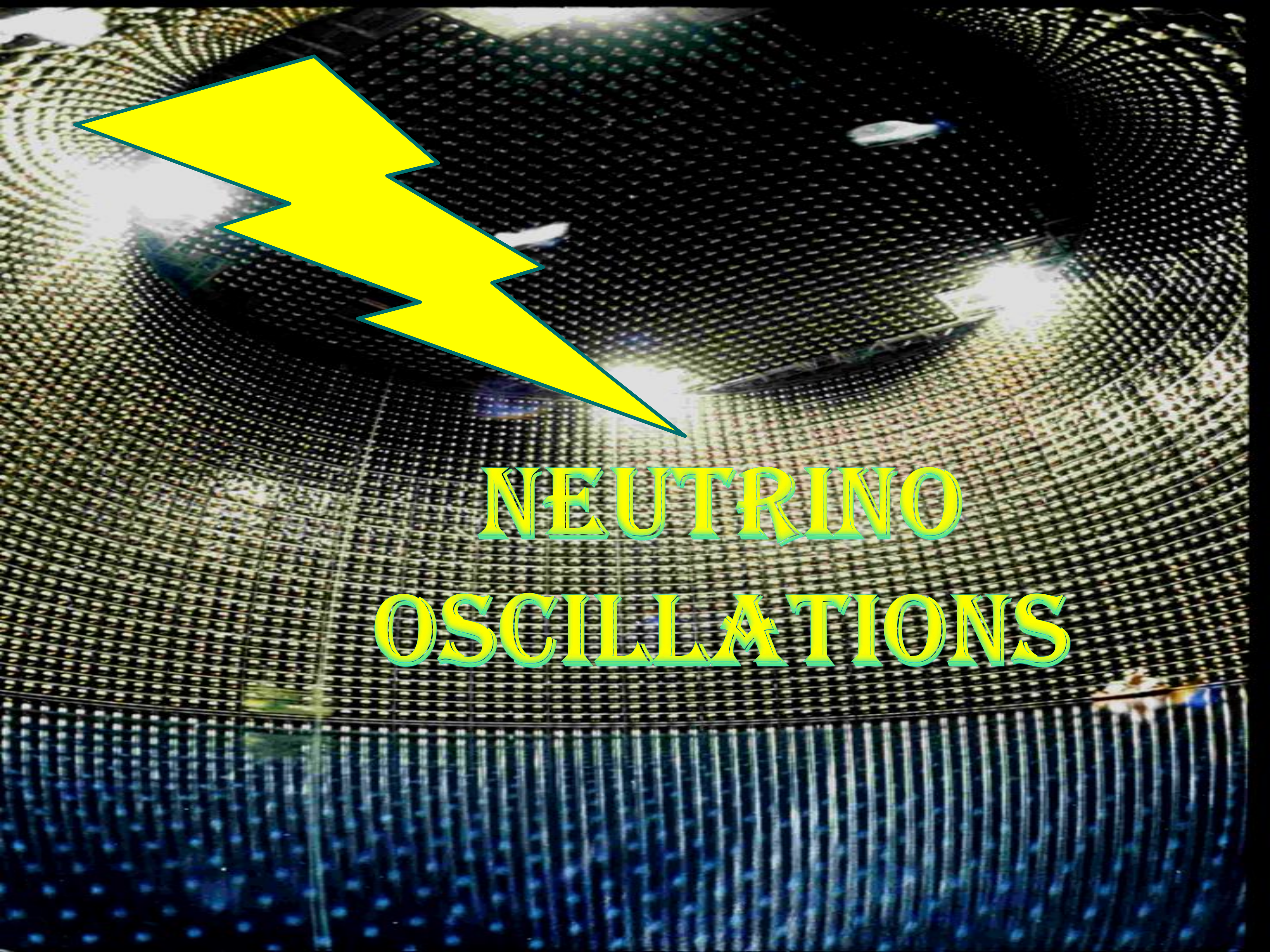
SUPERKAMIOKANDE INSTITUTE FOR COSMIC RAY RESEARCH UNIVERSITY OF TOKYO

NIKKEN SEKKEI

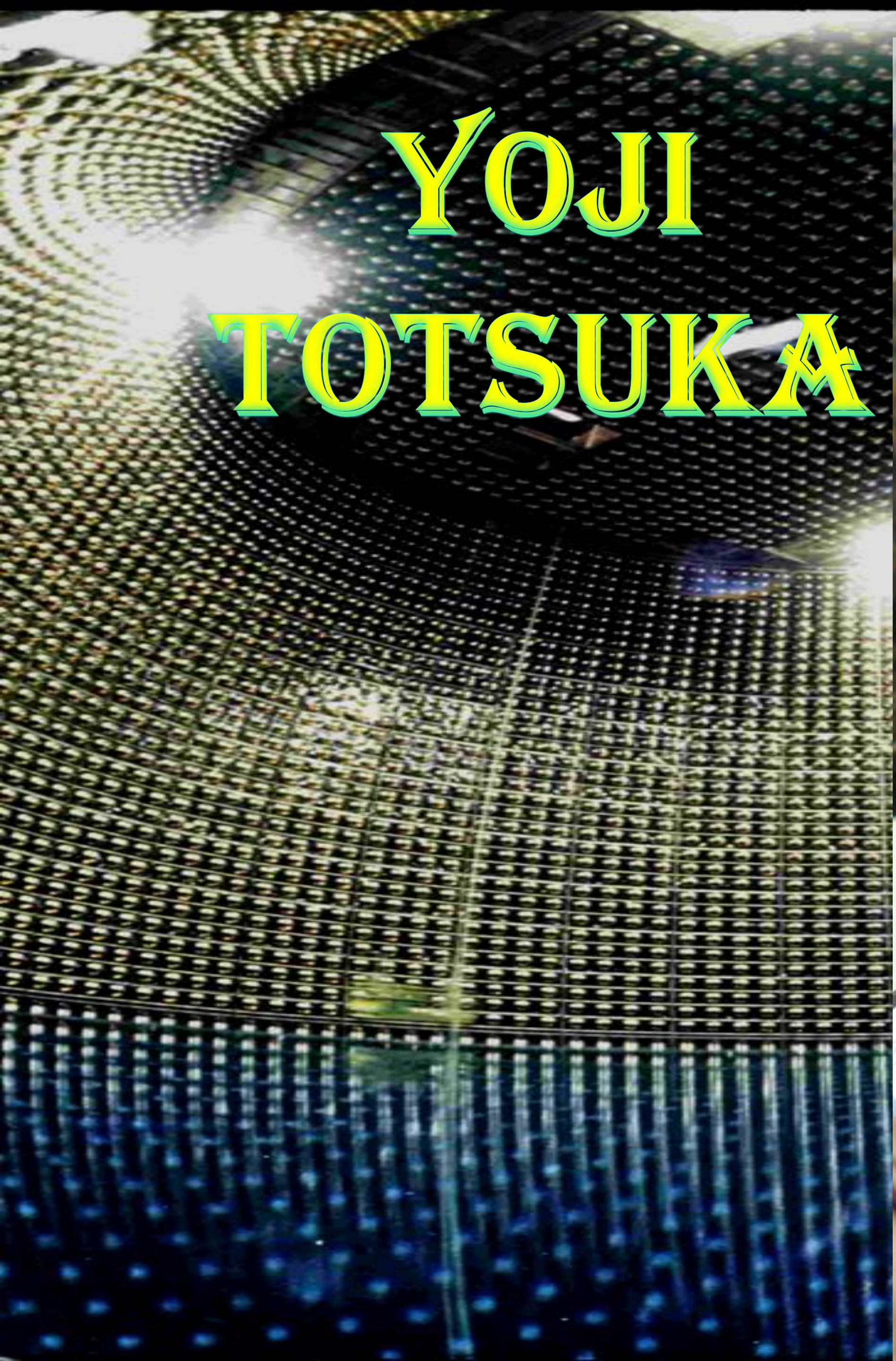


ATMOSPHERIC NEUTRINOS


NEUTRINO
OSCILLATIONS


NEUTRINO
OSCILLATIONS

Yoji Totsuka

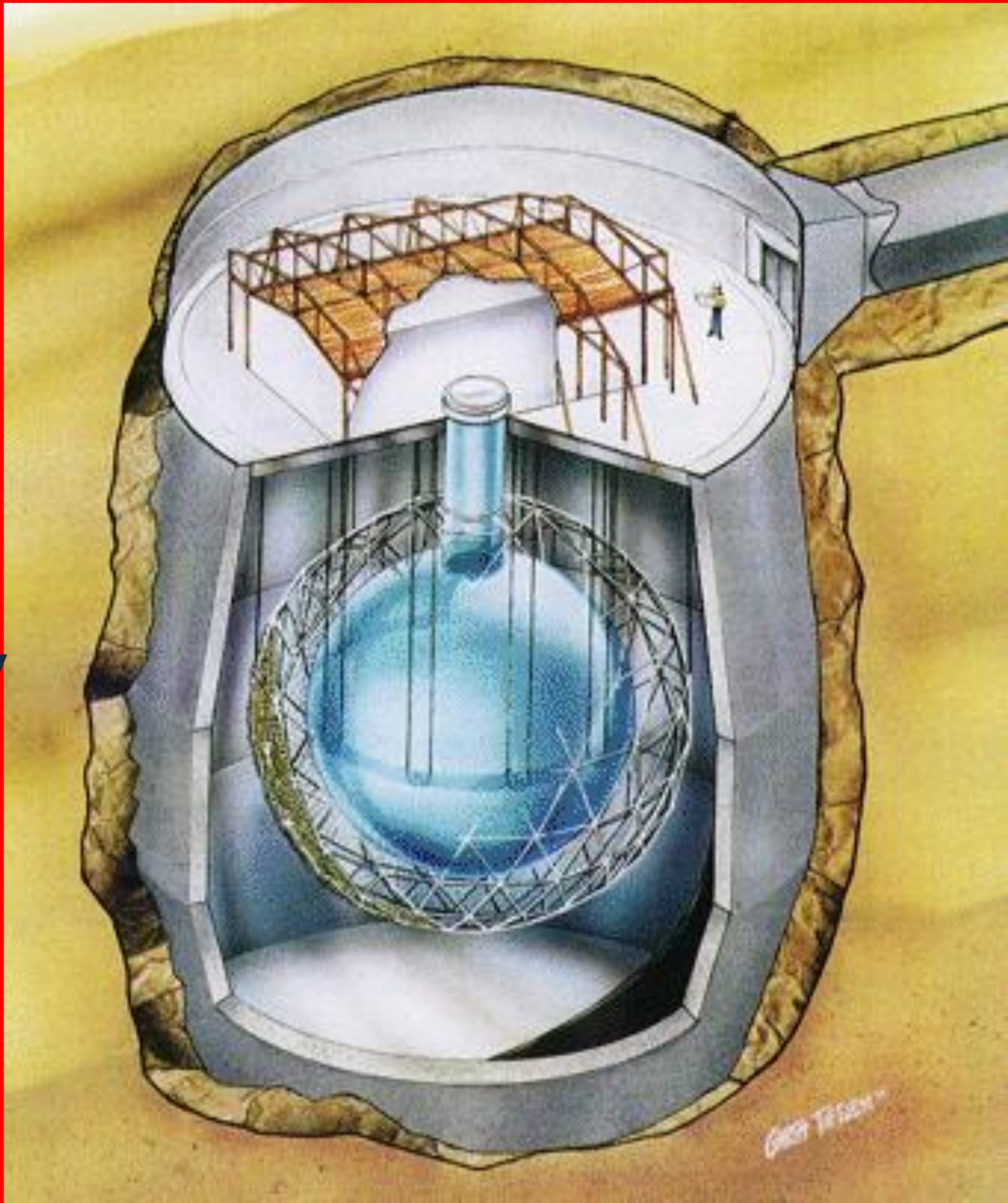


2001 =>

Canada

Sudbury Neutrino Obervatory

SNO

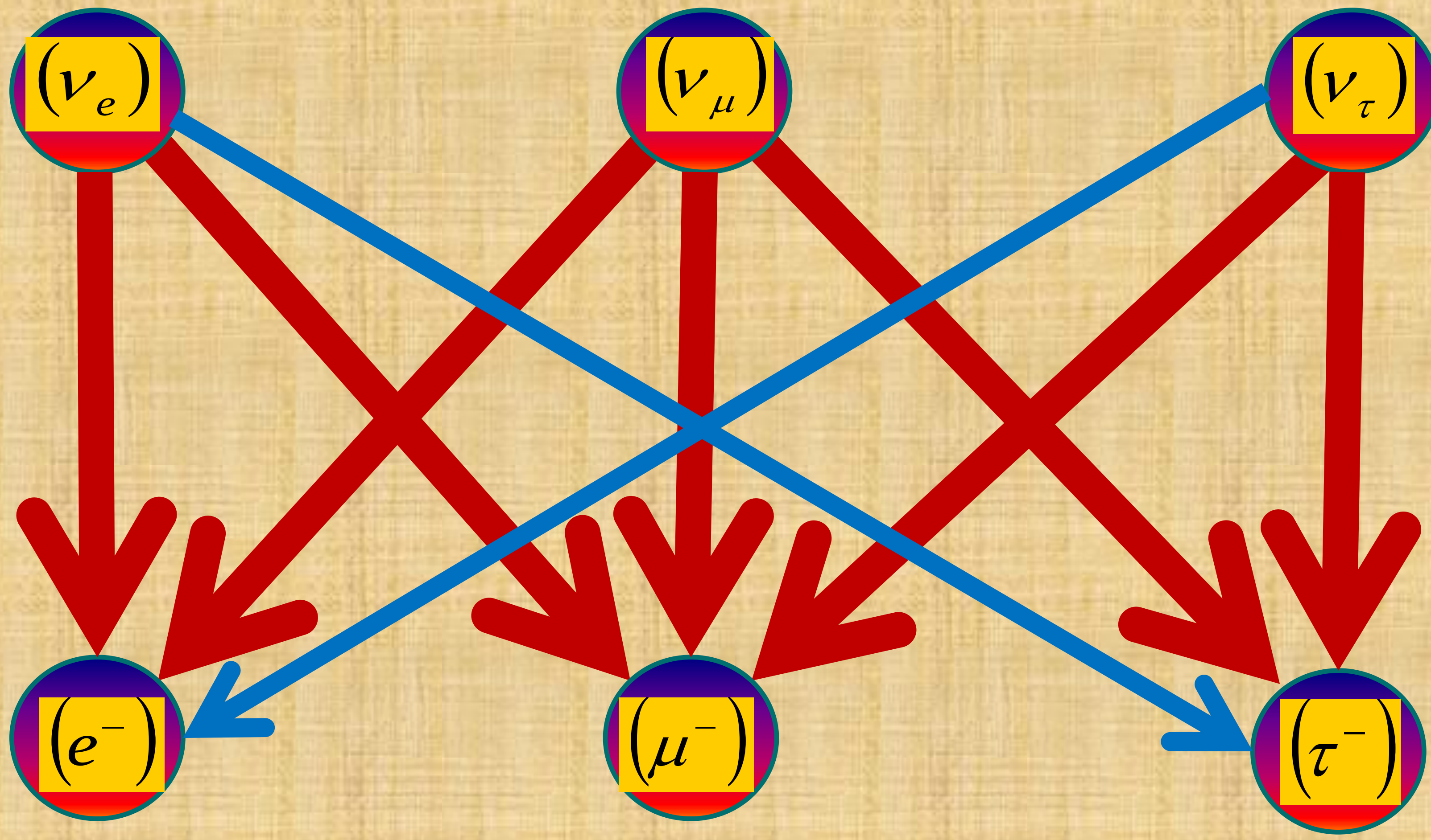


S N O

SOLAR NEUTRINOS

charged current

neutral current



FLAVOR MIXING
LEPTONS

neutrino mixing matrix

(==> CKM Matrix)

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

$$V = U P$$

$$P = \begin{bmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{U = \begin{bmatrix} \cos\theta_l & \sin\theta_l & 0 \\ -\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \bullet \begin{bmatrix} \cos\theta_\nu & -\sin\theta_\nu & 0 \\ \sin\theta_\nu & \cos\theta_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\theta_l \approx reactor - angle \quad \theta \approx \theta_{at} \quad \theta_\nu \approx \theta_{sun}$$

Fritzsch - Xing

Kamiokande - SNO

$$32^o \le \theta_{sun} \le 35^o$$

$$37^o \le \theta_{at} \le 40^o$$

$$\Delta m^2{}_{21} = 7.54^{+0.26}_{-0.22} \cdot 10^{-5} eV^2$$

$$\Delta m^2{}_{32} = 2.43^{+0.06}_{-0.10} \cdot 10^{-3} eV^2$$

texture zeros

$$\begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}$$

$$\tan\theta_{\nu} = \sqrt{\frac{m_1}{m_2}}$$

$$\tan\theta_{l} = \sqrt{\frac{m_e}{m_{\mu}}}$$

OBSERVATION

$$32^o \le \theta_{sun} \le 35^o$$

$$==> m_1 / m_2 = 0.39 \ldots 0.49$$

$$\Delta m_{21}^2 \approx 7.5 \cdot 10^{-5} eV^2$$

$$\Delta m_{32}^2 \approx 2.4 \cdot 10^{-3} eV^2$$

$$m_1 / m_2 \approx 0.44$$

==> neutrino masses

Neutrino Masses
(eV)

$$m_1 \approx 0.004$$

$$m_2 \approx 0.010$$

$$m_3 \approx 0.050$$

$$\theta_{at} \approx \arctan\sqrt{\frac{m_2}{m_3}} + \arctan\sqrt{\frac{m_\mu}{m_\tau}}$$

$$\theta_{at} \approx 24^0 + 14^0$$

$$\theta_{at} \approx 38^0$$

Neutrino Mixing Matrix

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

???

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

$$V_{e3} = \sin\theta_l \sin\theta_{at}$$

$$\tan\theta_l \approx \sqrt{\frac{m_e^0}{m_\mu^0}}$$

$$\boxed{V_{e3} = \sin\theta_l \sin\theta_{at}}$$

$$\tan\theta_l = \sqrt{\frac{m_e^0}{m_\mu^0}} \cong 0.22$$

$$38^o \le \theta_{at} \le 52^o$$

$$V_{e3} = \sin\theta_l \sin\theta_{at} = \sin\theta_{13}$$

$$\sin^2 2\theta_{13} = 0.090 \pm 0.025$$

$$\theta_{13} = 8.7^0 \pm 1.2^0$$

Daya Bay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RENO

$$\sin^2 2\theta_{13} = 0.090 \pm 0.025$$

$$\theta_{13} = 8.7^0 \pm 1.2^0$$

DAYA BAY - RENO

$$\sin^2 2\theta_{13} = 0.090 \pm 0.008 \pm 0.008$$

$$\theta_{13} \approx 8.7^0$$

Maximal CP-violation

$$U = U_{reactor} \bullet \begin{bmatrix} i & 0 & 0 \\ 0 & \cos\theta_{at} & \sin\theta_{at} \\ 0 & -\sin\theta_{at} & \cos\theta_{at} \end{bmatrix} \bullet U_{sun}$$

Experiments

(maximal CP-violation)

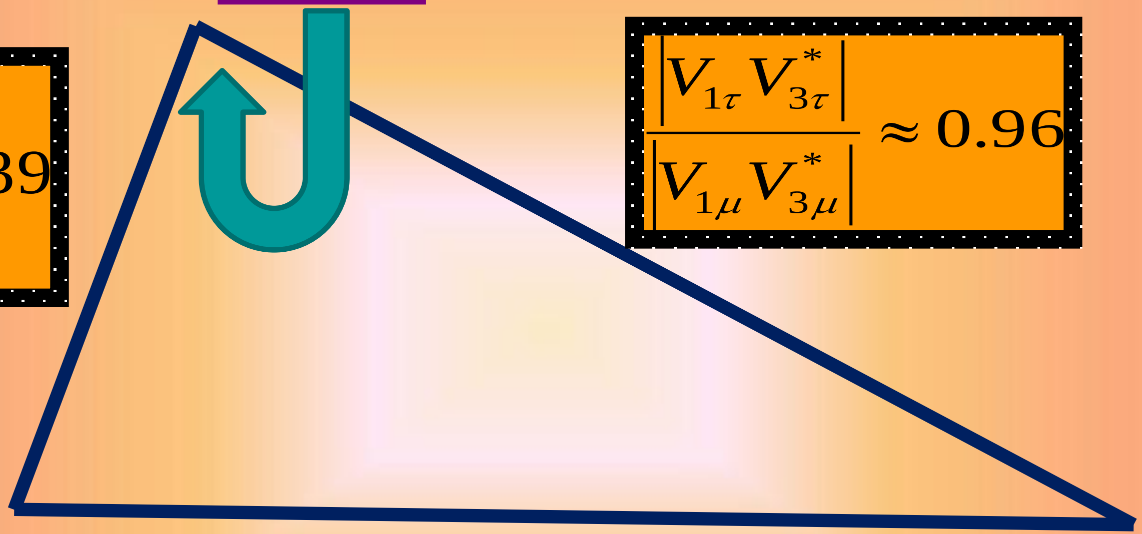
$$V \approx \begin{pmatrix} 0.81\cdot i+0.09 & -0.55\cdot i+0.14 & 0.14 \\ -0.18\cdot i+0.42 & 0.12\cdot i+0.62 & 0.63 \\ -0.36 & -0.53 & 0.77 \end{pmatrix}$$

quarks

$$V_{CKM} \approx \begin{pmatrix} 0.97 \cdot i + 0.02 & -0.20 \cdot i + 0.09 & 0.004 \\ -0.09 \cdot i + 0.20 & 0.02 \cdot i + 0.97 & 0.04 \\ -0.01 & -0.04 & 1 \end{pmatrix}$$

leptons

$$V \approx \begin{pmatrix} 0.81 \cdot i + 0.09 & -0.55 \cdot i + 0.14 & 0.14 \\ -0.18 \cdot i + 0.42 & 0.12 \cdot i + 0.62 & 0.63 \\ -0.36 & -0.53 & 0.77 \end{pmatrix}$$



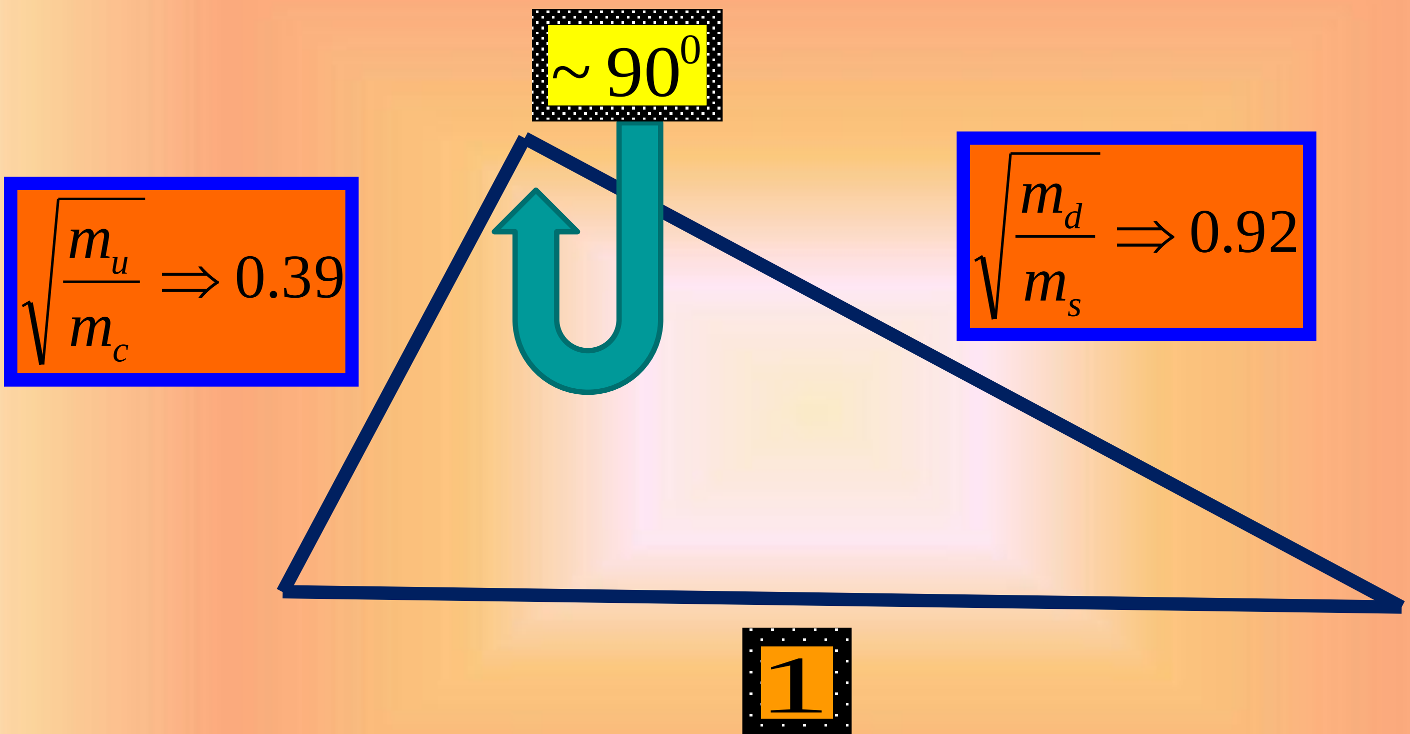
$\sim 84^0$

$$\frac{|V_{1e}V_{3e}^*|}{|V_{1\mu}V_{3\mu}^*|} \approx 0.39$$

$$\frac{|V_{1\tau}V_{3\tau}^*|}{|V_{1\mu}V_{3\mu}^*|} \approx 0.96$$

1

unitarity triangle
(leptons)



unitarity triangle

(quarks)

CP-violation

$$\boxed{P\left(\nu_{\alpha} \Rightarrow \nu_{\beta}\right) \neq P\left(\overline{\nu}_{\alpha} \Rightarrow \overline{\nu}_{\beta}\right)}$$

???????? ???????

neutrino masses

very small

???????? ???????

1906 => 1938

Dirac mass ?   Majorana mass ?

seesaw mechanism

$$M_{\nu} = \begin{bmatrix} 0 & D \\ D & M \end{bmatrix}$$

$$m_{\nu} \cong \frac{D^2}{M}$$

HISTORY SEESAW

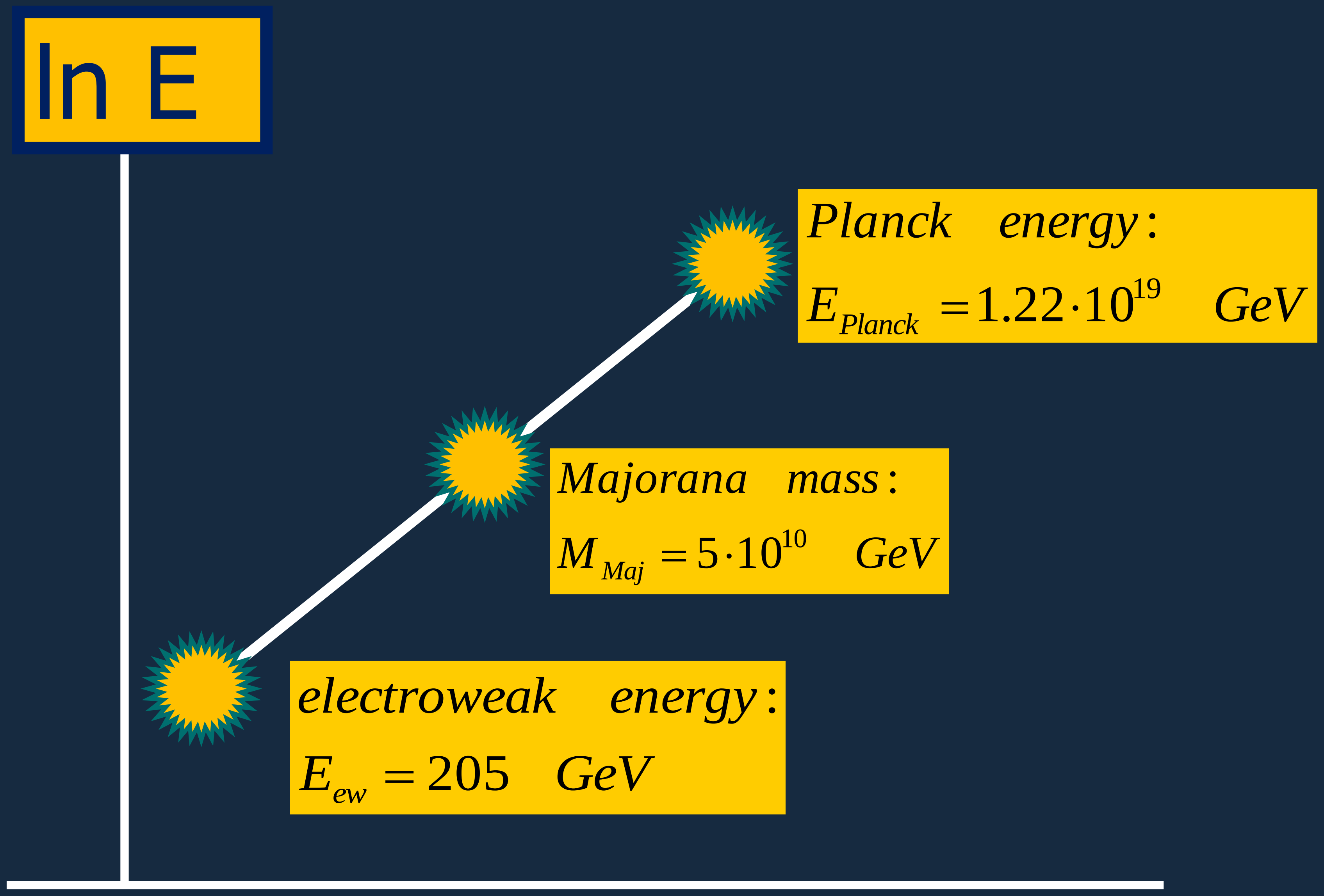
Footnote:
H. Fritzsch
M. Gell-Mann
P. Minkowski
PLB 59 (1975) 256

P. MINKOWSKI,
PLB 1976

T. Yanagida 1979
M. Gell-Mann, P. Ramond, R. Slansky 1979

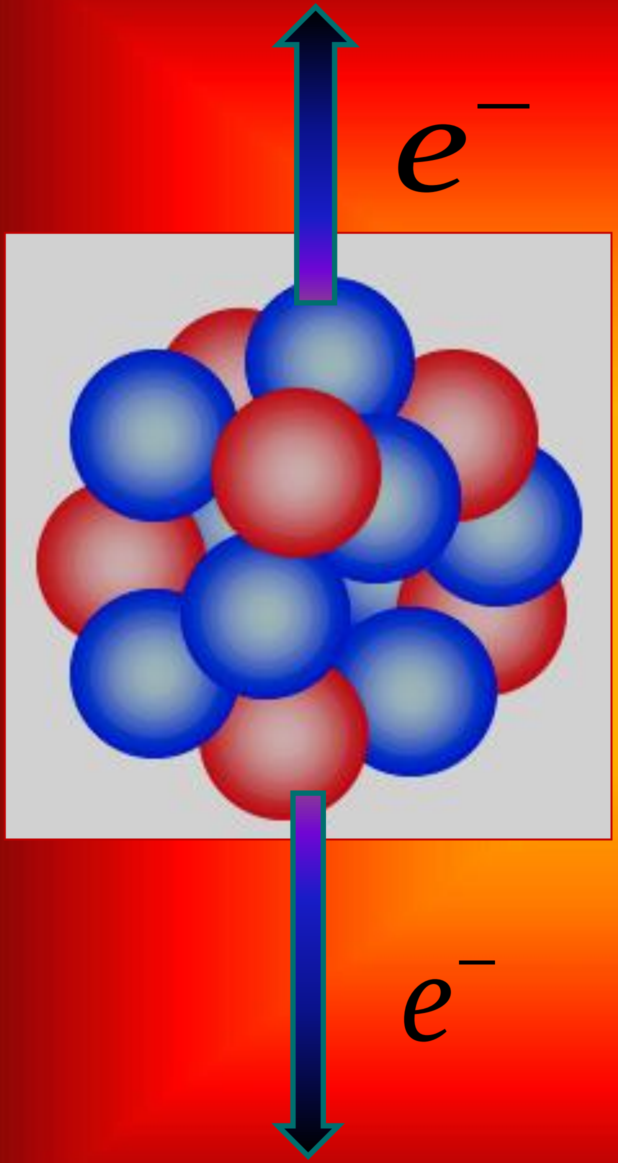
$$m_{\nu_\tau} = 0.050 \quad eV = \frac{m_\tau^2}{M}$$

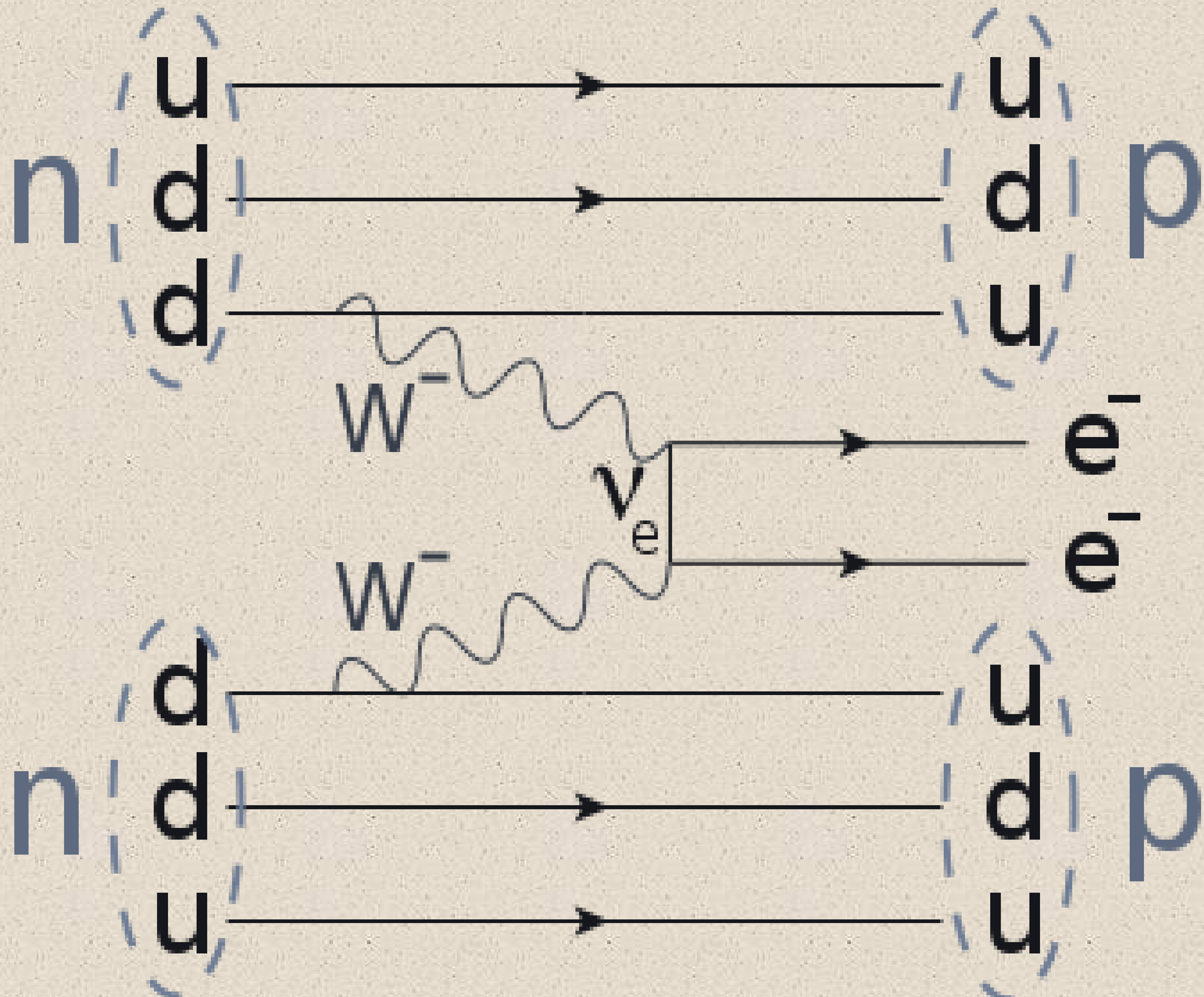
$$\boxed{M_{Maj} = 5 \cdot 10^{10}\, GeV}$$



ln E
$Planck \quad energy:$
$E_{Planck} = 1.22 \cdot 10^{19} \quad GeV$
$Majorana \quad mass:$
$M_{Maj} = 5 \cdot 10^{10} \quad GeV$
$electroweak \quad energy:$
$E_{ew} = 205 \quad GeV$

Neutrinoless Double Beta Decay



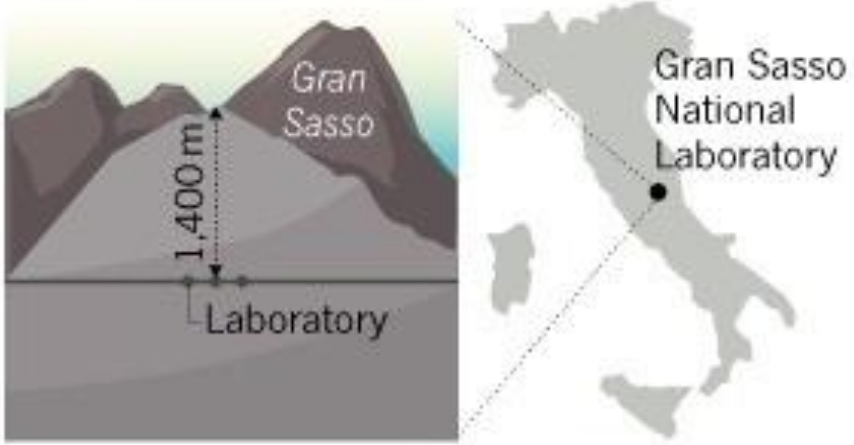


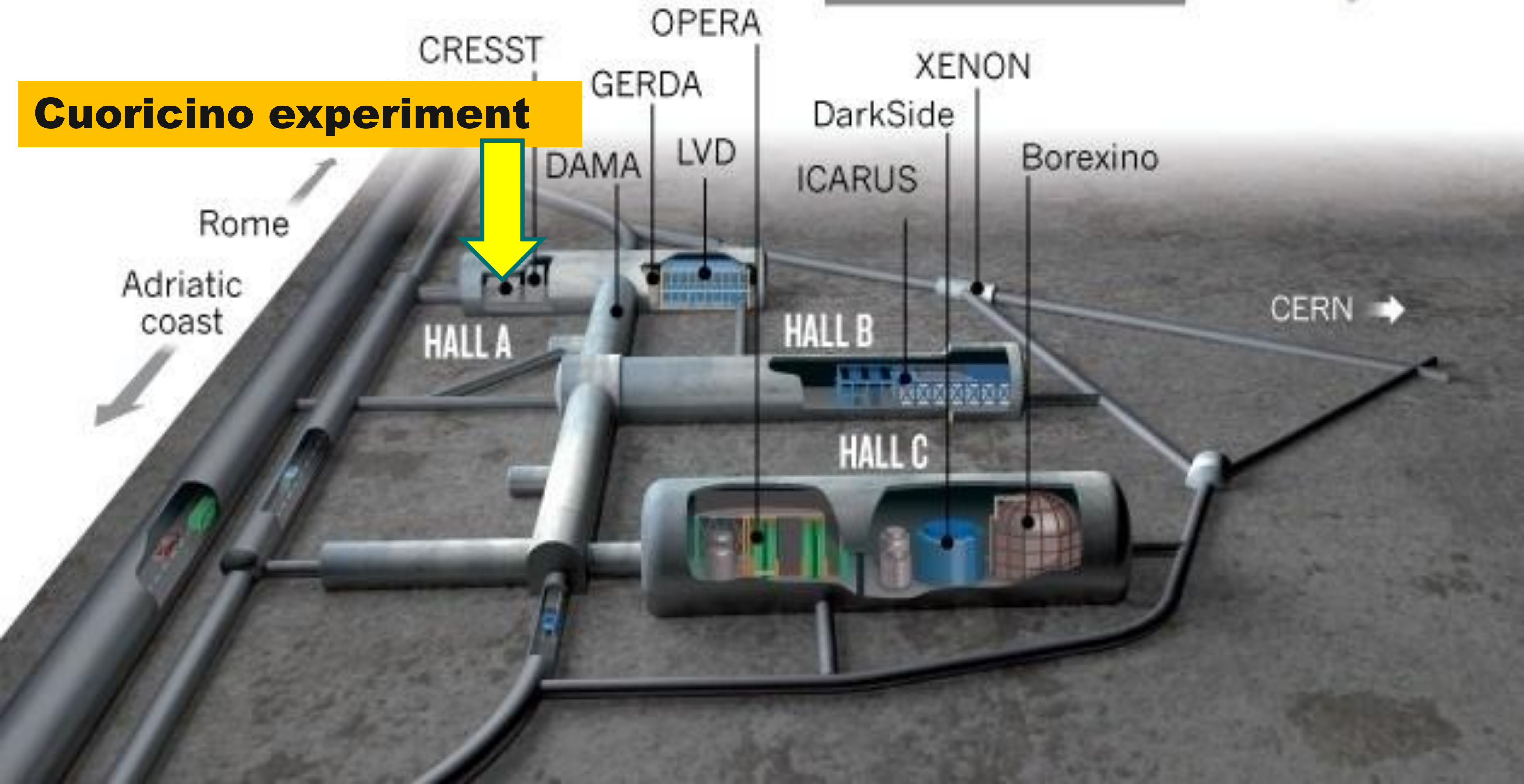
n
u
d
d
p
u
d
u
W⁻
ν
e
e⁻
e⁻
W⁻
n
d
d
u
p
u
d
u

Gran Sasso Laboratory

THE A, B AND C OF GRAN SASSO

Experiments at the Gran Sasso National Laboratory are housed in and around three huge halls carved deep inside the mountain, where they are shielded from cosmic rays by 1,400 metres of rock.





CUORICINO EXPERIMENT

$$\boxed{T_{1/2}^{0\nu}(^{130}Te) \geq 1.8 \cdot 10^{24}\ years}$$

Effective Majorana neutrino mass:

< 0.23 eV

relevant mass term:

$$m_{eff.} = | V_{1e}^2 \cdot m_1 + V_{2e}^2 \cdot m_2 + V_{3e}^2 \cdot m_3 |$$

EXPECTED:

$$\boxed{m_{eff.} \approx 0.005 eV}$$

OBSERVED:

$$\boxed{m_{eff.} < 0.23 eV}$$

CONCLUSIONS

$$\begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}$$

texture zeros

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}}$$

$$\tan\theta_d = \sqrt{\frac{m_d}{m_s}}$$

$$\theta_u \approx 5.0^o \pm 0.7^o$$

$$Exp: 5.4^o \pm 1.1^o$$

$$\theta_d \approx 13.0 \pm 0.4^o$$

$$Exp: 11.7^o \pm 2.6^o$$

Neutrino Masses (eV)

$$m_1 \approx 0.004$$

$$m_2 \approx 0.010$$

$$m_3 \approx 0.050$$

$$\theta_{at} \approx 38^0$$

$$\theta_{13} = 8.7^0 \pm 1.2^0$$

Cuoricino Experiment

EXPECTED:

$$\boxed{m_{eff.} \approx 0.005 eV}$$

Cuoricino Experiment

EXPECTED:

$$m_{eff.} \approx 0.005eV$$

OBSERVED:

$$m_{eff.} < 0.23eV$$